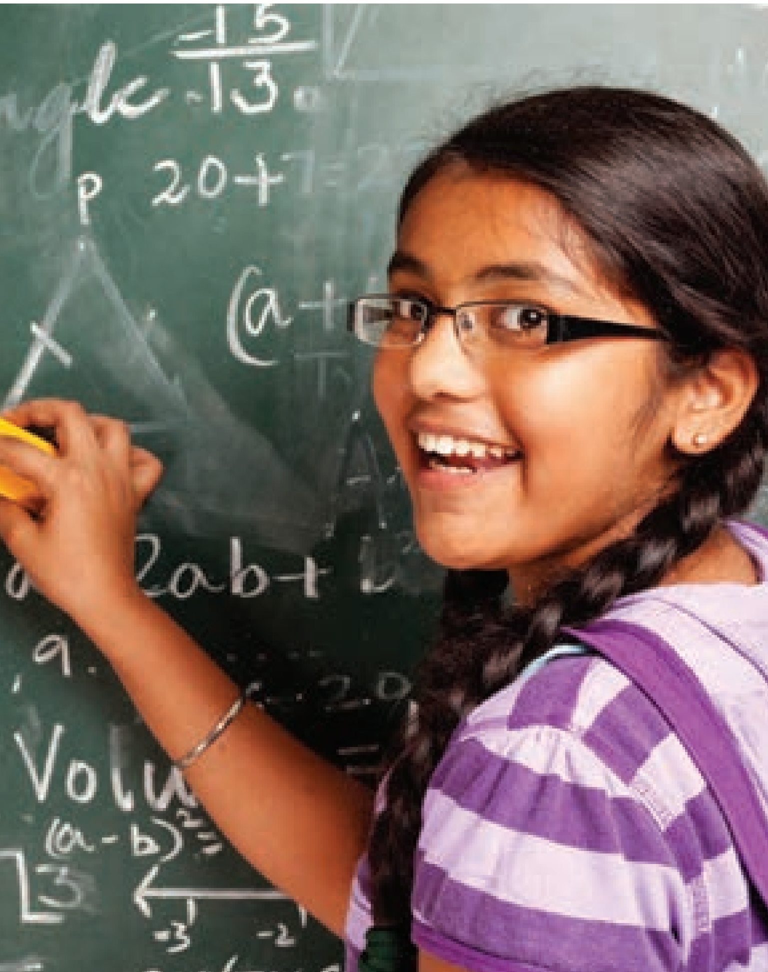
A Union of Professionals

Questions for Math Class

MIDDLE SCHOOL

An AFT Common Core Resource



A Union of Professionals

Our Mission

The **American Federation of Teachers** is a union of professionals that champions fairness; democracy; economic opportunity; and high-quality public education, healthcare and public services for our students, their families and our communities. We are committed to advancing these principles through community engagement, organizing, collective bargaining and political activism, and especially through the work our members do.

Questions for Math Class

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Questions similar to these should be considered as teachers implement the Common Core State Standards for Mathematics.

Many of these questions are useful in helping students develop and use the habits of mind referred to in the Standards for Mathematical Practice. Others are meant to focus students on important ideas in the mathematics. Both kinds of questions are necessary to develop mathematical proficiency.

These lists are not definitive, and teachers should add their own questions. Although these questions appear under specific headings, the thought processes and content knowledge required to answer them are intertwined. The answers elicited should integrate and connect ideas.

Questions and prompts to engage and build mathematical practices

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the … years.

—CCSSM, p. 8

1. Make sense of problems and persevere in solving them.

- Retell the problem in your own words.
- Can you think of similar problems we have solved? How might that help?
- We know ____. What do we not know?
- Where could you start?
- What have you already tried?
- Can you think of another strategy to try?
- Is there a simpler or special case that can help?
- Does this answer make sense? Why or why not?
- What should you do if you're stuck?
- What key variables do we need to control?

(Add your own)

2. Reason abstractly and quantitatively.

- Can you write an equation/expression for what you said?
- What do these numbers represent? What does ____ stand for?

- What is the relationship between ____ and ____?
- What does this quantitative relationship mean?
- Does your solution answer the question? How do you know?
- What is the unit of measure being used?
- How do you know that?
- Can you decontextualize the numbers to find a mathematical relationship?
- Does your abstract representation of the quantities make sense in context?

(Add your own)

3. Construct viable arguments and critique the reasoning of others.

- Convince us that your solution makes sense.
- Did you test whether your method worked? How?
- Does ____'s solution make sense to you? Why or why not? Do you have a question for him or her?
- What assumptions are you making?
- What things that you already know are helping you?
- Were you able to follow ____'s explanation? Can you explain it now? What wasn't clear?
- Can anybody explain it in a different way?
- Do these different solutions express the same or different ideas? Explain any connections.
- How can such different solutions both/all be correct?
- Tell us why you think this solution is wrong.
- Are these statements logical? Connected?
- Is the solution plausible?

- Have you sufficiently supported your answer and shown your work?
- Does anyone have a counterargument?

(Add your own)

4. Model with mathematics.

- Show me what you mean so I can see it.
- How could you visualize or organize your thinking? (For example, by using objects, diagrams/pictures, tables, graphs, equations, words, etc.)
- What are the important quantities/numbers in the problem?
- Can you write an equation to represent this situation?
- What mathematics applies to this situation and these data?
- What simplifications or approximations should you make in order to make a mathematical model of the phenomena/data/experience?
- What are the limits of this (or any) mathematical model?

(Add your own)

5. Use appropriate tools strategically.

- What math tools can you use to solve the problem?
- Which tools will be most efficient and effective?
- Which tool would be most helpful for you? Would a ____ or ____ make more sense?
- Can that tool model this problem?
- Which of the tools takes less time?
- Why did you decide to use ____?
- Is there a better tool available?
- How could estimation help you?

- How are spatial relationships, including space and dimension, used to draw, construct, model and represent real situations or solve problems?

(Add your own)

6. Attend to precision.

- Use precise math language and definitions in your explanation.
- What does the equal sign tell you here?
- Label quantities and axes so we all know what they refer to.
- Relate your answer to the problem's context or story.
- What is the appropriate degree of precision?
- Have you made the data, reasoning and conclusion sufficiently clear (for the audience and purpose)?
- Have you tested the accuracy of your answer? How?
- Will everyone understand the terms you used?

(Add your own)

7. Look for and make use of structure.

- Do you see a pattern as you work? What is it? Can it help you?
- Can you think of a rule or a property that could help us?
- We said some things in mathematics are always true. Can one of those relationships or structures help here?

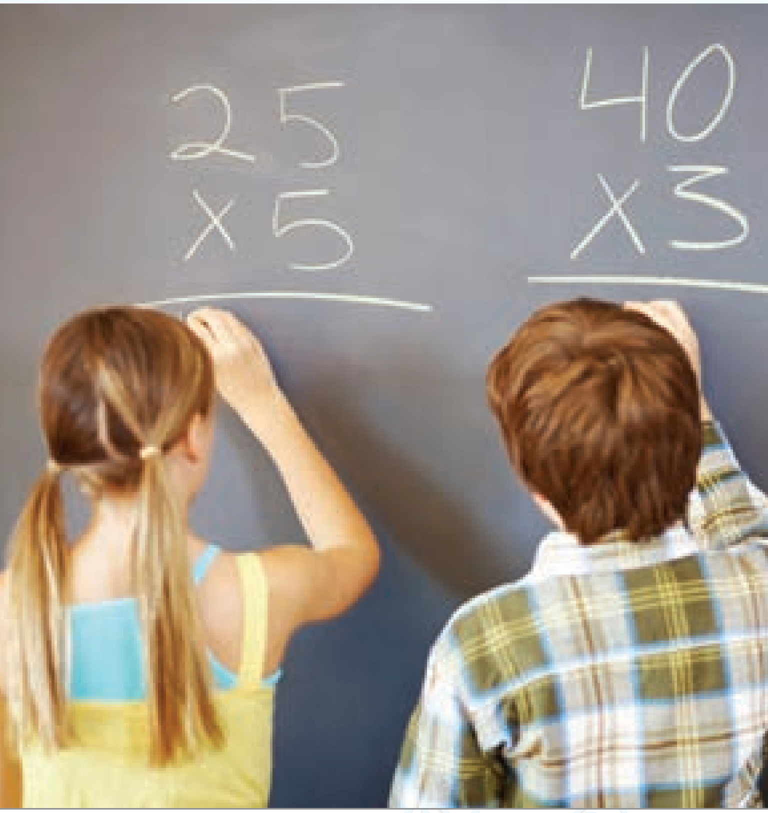
(Add your own)

8. Look for and express regularity in repeated reasoning.

- What pattern do you see in the calculation?
- What does that pattern/repetition tell you?

- Can you predict the next one? The *n*th one? Is it true in all cases?
- Can you make a conjecture about ___ that will always be true?
- Why does that always work?
- What regularities suggest a constant relationship?
- Is there a summary or shorthand way of expressing these recurring patterns?

(Add your own)



Questions and prompts focusing on middle school mathematics content

Division with Fractions

1. What does it mean to divide a number by a fraction (e.g., ¾)?
2. Make up a word problem for an expression (e.g., $4 \div \frac{1}{3}$) and explain what the result means.
3. Give us an example of a situation in which a fraction would be divided by a whole number.
4. What would happen to the quotient if the divisor is increased? Why?
5. What is the difference between increasing a fraction, increasing a numerator, and increasing a denominator? What effect does each have?
6. How do the factors compare to the product when you divide a whole number by a whole number? A whole number by a fraction? A fraction by a whole number? A fraction by a fraction? Why?
7. Can you illustrate that with a drawing or on a number line?
8. Show how this operation with fractions follows the properties of operations.

Expressions and Equations

1. Why are there parentheses here? What would change if they were not here?
2. What makes you think these expressions are or are not equivalent?
3. What tools can you use to show whether they are equivalent?
4. Write an equivalent expression.

5. Can you think of a situation this expression could represent?
6. Explain the difference between $2n$ and n^2.
7. Is there a common factor in each of these terms?
8. How can you prove whether two expressions are equivalent?
9. Can you write an expression or equation that goes beyond this situation/problem?
10. Give a situation and an equation, such as $h=3t+5$. What does the t represent? What does 3 mean in this situation? What is h in this situation?

Ratio and Proportional Relationships

1. What is the relationship between the quantities in the problem?
2. Describe what you know using ratio language.
3. How can you find the unit rate?
4. How many of x would you need to maintain this ratio if y doubles?
5. How can you find the missing quantity in a proportion?
6. How can you find equivalent ratios?
7. How can a multiplication table help you find equivalent ratios? Why does that work?
8. How can you tell whether two ratios are equivalent?
9. Explain what happens when you reduce a price by a certain percent and then raise it again by the same percent.
10. How can you convince us whether or not two ratios are proportional?
11. How are equivalent ratios and equivalent fractions different? Use words and pictures to explain.

12. How do you know by looking at this graph whether or not this is a proportional relationship?
13. What is the unit rate in the graph? Explain in terms of the situation.

Rational Number System

1. Where does this number fall on the number line? How do you know?
2. Explain the meaning of "the opposite of" a number.
3. Think of a situation where two opposites combine to make zero.
4. Explain what kind of numbers belong in each quadrant of the coordinate plane.
5. Is it possible for a larger digit to represent a smaller quantity? Justify why or why not, using an example.
6. How do you determine the location of coordinate pairs on a coordinate plane? Explain. Why?
7. Give an example of an account balance that is less than -$30. Explain what a balance of -$30 means.

Functions

1. Identify the independent and dependent variables. How are they related (in the context of the situation)?
2. What's the same about these functions? Can one situation be represented with multiple functions? Why or why not? How?
3. Explain what the line on this graph shows.
4. What is the shape of a linear expression?
5. What does having the same rate of change mean for both/all these functions?
6. Anne said ____ stayed the same each time. What does that mean for the function?

7. How can you tell whether a function is linear or quadratic?
8. Starting with this ____, represent this function with a ____, ____ and ____. (To develop flexibility, have students begin with different representations: tables, graphs, equations or verbal descriptions. Then have them create the other three from that one.)

Statistics and Probability

1. Describe the statistical reasoning process.
2. What math practice standards are involved in the process?
3. Explain, using at least two comparisons, why there is not a set rate of acceptable variability that can be used from situation to situation.
4. Do all measures of central tendency result in the same quantity? Explain. Give an example of when one measure (e.g., median, mean, interquartile range, etc.) might be better than another measure.
5. What kind of visual would you choose to present your/this data to a specific audience (e.g., parents, congressmen, school board members, students, scientists, funders, etc.)? Why?
6. In what ways can data be expressed so that their accurate meaning is concisely presented to a specific audience?
7. How can mathematics be used to provide models that help us interpret data and make predictions?
8. Explain the difference between an experiment that leads to the probability of an occurrence and one that results in only statistics. Why does one not tell probability?
9. What is random sampling? When and why should it be used?

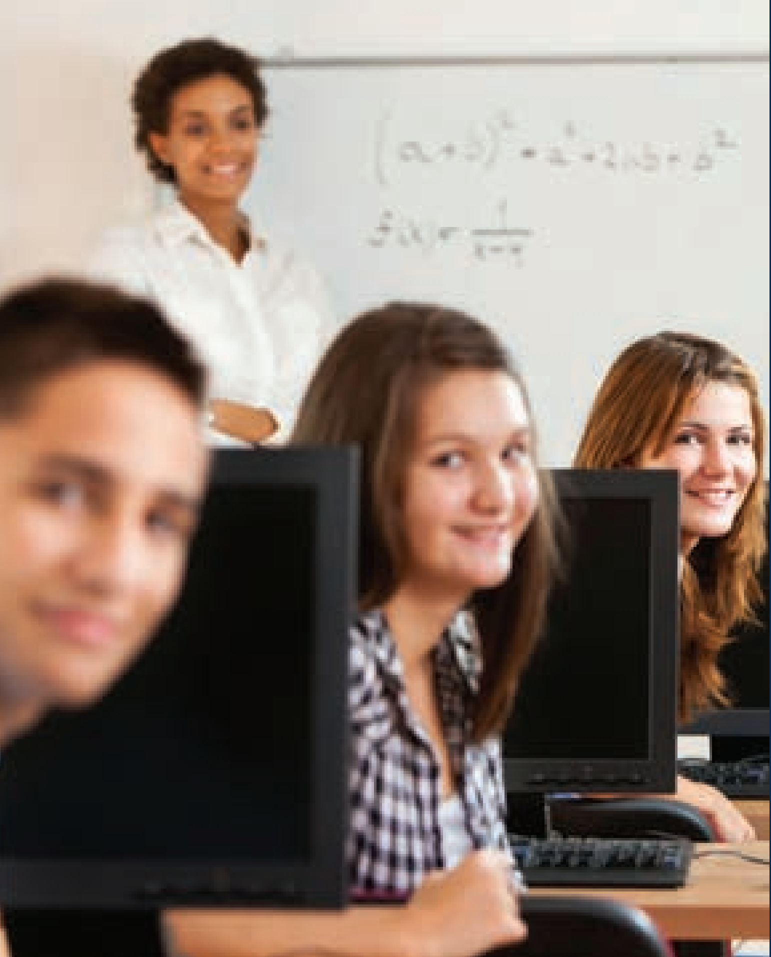
10. Explain possible differences in surveys of random and nonrandom samples.

11. Give an example of bivariate data that you could collect and explain what you might expect to learn from it. Include information about the line of fit, trends and/or associations, if appropriate.

12. What should you consider when interpreting data displayed in a table, graph, etc., or when making such a display?

The questions in this pamphlet are examples of the kinds of questions that invite thinking.

Neither well-written standards, nor tasks with high cognitive demand, nor questions by themselves guarantee that students will engage in high-level discussions or learn rigorous mathematics. Rigor involves weaving together conceptual understanding, procedural skill and fluency, and appropriate application to the world in which students live. What happens between students and teachers as they work with tasks makes all the difference.



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