

Gain-Scheduled Smith PID Controllers for LPV First Order plus Time Varying Delay Systems

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Abstract— Practical control problems often deal with parameter varying uncertain systems that can be described by a first order plus delay (FOPD) model. In this paper, a new approach to design gain scheduled robust linear parameter varying (LPV) PID controllers with pole placement constraints through LMI (Linear Matrix Inequalities) regions is proposed. The controller structure includes a Smith Predictor (SP) to deal with the delays. System parameter variations are measured on-line and used to schedule the controller and the SP. Although the known part of the delay is compensated with the “delay scheduling” SP, the proposed approach allows to consider uncertainty in the delay estimation. This uncertainty is taken into account in the controller design as an unstructured dynamic uncertainty. Finally, two applications are used to assess the proposed methodology: a simulated artificial example and a simulated physical system based on an open canal system used for irrigation purposes. Both applications are represented by FOPD models with large and variable delays as well as parameters which depend on the operating conditions.

Keywords: Linear parameter varying systems, PID controllers, Smith predictor.

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I. INTRODUCTION

ALTHOUGH the control community has developed new and, in many aspects, more powerful control techniques (as f.e. H_∞ robust control) during the last decades, the PID controller is still used in many of the real world control applications. The reason is the simplicity and the facility to tune using heuristic rules [28]. On the other hand, advanced controllers designed with the aid of H_∞ robust control techniques are usually of high order, difficult to implement and virtually impossible to re-tune on line. Furthermore, if implementation issues have been overlooked, they can produce extremely fragile controllers (small perturbations of the coefficients of the controller destabilise the closed-loop control system [15] [53]).

Since the 60's, the empirical (or classical) gain-scheduling (GS) control [33,34,29] has been used for controlling non-linear and time-varying systems. But, this control methodology achieves closed loop stability, without guarantees, for slowly varying parameters [32]. In order to overcome this deficiency, linear parameter-varying gain-scheduling (LPV GS) controllers are introduced to allow arbitrarily smooth or discontinuous variations of plant dynamics [34]. The LPV GS method guarantees closed loop stability based on the concept of quadratic stability (QS) [6,7,26] for all real parameter trajectories inside a given region. This methodology allows multi-objective criteria (H_∞ , H_2 , pole-placement) as well [1,2,32,38]. Under additional hypotheses, such kind of synthesis problems can be transformed to a convex optimisation problem involving linear matrix inequalities (LMI's). This results in a well behaved and computationally tractable problem [17,26]. For analysis, when the LMI conditions depend on the system parameter vector in a multi-affine way, it suffices to verify these conditions only at the vertices of the parameter polytope. In this case, a polytopic controller [24,38] should be implemented.

Usually for time invariant first order plus delay (FOPD) systems, a Smith predictor is an effective method to control the process, because the time delay is fixed [41]. Nevertheless, the Smith predictor has the inherent drawback that its performance is sensitive to the process model uncertainty, especially to the time delay. If a process model deviates from the process dynamics the system performance deteriorates.

Applications of the Smith predictor are, therefore, often limited in industrial processes. To alleviate this limitation, it is necessary to find a mechanism to compensate or take into account modeling errors. Significant research has been done in relation to the robustness issues of the Smith predictor system [23][27] [46] [47] [48] [49] [50] [51]. In [18], alternative, a adaptive Smith predictor is proposed that estimates on-line the time-delay. Other works following this approach are [40][41][42].

The contribution of this paper is to develop a methodology for that allows to design a Linear Parameter Varying PID controller plus Smith Predictor (LPV PID+SP) for first order plus delay (FOPD) linear parameter varying systems. The design methodology exploits the FOPD structure of the system in order to obtain a fixed order controller with PID structure. The varying parameters are measured (estimated) in real time and used to schedule PID parameters as well as the Smith predictor. An additional contribution of the paper is to consider that although the “*delay scheduling*” Smith predictor scheme compensates most of the estimated delay, there is still a remaining delay due to the estimation inaccuracy. The error in the delay estimation is taken into account in the proposed design methodology as unstructured dynamic uncertainty.

In the literature other works has addressed the problem involving LPV PID's [19,24]. But these references do not consider neither the delays and their estimation uncertainty nor assume a FOPD model structure for the system from which the design methodology can benefit. Because of the special structure of the plant model (FOPD), the basic idea in proposed approach is to tune the PID controller reformulating it as a convex state-feedback problem¹ [4,19,39,8]. Performance is quantified in an H_∞ sense and uncertainty as dynamic multiplicative covering the delay measurement error. Next, the PID controller design is formulated as a mixed sensitivity problem (MSP) with measured varying parameters and the addition of closed loop pole placement constraints. This can be solved by standard LPV theory using LMI optimization [20,38], considering LMI regions for pole placement. Due to the fact that the problem is affine in the parameters and that these live in a polytopic region, it can be solved by a finite number of LMI's, one for each vertex of the polytopic region. The controller is implemented as a PID

plus SP which are scheduled using the real time measurement of the operating conditions.

