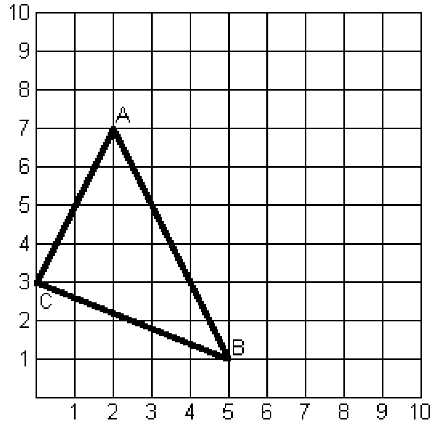
Find the area of ΔABC if A is (2, 7), B is (5, 1) and C is (0, 3)



Method 1: Using rectangle minus three triangles

$\Delta ABC =$ Rectangle $-_{\Delta}$ I $-_{\Delta}$ II $-_{\Delta}$ III

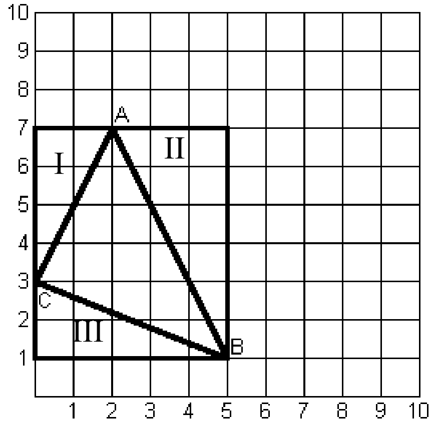
Area Rectangle $= 5 \bullet 6 = 30$

Area $_{\Delta}\text{I} = \frac{1}{2}(2)(4) = 4$

Area $_{\Delta}\text{II} = \frac{1}{2}(3)(6) = 9$

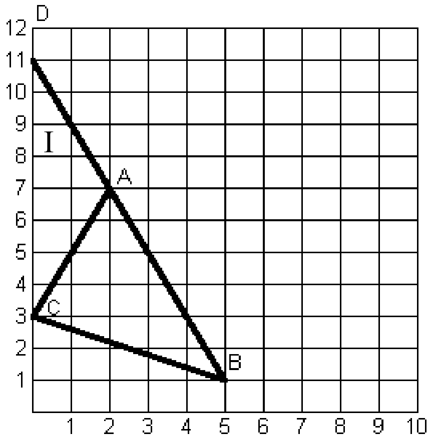
Area $_{\Delta}\text{III} = \frac{1}{2}(2)(5) = 5$

Area $\Delta ABC = 30 - 4 - 9 - 5 = 12$



Method 2: Subtracting two triangles

Extend segment AB so that the y-intercept is D. Find D.



$$\frac{y-7(11-3)}{0-2} = \frac{7-1}{2-5}$$

$$\frac{y-7}{-2} = \frac{6}{-3} = \frac{-2}{1}$$

$$y - 7 = 4$$

$$y = 11$$

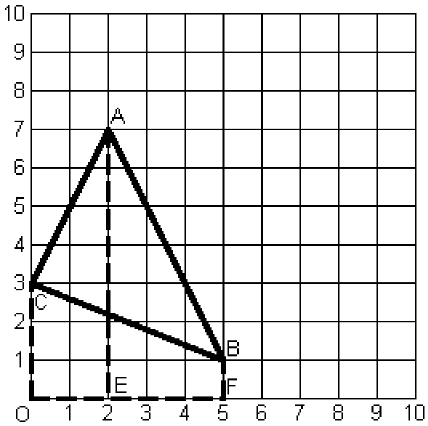
area ΔABC = area ΔDCB $-$ area ΔI

area $\Delta\text{DCB} = \frac{1}{2}\ (11-3)(5) = 20$

area $\Delta\text{I} = \frac{1}{2}(11-3)(2) = 8$

area ΔABC $= 20 - 8 = 12$

Method 3: Two trapezoids minus one trapezoid



$$\text{area } \Delta ABC = \text{area } AEOC + \text{area } ABFE - \text{area } CBFO$$

