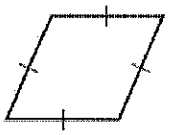
Key

Essential Question

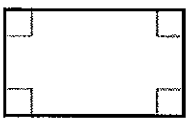
What are the properties of the diagonals of rectangles, rhombuses, and squares?

Core Concept

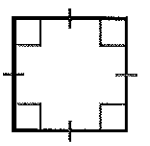
Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.



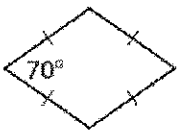
A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

Classify the special quadrilateral.

Explain your reasoning.



The quadrilateral has 4 congruent sides. By the Rhombus Corollary, the quadrilateral is a rhombus. Because one of the angles is not a right angle, the rhombus cannot be a square.

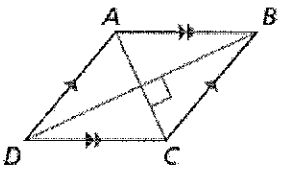
Theorems

Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof p. 390; Ex. 72, p. 395

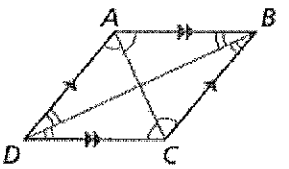


Theorem 7.12 Rhombus Opposite Angles Thoerem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

Proof Exs. 73 and 74, p. 395

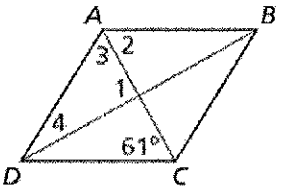


Find the measures of the numbered angles in rhombus *ABCD*.

$m\angle 1 = 90°$

$m\angle 2 = 61°$

$m\angle 3 = 61°$



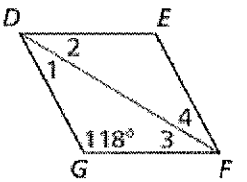
$m\angle 1 + m\angle 3 + m\angle 4 = 180°$

$90° + 61° + m\angle 4 = 180°$

$m\angle 4 = 29°$

4. In Example 3, what is $m\angle ADC$ and $m\angle BCD$?

$m\angle ADC = 58°$ $m\angle BCD = 122°$



5. Find the measures of the numbered angles in rhombus *DEFG*.

$m\angle 1 = 31°$ $m\angle 3 = 31°$

$m\angle 2 = 31°$ $m\angle 4 = 31°$

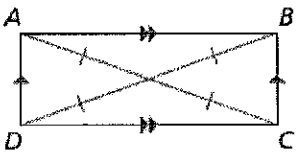
Theorem

Theorem 7.13 Rectangle Diagonals Theorem

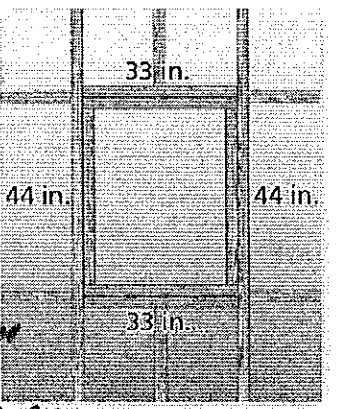
A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof Exs. 87 and 88, p. 396



You are building a frame for a window. The window will be installed in the opening shown in the diagram.



a. The opening must be a rectangle. Given the measurements in the diagram, can you assume that it is? Explain.

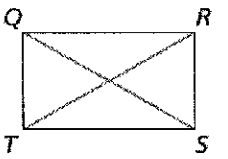
No. The boards are the same length so they form a parallelogram but you do not know if the angles are 90°

b. You measure the diagonals of the opening. The diagonals are 54.8 inches and 55.3 inches. What can you conclude about the shape of the opening?

The diagonals are not congruent so the boards do not form a rectangle.

In rectangle $QRST$, $QS = 5x - 31$ and $RT = 2x + 11$.

Find the lengths of the diagonals of $QRST$.



$5x - 31 = 2x + 11$
$-2x \quad -2x$
$3x - 31 = 11$
$+31 \quad +31$
$3x = 42$
$x = 14$

$QS = 5x - 31$
$= 5(14) - 31$
$= 39$

$RT = 2x + 11$
$= 2(14) + 11$
$= 39$

6. Suppose you measure only the diagonals of the window opening in Example 4 and they have the same measure. Can you conclude that the opening is a rectangle? Explain.

No. The quadrilateral might not be a parallelogram.

7. WHAT IF? In Example 5, $QS = 4x - 15$ and $RT = 3x + 8$. Find the lengths of the diagonals of $QRST$.

$4x - 15 = 3x + 8$
$-3x \quad -3x$
$x - 15 = 8$
$x = 23$

$QS = 4x - 15$
$= 4(23) - 15$
$= 92 - 15$
$= 77$