The structure of the paper is the following: In *Section II*, the background on LPV theory and on the design of a PID control problem as a state feedback are presented. The formulation, synthesis and implementation of the PID control of a FOPD LPV system in LPV framework using a “*delay scheduling*” Smith predictor (Smith LPV PID) are presented in *Section III*. In *Section IV*, the proposed methodology is applied to two different processes: a simulated artificial example and a simulated open canal system. In *Section V*, final conclusions are drawn.

II. BACKGROUND

A. LPV General Framework

Definitions and notations

Mainly, there are two types of LPV systems widely used in the control literature: 1) polytopic LPV systems [2] and 2) LFT²/LPV systems [2]. In this paper the polytopical LPV approach is used and is shortly presented in the following:

$$\begin{aligned}\dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) \\ y(t) &= C(\theta)x(t) + D(\theta)u(t) . \\ \theta(t) &\in \Theta, \quad \forall t \geq 0\end{aligned}\tag{1}$$

where u is the plant’s input, x are the plant’s states, y are the plant’s outputs and A , B , C and D are matrices that depend on time varying parameters $\theta(t)$ that belong to a convex polygon Θ of vertices v_1, v_2, \dots, v_r , that is³,

$$\theta(t) \in \Theta := Co\{v_1, v_2, \dots, v_r\} := \left\{ \sum_{i=1}^r \lambda_i v_i : \alpha_i > 0, \sum_{i=1}^r \lambda_i = 1 \right\} .\tag{2}$$

¹ For a general LPV system case, the design of a LPV PID controller should be formulated as an output feedback control that usually derives in solving a non-convex optimisation problem based on BMI’s [24].

²Linear Fractional Transformation

³ Co{.} notation represents the convex hull of the set {.}.

These vertices represent the limit values of the parameters can be represented with an $(r=2^l)$ -th order polytopic way where v_i is the l -th dimension.

LPV models can be determined directly from physical laws or by LPV identification. In the former case the model is obtained by first-principle laws taking into account that the model parameters vary according to the system states, the operating conditions and/or external factors [35, 21, 11]. In the latter case, there are two procedures to carry out the LPV identification:

- 1) Locally: Since a LPV model is essentially a parameterized family of LTI models, a possible identification scheme is to fix each operating point, and collect enough data to identify the LTI model at that point [45]. The identified LTI coefficients can then be used as interpolation points to find the coefficients as parametric functions of the scheduling variables.
- 2) Globally: The identification can be carried out in ‘one shot’ as it is proposed in [43, 44]. In this procedure, it is assumed that the inputs, outputs and scheduling parameters are directly measured, and a form of the functional dependence of system coefficients on the parameters is known. This identification problem can be reduced to a linear regression and provide compact formulae for the corresponding Least Mean Square algorithms. The first method would produce a model similar to the second but it would require identifying a set of LTI models at different operating points.

The synthesis technique for LPV systems is based on the following results: 1) Quadratic H_∞ performance [1] [2] [7] [26]. 2) Robust and Quadratic D–Stability [13]. These two results are detailed in *Theorems 1* and 2, respectively.

Theorem 1 (*Quadratic H_∞ Performance*). *The LPV system given by Eq.(1) is quadratically stable (QS) and has quadratic H_∞ performance if there exists a positive definite matrix $X>0$ such that*

$$B_{[A(\theta),B(\theta),C(\theta),D(\theta)]}^0(X,\gamma) := \begin{bmatrix} A^T(\theta)X + XA(\theta) & XB(\theta) & C^T(\theta) \\ B^T(\theta)X & -\gamma I & D^T(\theta) \\ C(\theta) & D(\theta) & -\gamma I \end{bmatrix} < 0$$

for all admissible values of the parameter θ . See Theorem 3.1 and Definition 3.2 in [2].

Definition 1 (*LMI-Region*) [13]. A subset D of the complex plane is called an LMI-Region if there exists a symmetric matrix $\alpha = [\alpha_{kl}] \in \Re^{m \times m}$ and a matrix $\beta = [\beta_{kl}] \in \Re^{m \times m}$ such that $D = \{z \in \mathbb{C} : f_D(z) < 0\}$, with

$$f_D(z) := \alpha + z\beta + \bar{z}\beta^T = [\alpha_{kl} + \beta_{kl}z + \beta_{kl}\bar{z}]_{1 \leq k, l \leq m}.$$

These regions make up a dense subset in the set of regions of the complex plane, symmetric with respect to the real axis. This makes them appealing for specifying pole placement design objectives.

Theorem 2 (*Quadratic D stability*). Consider the LPV system $\dot{x} = A(\theta)x$ with parameter θ , when θ is a fixed value (“frozen” time). Its pole location in the LMI-Region D at each time t (“frozen” time) can be described by: $M_D = [\alpha_{kl}X + \beta_{kl}A(\theta)X + \beta_{lk}XA(\theta)^T]_{1 \leq k, l \leq m}$, where X is a positive definite matrix, and $M_D[A(\theta), X]$ and $f_D(z)$ can be related by the following substitution, $[X, A(\theta)X, XA(\theta)^T \leftrightarrow (1, z, \bar{z})]$. Then, the matrix $A(\theta)$ is quadratic D stable if and only if there exists a symmetric positive definite matrix X such that $M_D[A(\theta), X] < 0$ for all admissible values of the parameter θ . See Theorem 2.2 in [13].