$$\text{area } AEOC = \frac{1}{2}(2)(3+7) = 10$$

$$\text{area } ABFE = \frac{1}{2}(3)(7+1) = 12$$

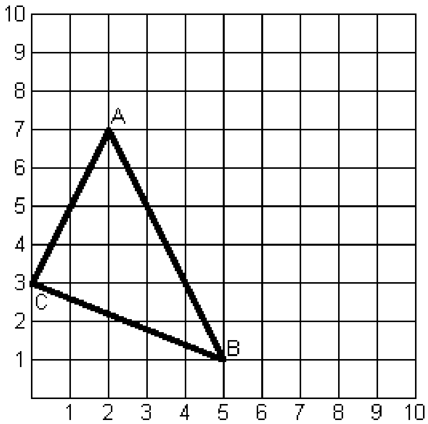
$$\text{area } CBFD = \frac{1}{2}(5)(3+1) = 10$$

$$\text{area } \Delta ABC = 10 + 12 - 10 = 12$$

Method 4: Heron's Formula where s is the semiperimeter and a, b and c are the sides of ΔABC.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$



$$AB = \sqrt{(7-1)^2 + (2-5)^2} = 6.7082$$

$$BC = \sqrt{(5-0)^2 + (1-3)^2} = 5.3852$$

$$AC = \sqrt{(7-3)^2 + (2-0)^2} = 4.4721$$

$$s = 8.2875$$

$$s - a = 1.57455$$

$$s - b = 2.89755$$

$$s - c = 3.81065$$

$$A = \sqrt{143.9995} \approx 12$$

Method 5: Coordinate Method

Heron's Formula is derived from the following formula that is often simpler to use than the formula. The order of the coordinates is not important.

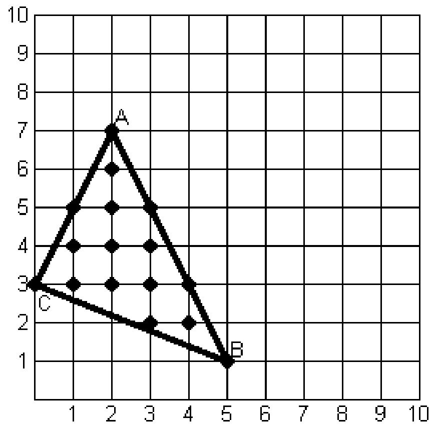
$$A = \left| \frac{1}{2}\left[x_1(y_3 - y_2) - x_2(y_3 - y_1) + x_3(y_2 - y_1)\right] \right|$$

$A = (2,7)$ $\quad B = (5,1)$ $\quad C = (0,3)$

$$\text{Area} = \left| \frac{1}{2}\left[2(3-1) - 5(3-7) + 0(1-7)\right] \right| = \frac{1}{2}(4 + 20 + 0) = 12$$

Have students derive this formula using method 3.

Method 6: Pick's Formula



Area = $\mathrm{I}+\frac{1}{2}B-1$

I = Inside points
B = Border points
I = 10
B = 6
Area = $10+\frac{1}{2}(6)-1=12$

Method 7: Distance from a point to a line.
How far is point C from $\overline{\mathrm{AB}}$? (This would be the height and AB would be the base.) The formula for the distance from (x_1,y_1) to the line $ax + by + c = 0$ is:

$$D=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

AB has equation $2x + y - 11 = 0$

Distance from (0,3) to $2x + y - 11 = 0$ is $\left|\frac{2(0)+3-11}{\sqrt{4+1}}\right|=-\frac{8}{\sqrt{5}}$

AB = $\sqrt{(7-1)^2+(2-5)^2}=\sqrt{45}$

Area = $\frac{1}{2}\cdot\frac{8}{\sqrt{5}}\cdot\sqrt{45}=12$

Using Coordinate Geometry to Find the Area of a Triangle

Divide the class into 7 groups. Number each group. Give coordinates of ΔABC and have each group use a particular method to find the area. Rotate methods so that each group works 4 problems 4 ways.

1. A (–1, 10) B (5, 2) C (9, 5)
2. A (0, 5) B (7, 1) C (–3, –2)
3. A (1, 1) B (1, 0) C (7, –3)
4. A (10, 4) B (–2, 4) C (3, –2)

Discuss which method is the easiest. *The coordinate method is the easiest most often.*

Ask if any of the students noticed that problem 1 is a right triangle with area ½ leg * leg.

Why is it a right triangle? *Slopes of segments AB and BC are negative reciprocals.*