


The Pythagorean Theorem

Part 1:

- State and illustrate the Pythagorean theorem. Discuss and interpret the meaning of the algebraic equation, $a^2 + b^2 = c^2$. What do a , b and c represent? What do a^2 , b^2 and c^2 represent?
- Create a custom tool for the construction of a right triangle if you have not already done so. Then use your custom tool to construct a right triangle in your sketch window. (**Instructor Note:** A *Sketchpad* file illustrating the construction of a right triangle has been saved as [righttri.gsp](#).)
- Does the Pythagorean theorem hold true for your triangle? It will probably be easier for you to relabel your triangle's sides and vertices to follow the usual conventions (i.e., vertices A , B , C where the sides opposite these vertices are a , b , c ; and c is the hypotenuse). Using the Sketchpad measurement tools and calculator, *numerically* verify the Pythagorean theorem.

To relabel an already labeled figure:

- Select the **Text Tool**  from the toolbox.
- Position the hand pointer on any label.
- Double click and type the new label in the **Label** dialog box.

- Drag a vertex (or side) to further verify the theorem.
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- Measure the area of your right triangle.

To construct the polygon interior of a closed figure:

- Select the vertices of the closed figure, in order.
- Choose from the **Construct** menu, the **Polygon Interior** command.
- Click anywhere inside the sketch window to de-select the polygon interior.

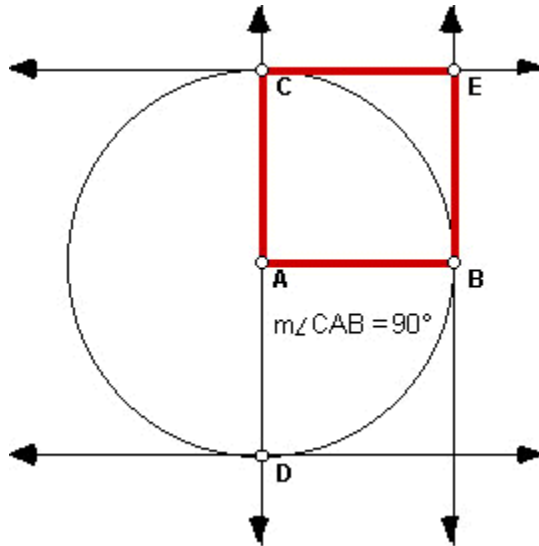
To measure the area of a polygon:

- Select the polygon interior.
- From the **Measure** menu, select the **Area** command.

- Manipulate the triangle around your sketch window by dragging one of its vertices. Observe changes in the lengths of sides and area of the triangle. Use these measurements to verify the formula for the area of a right triangle.

Part 2:

- Discuss how can you think about the Pythagorean theorem in terms of areas of squares.
- To construct squares on each side of your right triangle, it would be helpful to have a custom tool for the construction of a square. In a **New Sketch** window, create a custom tool for the construction of a square. **Hint:** Combine your techniques from constructing an isosceles triangle and a right triangle. (**Instructor Note:** A *Sketchpad* file illustrating the construction of a square has been saved as [square.gsp](#).)



- Manipulate your square. How do each of the vertices move when you drag them around your sketch window? Which vertices are the most restricted and which are the least restricted? As you manipulate your figure, does it always stay a square? How does your construction guarantee that your square will remain a square?
- Use your custom tool to create several squares in your sketch window. Pay close attention to how the custom tool creates a square.
- Now we are ready to construct squares on the sides of a right triangle. In a **New Sketch** window and using your custom tool, construct a right triangle. Further construct squares (that do not overlap your right triangle) on each side of your right triangle. If a square overlaps the triangle, don't worry. You have selected the two vertices in the wrong order. Under the **Edit** menu, select **Undo Square**. Try using your custom tool again,

Exercise 3

keeping in mind that the order of your vertices is important.

- Measure the area of each of the three squares. What do you notice about the areas? Manipulate your triangle. What conclusions can you draw from your observations? (**Instructor Note:** A *Sketchpad* file illustrating the Pythagorean Theorem has been saved as [pythag.gsp](#).)

To construct the polygon interior of a quadrilateral:

- Select the vertices of the quadrilateral, in order.
- Choose from the **Construct** menu, the **Quadrilateral Interior** command.
- Click anywhere inside the sketch window to de-select the quadrilateral interior.

To measure the area of a polygon:

- Select the polygon interior.
- From the **Measure** menu, select the **Area** command.

- Formally state and prove the Pythagorean theorem.

Part 3:

- Under the **File** menu choose **Open...** In the new window, look in the **C:.** Double-click on **Program Files » Sketchpad » Samples » Sketches » Geometry » Pythagoras.gsp.**
- Click on the **Behold Pythagoras!** button in your sketch window. The twelfth century Hindu scholar, Bhaskara demonstrated the Pythagorean theorem with a similar figure to the one found in your sketch window. The only text accompanying the figure was the word “Behold!”. Drag point D and observe what happens to the right triangles in the square on the left. Do the interior figures change shape? How is the hypotenuse of one of the triangles related to the side of the original square? Can you describe the area of this square in terms of the hypotenuse? Verify algebraically that the area of the five figures sum to the area of the larger square. Hint: To start, write an expression for (1) the area of the whole square in terms of c ; (2) the areas of the four right triangles in terms of a and b ; and (3) the area of the interior square. You should be able to write an equation involving a , b , and c .
- Click on the **Contents** button and then the **Leonardo da Pythagoras** button in your sketch window. Leonardo da Vinci (1452-1519) was a great Italian painter, engineer, and inventor during the Renaissance. He was also an amateur mathematician credited with the following proof of the Pythagorean theorem. Before you click on any of the buttons, describe what is displayed in the sketch window. What do each of the regions of the figure represent? Double click on the “reflect top” button. What happened? Did the area of the regions change? Double click on the “turn Δ ’s” button. Again, describe what happened. Further explore this sketch and interpret in your own words Leonardo’s proof of the Pythagorean theorem.
- Click on the **Contents** button and then the **Puzzled Pythagoras** button in your sketch

Exercise 3

window. Follow the directions within the sketch window. Can you explain how this demonstrates that the area of the square on the hypotenuse is equal to the sum of the areas on the squares on the other two legs of the triangle? This dissection demonstration convincingly illustrates the truth of the Pythagorean theorem, but does not provide a formal proof of the theorem. Using the “Puzzled Pythagoras” sketch and an argument of congruent triangles, formally prove the Pythagorean theorem.

- Click on the **Contents** button and then the **Shear Pythagoras!** button in your sketch window. *Shearing* is a transformation which translates every point in a figure in a direction parallel to a given line by a distance proportional to a point’s distance from the line. In your sketch window, drag point P to shear the parallelogram back and forth. Note that the parallelogram’s area doesn’t change as you change the figure’s shape. Finish the shear by dragging P to lie on the red line. Explain why shearing does not affect the area of the parallelogram. Shear the other parallelogram on the other leg of the triangle. Drag until point Q is on the line. Measure the area of each of the three squares, and then follow the directions given. Comment on your observations. Prove that the two shaded figures are congruent.

Extensions:

- State the converse of the Pythagorean theorem. How could you investigate the converse of the Pythagorean theorem? One way is by constructing squares on the sides of an arbitrary triangle. Measure the areas of the three squares and calculate the sum of two of them. Then drag a vertex until the sum equals the area of the third square. What kind of triangle do you have? Try to formally prove the converse of the Pythagorean theorem.
- Construct a right triangle and place a non-overlapping equilateral triangle on each of its sides. Conjecture a relationship between the areas of the equilateral triangles. Measure the areas to assess your conjecture. Formally prove your conjecture. Try placing other figures (e.g., pentagons, hexagons, semicircles, etc.) on the sides of a right triangle. Conjecture a relationship between the areas outside of the right triangle. Then measure the areas to assess your conjecture.