Based on the fact that a finite set of LMI can be solved in the multi-affine case when the parameters vary in a polytope, a computationally feasible solution to the problem exists, first formulated in [7], as follows.

Theorem 3. (*Vertex Property*). Consider a polytopic linear parameter-varying plant as in Eq. (1), where

$$\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} \in \Theta := \text{Co} \left\{ \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}, i = 1, \dots, r \right\}$$

and assume A, B, C, D are affine functions of θ , and (A_i, B_i, C_i, D_i) are the matrices for each vertex $i = 1, \dots, r$, then the following items are equivalent:

- i. The system is quadratic D -stable with Quadratic H_∞ performance γ .
- ii. There exists a positive definite matrix $X > 0$, which satisfies the following LMI's:

$$\begin{aligned} M_D(A_i, X) &< 0 \\ B_{[A_i, B_i, C_i, D_i]}^0(X, \gamma) &< 0, \quad i = 1, 2, \dots, r \end{aligned}$$

See Theorem 3.3 in [2].

If *Theorem 3* is fulfilled, the verification of *Theorems 1* and *2* only on the vertices of the parameter polytope Θ is sufficient for verifying such a condition for all $\theta \in \Theta$. This implies that the number of inequalities needed to test the analysis conditions of these theorems can be reduced to a finite one, which makes such an approach appealing.

Self-Scheduled H_∞ Control of LPV Systems

Following the terminology of [1] and considering mapping exogenous inputs w and control inputs u to controlled outputs q and measured outputs y , an LPV system can be described by state-space equations of the form:

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) + B_w(\theta)w(t) \\ z(t) &= C_z(\theta)x(t) + D_{zu}(\theta)u(t) + D_{zw}(\theta)w(t) \\ q(t) &= C_q(\theta)x(t) + D_{qu}(\theta)u(t) + D_{qw}(\theta)w(t) \end{aligned} \quad (3)$$

where $x \in \mathfrak{R}^n$ is the state vector with n is state vector order, $u \in \mathfrak{R}^{m_1}$ and $w \in \mathfrak{R}^{m_2}$ are the control and disturbance input vectors with m_1 is input vector order and m_2 is output vector order, respectively, $z \in \mathfrak{R}^{p_1}$ and $q \in \mathfrak{R}^{p_2}$ are the measured and controlled output vectors, respectively. $A(\cdot)$, $B(\cdot)$, $B_w(\cdot)$, $C_z(\cdot)$, $C_q(\cdot)$, $D_{qu}(\cdot)$, $D_{zu}(\cdot)$, $D_{zw}(\cdot)$, $D_{qw}(\cdot)$ are continuous matrix of appropriate dimensions, bounded functions that depend on the l -th order time varying parameter vector $\theta(t) \in \Theta \subset \mathfrak{R}^l$, Θ being a polytope with r vertices. We assume the time varying parameters $\theta(t)$ can be measured (or estimated in the case of quasi-LPV models) in real time as in [2] [7] [26]. Performance is defined as requiring a bounded output $q(t)$ for any bounded external signal $w(t)$, both measured by their energy integral. That is,

$$\begin{pmatrix} A(\theta(t)) & B(\theta(t)) & B_w(\theta(t)) \\ C_z(\theta(t)) & D_{zu}(\theta(t)) & D_{zw}(\theta(t)) \\ C_q(\theta(t)) & D_{qu}(\theta(t)) & D_{qw}(\theta(t)) \end{pmatrix} \in \Theta = Co \left\{ \begin{pmatrix} A_i & B_i & B_{wi} \\ C_{zi} & D_{zui} & D_{zwi} \\ C_{qi} & D_{qui} & D_{qwi} \end{pmatrix}, i = 1, 2, \dots, r \right\}, \quad (4)$$

where A_i, B_i, \dots , denote the values of $A(\theta(t)), B(\theta(t)), \dots$, at the vertices of the parameter polytope (this

formulation is equivalent to Eq. (1)).

We seek a LPV controller of the form

$$\begin{aligned}\dot{x}_k(t) &= A_k(\theta)x_k(t) + B_k(\theta)y(t) \\ u(t) &= C_k(\theta)x_k(t) + D_k(\theta)y(t)\end{aligned}\tag{5}$$

that guarantees the closed loop system is Quadratically Stable and satisfies Quadratic H_∞ Performance [1] [2] [7] [26], and Robust and Quadratic D–Stability [13]. These two results are detailed in previous *Theorems 1* and *2*, respectively.

Remark 1. According to the self-scheduled H_∞ control synthesis problem for LPV systems developed by [1], a control design which guarantees the Quadratic H_∞ performance for the closed-loop system, should fulfil the following necessary and sufficient conditions:

- (i) $D_{qu}(\theta) = 0$ or equivalently $D_{qu_i} = 0$ for $i=1,2,\dots,r$.
- (ii) $B(\theta), C_q(\theta), D_{zu}(\theta), D_{qw}(\theta)$ are parameter-independent or equivalently

$$B_i = B, C_{q_i} = C, D_{zu_i} = D_{zu}, D_{qw_i} = D_{qw} \text{ for } i=1,2,\dots,r.$$

- (iii) The pairs $(A(\theta), B)$ and $(A(\theta), C_q)$ are quadratically stabilizable and detectable over Θ , respectively.

B. PID LPV control as a state feedback control problem

As discussed in the introduction, for a general LPV system case, the design of a LPV PID controller, $K(s, \theta)$, should be formulated as an output feedback control that usually derives in solving a non-convex optimisation problem based on BMI's [24]. However, a convex state feedback problem can be formulated if only if the system to be controlled can be represented by (see [4,19,39,8] for details):

$$G(s, \theta) = \frac{b_0(\theta)}{s^2 + a_1(\theta)s + a_0(\theta)} \quad (6)$$

where $a_0(\theta)$, $a_1(\theta)$ and $b_0(\theta)$ are varying-parameters. According to [15], a PID controller:

$$G_c(s, \theta) = K_p(\theta) + \frac{K_I(\theta)}{s} + K_D(\theta)s$$

is adequate for such a kind of process. The feedback system is transferred from s -domain to time domain and can be expressed in the state space description:

$$\begin{aligned} \dot{x} &= A(\theta)x + B(\theta)u + B_r r, \\ u &= -K(\theta)x + K_P(\theta)r + K_D(\theta)\dot{r}, \\ y &= Cx, \end{aligned} \quad (7)$$

where y is the system output, $x = [x_1 \ x_2 \ x_3]^T$ the state with variables defined by $x_1 = y$, $x_2 = \dot{x}_1$, $x_3 = -\int edt$, $e = r - y$, r the reference input, and

$$\begin{aligned} A(\theta) &= \begin{bmatrix} 0 & 1 & 0 \\ -a_0(\theta) & -a_1(\theta) & 0 \\ 1 & 0 & 0 \end{bmatrix}, & B(\theta) &= \begin{bmatrix} 0 \\ b_0(\theta) \\ 0 \end{bmatrix}, & B_r &= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, & C &= [1 \ 0 \ 0], \\ K(\theta) &= [K_P(\theta) \ K_D(\theta) \ K_I(\theta)]. \end{aligned} \quad (8)$$

In this state-space model, the PID controller design becomes a static state feedback controller, and the static feedback gain $K(\theta)$ simply contains all the PID controller parameters. Note also that there are three varying parameters in Eq.(6) and Eq.(8) [19].

III. MAIN RESULTS

A. Smith LPV PID control problem set-up

Let us consider the following FOPD LPV system

$$G(s, \theta) = G_{dyn}(s, \theta)e^{-s\tau(\theta)} \quad (9)$$

with

$$G_{dyn}(s, \theta) = \frac{b(\theta)}{s + a(\theta)} \quad (10)$$

whose parameters are fixed functions of some vector of varying parameters $\theta(t)$ that can be measured on-line as in the case of general LPV systems presented in *Section II.A*. The parameter range Θ is a box defined by $[b_{\min}, b_{\max}]$ for the gain $b(\theta)$, $[a_{\min}, a_{\max}]$ for pole $a(\theta)$ and $[\tau_{\min}, \tau_{\max}]$ for the time delay $\tau(\theta)$. Our objective is to design a gain-scheduling PID controller using LPV theory [1,2,7,26] for the plant model described by Eq.(9) which is an usual representation of many industrial and environmental processes⁴. By including the parameter measurements, this controller adjusts to the variations in the plant dynamics in order to maintain stability and high performance along all trajectories $\theta(t)$. In other words, the controller is ‘self-scheduled’, that is automatically gain-scheduled with respect to $\theta(t)$. The variable delay in Eq. (9) can be handled in two different ways:

- As an LTI dynamic uncertainty covered conveniently by a weight W_{Δ} as in [27,36,31].
- As a time-varying parameter which updates a Smith Predictor.

The first approach could be conservative, and unnecessarily decrease the overall performance. On the other hand, the second approach could provide a far better performance, but it does not take into account the measurement error of the time-varying delay $\tau(\theta)$. In this paper, it is proposed to combine both approaches by assuming that a real time measurement $\hat{\tau}(\theta)$ of the delay is available, which will be used to update a Smith Predictor (Fig. 1). The difference between the actual and the estimated delay is considered as global dynamic uncertainty as in [27,36,31], and is used in the design and robustness conditions. Therefore we assume that the time delay dynamics has a time varying nature although its measurement error dynamics is time invariant, with a constant bound. The latter can be explained as follows: sensors are usually modelled as time invariant systems, with a bounded error provided by the manufacturer, as we have assumed here. The dependence of the delay with the operating point can be determined by physical modelling [21,11] or identification [31,30] and is measured (estimated) in real time. Proceeding in such a way, most of the delay is compensated and the remaining portion, denoted as

$$\Delta\tau(\theta) = \tau(\theta) - \hat{\tau}(\theta) \quad (11)$$

⁴ Note that the plant of Eq.(5) is approximated by a second order transfer function. We explain in *Section III.B*.

can be covered by LTI unstructured uncertainty. This measurement error is always smaller than the actual delay, therefore the uncertainty is less conservative, which in turn has a lower impact on performance. This uncertainty is handled here as multiplicative output uncertainty and the following weight “covers” the delay measurement error frequency response as tightly as possible (see Chapter 11, [31], [52]):

$$W_{\Delta}(s, \Delta\tau) = \frac{2.05\Delta\tau_{max}s}{\Delta\tau_{max}s + 1} \quad (12)$$

with $\Delta\tau(\theta) \leq \Delta\tau_{max}$.

Although the delay is time varying, by assuming that the delay measurement error is time invariant, the same robust stability analysis of the Smith predictor can be performed, following the approach proposed in [27,36,31] for the LTI case. This is due to the fact that the remaining system, after the cancellation of delay with the use of its estimation, can be considered as finite dimensional LTI, according to this assumption. Therefore, the *delay scheduled* Smith Predictor eliminates the infinite dimensional as well as the time varying nature of the delay, reducing it to a LTI dynamic uncertainty. This is an important contribution of this work as compared to previous approaches [19,24].

B. Statement of the Smith LPV PID controller

The LPV PID controller design of the system described by Eq.(9) will now be formulated as a state feedback problem as described in *Section II.B* and will be embedded in a self-scheduled LPV control problem as developed by [1,7,26], briefly summarised in *Section II.A*.

The control design specifications that will be considered are a mixture of performance and robustness objectives arranged as a MSP [16] (see Fig.2), as follows:

$$\| [W_e S \quad W_u K S \quad W_{\Delta} T]^T \|_{\infty} < \gamma \leq 1. \quad (13)$$

Here S is the sensitivity and T is the complementary sensitivity functions. These transfer functions represent weighted tracking error (or disturbance rejection), weighted control action and robust stability, respectively. In order to limit the control energy and bandwidth of the controller, a weight W_u is included in the design. Such weight is a transfer function with a crossover frequency approximately equal to that

of the desired closed-loop bandwidth. The weight for the complementary sensitivity, W_{Δ} , captures the uncertainty of the plant model (in this case coming from the delay measurement error) and also limits the closed loop bandwidth. Typically, a disturbance in the system output is a low frequency signal, and therefore it will be successfully rejected if the minimum value of S is achieved over the same frequency band. This is performed by selecting a weight W_e , with a bandwidth equal to that of the disturbance in the controller design specifications.

Robustness is presented as an H_{∞} bound and is related with the dynamic uncertainty coming from the real time delay estimation error. Performance is a combination of weighted error and control action minimization measured in terms of the energy integrals of the input and output signals involved. A PID controller is a good approximation of a robust high order controller at low frequencies, especially because of the inclusion of the integral action. Then, the resulting PID controller is expected to preserve the disturbance rejection performance of a high-order controller. Furthermore, the time response is tuned via a selected closed loop pole placement LMI region [12]. This control design problem will be solved using the notion of QS and closed loop pole placement applied to a MSP, considering the delay measurement error as multiplicative dynamic uncertainty (see *Section III.A*). A MSP can always be formulated as a Linear Fractional Transformation (LFT), and solved recasting two previous theoretical results (see *Section II.A*): 1) Quadratic H_{∞} performance [1,2,7,26]. 2) Robust and Quadratic D–Stability [13].

The problem statement is as follows:

Problem 1. *Given the system in Eq. (3), find a gain-scheduling PID controller using an augmented LPV plant, guaranteeing QS and an H_{∞} norm bound less than a positive number γ on the w - z input-output channel $\forall \theta \in \Theta$, and pole placement requirements applied to the MSP in Eq. (13).*

The control design scheme proposed for *Problem 1*, which combines measured (estimated) LPV parameters and unstructured output uncertainties is presented in Fig.2, and represented as a LFT in Fig.3.

In such a LFT representation, the FOPD LPV model in Eqs. (9)-(10) is represented by Eq. (3). To

achieve a PID controller as a state feedback, it is necessary to consider the following issues:

1. The performance and control effort weight functions need to be constants ($W_e = D_e$, $W_u = D_u$), so that the order of the augmented model is the same as the one in *Section II.B*, and a PID controller can be designed.
2. In order to achieve a plant with second order dynamics presented in Eq.(6) with the objective to obtain a fixed order controller with PID structure (see *Section II.B*), the model of Eq. (10) should be connected in series with the following low-pass filter [2]

$$G_f(s) = \frac{K_f}{T_f s + 1}, \quad (14)$$

The latter should not modify the gain and phase in the working frequencies. Furthermore, an additional use of this series connection is to filter high frequency noise.

The modified dynamics of the plant is the following second order transfer function (Eq.(6)), see *Section II.B*:

$$\tilde{G}_{dyn}(s, \theta) = \frac{b(\theta)}{(s + a(\theta))} \frac{K_f}{(T_f s + 1)}. \quad (15)$$

In order not to increase the augmented model's order, the uncertainty weight (Eq.(12)) is modified as follows:

$$\tilde{W}_\Delta(s, \Delta \tau) = 2.05 \Delta \tau_{max} s, \quad (16)$$

so that $W_\Delta = \tilde{W}_\Delta G_f$.

Considering all these assumptions and using a Smith Predictor scheme (Fig. 1) and the uncertainty weight introduced in Eq. (16) bounding the delay measurement error in Eq. (11), the following LFT LPV system representation is obtained:

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) + B_{u_\Delta}(\theta)u_\Delta(t) \\ z(t) &= C_z(\theta)x(t) + D_{zu}(\theta)u(t) + D_{zu_\Delta}(\theta)u_\Delta(t) \\ q(t) &= C_q(\theta)x(t) + D_{qu}(\theta)u(t) + D_{qu_\Delta}(\theta)u_\Delta(t) \end{aligned} \quad (17)$$

with $x = [x_1 \ x_2 \ x_3]^T = [y \ x_\Delta \ x_I]^T$, $\underline{u} = [u_\Delta \ u]^T$, $z = [y_\Delta \ \tilde{u} \ \tilde{e}]^T$.

Here

$$\begin{aligned}
A(\theta) &= \begin{bmatrix} 0 & 1 & 0 \\ -a(\theta)/T_f & -(a(\theta)+1/T_f) & 0 \\ 1 & 0 & 0 \end{bmatrix}, & B(\theta) &= [0 \quad b(\theta)/T_f \quad 0]^T, & B_{u_\Delta}(\theta) &= [0 \quad 0 \quad 0]^T, \\
C_z(\theta) &= \begin{bmatrix} D_\Delta & C_\Delta & 0 \\ 0 & 0 & 0 \\ -D_e & 0 & 0 \end{bmatrix}, & C_q(\theta) &= [-1 \quad 0 \quad 0], & D_{zu}(\theta) &= [0 \quad D_u \quad 0]^T, & D_{zu_\Delta} &= [0 \quad 0 \quad -D_e]^T, \\
D_{qu_\Delta} &= -1, & D_{qu} &= 0.
\end{aligned} \tag{18}$$

C. Implementation of the Smith LPV PID controller

Since the gain, $b(\theta)$ of the system in Eqs. (9)-(10) varies with parameter θ , to fulfil hypothesis (ii) associated to *Remark 1* in *Section II.A*, the time varying gain of the system can be compensated in the following way. First, the LPV gain-scheduling PID controller $K(s, \theta) = K[s, a(\theta)]$ is designed taking into account only the variation of the parameter $a(\theta)$ and assuming that the parameter $b(\theta)$ has a nominal value b_{nom} . Finally, keeping the same inner loop through equation

$$\tilde{K}[s, a(\theta), b(\theta)] = K[s, a(\theta)] \frac{b(\theta)}{b_{nom}} \tag{19}$$

the variation of parameter $b(\theta)$ is considered in the design of the controller.

Due to the fact that the time varying parameters enter affinely in the augmented model equations (see Eqs. (17) and (18)), the parameter region is polytopic and since condition (ii) is fulfilled through the transformation introduced by Eq. (19), the model of the LPV system can be represented by:

$$\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} = \sum_{i=1}^r \lambda_i \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}.$$

The delay $\tau(\theta)$ has already been considered as a scheduled (time varying) parameter in the Smith Predictor implementation, and the delay estimation error bounded by a multiplicative uncertainty in the design process, as explained in *Section III.A*. Next compute a static time varying state feedback

controller, which satisfies QS and the quadratic H_∞ performance specifications. Such a controller can be transformed by the equivalence introduced in *Section II.B*, in a PID controller as in Eq. (7).

This controller schedules the open loop pole parameter $a(\theta)$, and by means of the transformation in Eq. (19), the scheduling of parameter $b(\theta)$ is added, while preserving inner loop dynamics. This controller guarantees QS and Quadratic H_∞ Performance, as well as (“frozen”) closed loop pole location inside the desired LMI region. Since the plant is polytopic, the controller $K(s, \theta) = K(\theta)$ is designed as a polytopic model and implemented according to:

$$K(\theta) \in Co\{K(v_1), K(v_2), \dots, K(v_r)\} := \left\{ \sum_{i=1}^r \lambda_i K_i; \lambda_i \geq 0, \sum_{i=1}^r \lambda_i = 1 \right\} \quad (20)$$

This technique is known as a convex decomposition technique, and Co is the function that generates the convex hull of the polytope vertices (see Eq. (2)). The polytopic coordinates are calculated in such a way that each vertex $v_i, i=1, \dots, r$ has coordinates:

$$\lambda_i = \prod_{j=1}^l \tilde{g}_j^i \quad \text{with} \quad \tilde{g}_j^i = \begin{cases} g_j^i & \text{if } \underline{g}_j^i \text{ is a coordinate of } v_i \\ 1 - g_j^i & \text{if } \bar{g}_j^i \text{ is a coordinate of } v_i \end{cases} \quad \text{where } \tilde{g}_j^i = \frac{(\bar{\theta}_j^i - \theta_j^i)}{(\bar{\theta}_j^i - \underline{\theta}_j^i)}, j=1, \dots, l, \quad (21)$$

and $(\underline{\theta}_j^i, \bar{\theta}_j^i)$ represent the upper and lower bounds of θ_j^i , and l fulfils that $r=2^l$.

Finally, the closed-loop system is $\dot{x}_{cl} = A_{cl}(\theta)x_{cl} + B_{cl}w$, with matrices $A_{cl}(\theta)$ and $B_{cl}(\theta)$ that depend on the parameter vector θ described as follows (for more details see [2]):

$$A_{cl} = \left\{ \sum_{i=1}^r \lambda_i (A_i + B K_i C_z) \right\}, B_{cl} = \left\{ \sum_{i=1}^r \lambda_i (B_w + B K_i D_{zw}) \right\} \quad (22)$$

D. Summary of Smith LPV PID controller design

The steps to obtain the proposed LPV PID controller can be summarised as follows:

1. Obtain a FOPD structure for a non-linear system as a LPV model on a l^{th} -dimensional scheduling parameter vector θ which should be measured in real time. This first step can be carried out by LPV modelling [35] or identification [25,5]. Consider the gain, pole and delay as time varying parameters which depend on $\theta(t)$.
2. Estimate or measure the time varying delay and implement it as part of a Smith Predictor control structure.
3. Quantify the measurement/estimation error for the delay, given by Eq. (12). Obtain its model as an unstructured LTI uncertainty with the weight function W_{Δ} (see *Section III.A*)
4. Specify the weights W_e (performance weight), W_u (control effort weight) as constant values according to the specifications of the problem.
5. Build the augmented system (17)-(18), considering the issues explained in *Section III.A*.
6. Impose the Quadratic H_{∞} performance and choose the desired closed-loop poles region to guarantee closed-loop damping and avoid the fast dynamics imposed by the LPV design. In order to achieve both purposes, (X, γ) are computed solving the LMIs in *Theorem 1* and *2* only at (all) the vertices of the parameter polytope, according to *Theorem 3*.
7. Solve the LMI's obtained after *Step 6* and obtain the vertex controllers $K_i(\theta) = (K_{P_i}(\theta), K_{D_i}(\theta), K_{I_i}(\theta))$ taking into account that the PID is formulated as a state feedback control (see *Section II.B*)
8. Modify the controller to incorporate the scheduling of parameter $b(\theta)$ (Eq.(19)).
9. Implement the LPV PID controller $K(\theta) = (K_P(\theta), K_D(\theta), K_I(\theta))$ as an 'interpolation' of the vertex controllers $K_i(\theta) = (K_{P_i}(\theta), K_{D_i}(\theta), K_{I_i}(\theta))$ calculated (off-line) in *Step 7*.

The gain-scheduled controller $K(s, \theta)$ is updated on-line in real-time based on the measurement of parameter $\theta(t)$ and its decomposition (λ_i) given by Eqs. (20)-(21), enforcing the expected quadratic performance over the entire parameter polytope and along arbitrary (and arbitrarily fast) parameter trajectories.

IV. APPLICATIONS

Finally, in order to show the effectiveness of the proposed approach, two applications based on an artificial simulation example and a simulated physical systems based on an open canal system will be used, respectively.

A. Artificial Simulation Example

First, in order to show the effectiveness of the proposed approach, an artificial simulation example is used. The proposed LPV system dynamics is described by the following FOPD LPV model:

$$G(s, \theta) = \frac{Y(s)}{U(s)} = \frac{k(\theta_1)}{T(\theta_1)s + 1} e^{-\tau(\theta_2)s} \quad (23)$$

The time varying components of this model are: the steady-state gain $k(\theta_1) \in [0.57, 1.26]$, the time constant $T(\theta_1) \in [1.13, 63.25]$ (s), and the time delay $\tau(\theta_2) \in [50, 100]$ (s). The operating range of the control signal is $u \in [-10, 10]$ and the varying parameter vector is $\theta = [\theta_1, \theta_2]$ where θ_1 and θ_2 are both scheduling variables. It is assumed that they are external system variables.

Variable large delays should be carefully taken into account so that instabilities in the closed loop system are avoided. The “*delay scheduling*” Smith predictor presented in Fig. 1 is used to compensate this effect. The delay used by such a predictor is estimated by LPV identification (the first case explained shortly in the *Section III.3*) by a second order polynomial that depends on the parameter θ_2 according to: $\hat{\tau}(\theta_2) = a\theta_2^2 + b\theta_2 + c$ with $a = 30$, $b = 20$ and $c = 50$ [5]. The estimation error $\Delta\hat{\tau}(\theta_2) \in [0.1 \ 1]$ (s) is taken into account in the control design as an LTI unstructured multiplicative uncertainty

$W_\Delta(s, \Delta\hat{\tau}) = \frac{2.05s}{0.25s + 1}$ (Fig. 4). This weight is decomposed as a modified weight $\tilde{W}_\Delta(s) = 2.05s$ and a

filter $G_f = \frac{1}{0.25s + 1}$, according to *Section III*, in order to the PID controller could be designed as a state-

feedback control. This modified weight does not modify significantly the system bandwidth, so that it does not degrade the performance of the control system.

Once the main time varying delay has been compensated by the Smith Predictor (Fig.1) and the remaining delay error considered as the weight $W_{\Delta}(s, \Delta \hat{\tau})$ of a multiplicative dynamic uncertainty, a PID controller is designed as a state feedback. This controller should guarantee closed loop stability and the following (step response) performance specifications:

- tracking error of 0.1
- control signal within ± 10
- closed-loop damping of $\xi \geq 0.5$ and settling time in $50 \leq t_{ss} \leq 700$ (s),

for any arbitrarily fast parameter variation. The tracking error and the bounded control signal are represented by performance weights $W_e = 10$ and $W_u = 0.1$, respectively. Furthermore, to achieve this desired transient behaviour and prevent controller fast dynamics, a pole clustering constraint is added. To this end, the LMI region $S(h_1, h_2, \alpha)$ represented in Fig. 5 is required, that is a combination of three subregions:

1. A conic sector with apex at $x = 0$ and angle $\alpha = 3\pi/4$, which captures the closed-loop damping constraint $\xi \geq 0.5$.
2. Left half plane that guarantees the maximum settling time ($h_1 = -0.05$).
3. Left half plane that guarantees the minimum settling time ($h_2 = -2$).

The two previous regions are associated with the real value of the dominant poles.

To illustrate the advantages of the robust LPV PID controller with time varying Smith Predictor (LPV PID+SP), it is compared to two different controllers:

- A LTI H_{∞} PID controller with a standard Smith Predictor (LTI PID+SP) designed for the *worst case* set of parameters, i.e. k_{\max} , T_{\min} and τ_{\max} .
- A LTI H_{∞} PID controller designed for the *worst case* set of parameters, k_{\max} and T_{\min} , and a delay scheduled Smith Predictor, with no measurement error in the time varying delay (LTI PID+TV SP).

All controller parameters are shown in Table 1.

First, the closed loop responses of the (k_{\max}, T_{\min}) vertex ($\Delta\tau = 1\text{s}$) using the three previous PID designs are shown in Fig. 6, where it can be observed that when the measurement error is not considered (LTI PID+TV SP) the control response is unstable. On the other hand, using the LPV PID+SP and LTI PID+SP designs, the control response is stable although in the latter case, the control objectives are not achieved.

Next, in order to validate the controller design since the case LTI PID+TV SP is unstable, closed loop step responses for only the LPV PID+SP and the LTI PID+SP controllers will be considered. In Figs. 7 and 8, these responses for all parameter space vertices are presented. These results show that the desired time specifications are met in the admissible operating range, in the case of the LPV controller. But in some cases, e.g. settling time and overshoot, the LTI control does not fulfil them.

Finally, the LPV PID+SP and LTI PID+SP designs are tested when the parameters in Eq.(9) follows the trajectory in Fig. 9:

$$k(t) = 0.87 + 0.33 e^{-\omega_2 t} \cos(\omega_1 t), T(t) = 47 + 13e^{-\omega_0 t} \cos(\omega_1 t), \tau(t) = 0.48 + 0.505e^{-\omega_2 t} (\sin(\omega_1 t)),$$

with $\omega_0=0.2$, $\omega_1=0.3$ and $\omega_2=0.04$ (rad/s). The parameters of this trajectory are sampled every 0.1 s.

From Fig. 10, it can be observed that the step response when using the LPV controller when parameters follow this trajectory is stable and the specifications are fulfilled no matter how smooth or abrupt are these variations inside the large operation range. On the other hand, in spite the LTI controller provides a stable closed loop, it does not fulfil the desired overshoot requirement. However, initially, the flow control signal used by the LPV control is larger than that of the LTI, although it remains inside the operation range. Nevertheless for the LPV case, the step response has a smaller overshoot than in the LTI case. The overall performance of the LPV controller behaves as predicted by the theory meeting tight specifications with little apparent conservatism. In addition, it guarantees closed loop stability and performance for any arbitrary parameter variation trajectory, which cannot be assured in the LTI controller case.

B. Open Flow Canal System

The second application example is an open flow canal⁵ composed by a single pool equipped with an upstream sluice gate and a downstream spillway (Fig. 11). A servomotor is used to drive the control gate position and there are two level sensors located upstream (y_{ups}) and downstream of the gate. Upstream of this gate there is a dam of constant level $H=3.5$ m. The total length of the pool is $L = 2$ km, with an initial flow $Q_0=1$ m³/s, a gate discharge coefficient $C_{dg} = 0.6$, a Manning roughness coefficient $n=0.014$, a gate and canal widths of $b=2.5$ m and $B=2.5$ m, respectively. The downstream spillway has height $y_s = 0.7$ m, the spillway coefficient is $C_{ds} = 2.66$, and the bottom slope has $I_0= 5 \cdot 10^{-4}$.

Open-flow canals involve mass energy transport phenomena which behave as intrinsically distributed parameter systems. Their complete dynamics are represented by non-linear partial differential hyperbolic equations (PDE) that are function of time as well as of spatial coordinates: the Saint-Venant's equations. This equation system has no known analytical solution in real geometry and has to be solved numerically (characteristic method, