Solution Manual

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1 (Chapter 1 yet to be named)

2 Foundations of Bayesian Analysis

2.2 Solution

We know that $H_0 : N(0,1)$ and $H_1 : N(2,1)$.

(a) $z = \frac{x-0}{1} \Rightarrow x < 1.645$ (**critical region**)
power $= k(\mu) = 1 - \Phi(\frac{1.645-2}{1}) \approx 1 - \Phi(0.355) \approx 1 - \Phi(0.36) \approx 1 - 0.6406 \approx 0.3594$

(b) $\frac{L(x,\theta_1)}{L(x,\theta_0)} \times \frac{P(H_1)}{P(H_0)} \Rightarrow \frac{e^{\frac{-(x-2)^2}{2}}}{e^{\frac{-(x-0)^2}{2}}} \times \frac{0.25}{0.75} > 1 \Rightarrow x > 1.549$ (**reject** H_0)

2.3 Solution

(a) $\frac{L(x,\theta_1)}{L(x,\theta_0)} \times \frac{P(H_1)}{P(H_0)} = \frac{2}{x} \times \frac{0.5}{0.5} = \frac{2}{x}$
$\frac{2}{x} > 1 \Rightarrow x < 2$ (**reject** H_0)
$\frac{2}{x} > 3 \Rightarrow x < \frac{2}{3}$ (**reject** H_0)
$\frac{2}{x} > 10 \Rightarrow x < \frac{2}{10}$ (**reject** H_0)

(b) $\frac{L(x,\theta_1)}{L(x,\theta_0)} \times \frac{P(H_1)}{P(H_0)} = \frac{2}{x} \times \frac{1/3}{2/3} = \frac{1}{x}$
$\frac{1}{x} > 1 \Rightarrow x < 1$ (**reject** H_0)

(c) $\frac{L(x,\theta_1)}{L(x,\theta_0)} \times \frac{P(H_1)}{P(H_0)} = \frac{2}{x} \times \frac{2/3}{1/3} = \frac{4}{x}$
$\frac{4}{x} > 1 \Rightarrow x < 4$ (**reject** H_0)

2.4 Solution

We know that,

$$L(\theta) = \frac{\lambda^{\sum x_i e^{-\lambda n}}}{\prod x_i!}$$

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$$K(\lambda) = \frac{1}{\theta} e^{-\frac{\lambda}{\theta}}$$

Thus,

$$\frac{\lambda^{\sum x_i} e^{-\lambda n - \lambda/\theta}}{\theta \prod x_i!} \propto \lambda^{13} e^{-\lambda n - \lambda/\theta} \propto \lambda^{13} e^{\lambda(n+1/\theta)} \Rightarrow \text{gamma}(14,\ 2/11)$$

Therefore, $\mu = 28/11 = \lambda$.

2.5 Solution

(a)

$$\frac{\lambda^{\sum x_i} e^{-\lambda n}}{\prod x_i!} \frac{\lambda^{\alpha-1} e^{-\lambda/\theta}}{\Gamma(a)\theta^a} \propto \lambda^{\sum x_i} e^{\lambda(n+1/\theta)} \lambda^{\alpha-1} \propto \lambda^{15} e^{-6\lambda} \Rightarrow \text{gamma}(16,\ 1/6)$$

(b) gamma(16, 1/6)
$\theta = \text{mean} = \alpha\theta = 16/6$.

(c)

$$L(\lambda) = \frac{\lambda^{\sum x_i} e^{-\lambda n}}{\prod x_i!}$$

$$lnL(\lambda) = \sum x_i ln(\lambda) - \lambda n - ln(\prod x_i!)$$

$$\frac{\partial}{\partial \lambda}(lnL(\lambda)) = \sum x_i (1/\lambda) - n$$

Therefore, $n = \frac{\sum x_i}{\lambda}$ and $\lambda = \frac{\sum x_i}{n} = 2.8$.

3 Background for Markov Chain Monte Carlo

3.1 Solution

Mean = 0.4292 (Answer may vary)

Variance = 0.1423 (Answer may vary)

R Code:

```
#cumulative density function
F_x = function(x){1-exp(-2*x)}

#inverse of cumulative density function
iF_u = function(u){(-1/2)*log(1-u)}
```

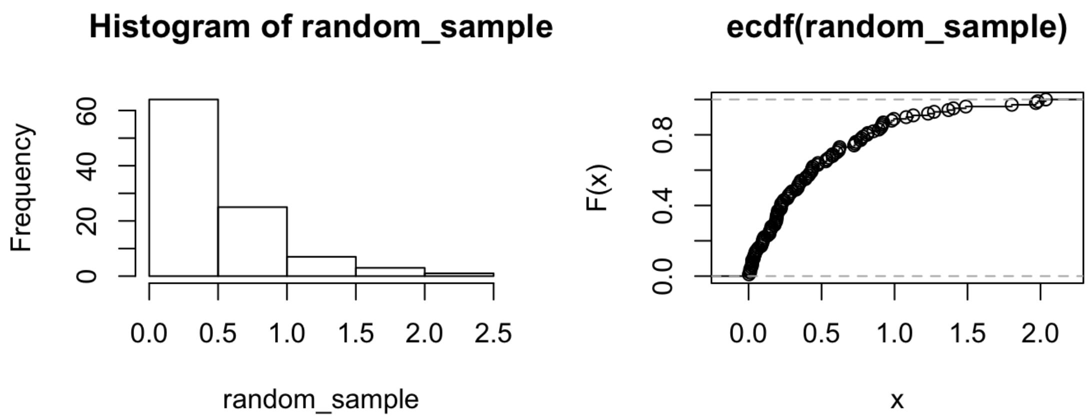

Figure 1: Histogram and ECDF Plot

```
#generate 100 random uniform values
random = runif(100)

#generate transformed values
random_sample = sapply(random, iF_u)

hist(random_sample)
mean(random_sample)
var(random_sample)
plot(ecdf(random_sample), lab = 'F(x)', pch = 1)
```

3.3 Solution

R Code:

```
f_u = function(u){3*u*exp(-3*u)}

random = runif(1000)

f_u_hat = sapply(random, f_u)

mean(f_u_hat)
```

3.9 Solution

We know that,

$$P = \begin{pmatrix} 0.3 & 0.3 & 0.4 \\ 0.2 & 0.5 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}$$

(a)

$$P3 = \begin{pmatrix} 0.174 & 0.372 & 0.454 \\ 0.178 & 0.380 & 0.442 \\ 0.166 & 0.372 & 0.462 \end{pmatrix} P4 = \begin{pmatrix} 0.172 & 0.374 & 0.454 \\ 0.174 & 0.376 & 0.450 \\ 0.170 & 0.374 & 0.455 \end{pmatrix} P5 = \begin{pmatrix} 0.172 & 0.375 & 0.453 \\ 0.172 & 0.375 & 0.452 \\ 0.172 & 0.375 & 0.454 \end{pmatrix}$$

R Code:

```
data = c(0.3, 0.2, 0.1, 0.3, 0.5, 0.3, 0.4, 0.3, 0.6)
P = matrix(data, nrow = 3)
P3 = P %*% P %*% P
P4 = P %*% P %*% P %*% P
P5 = P %*% P %*% P %*% P %*% P
```

(b) To be continued...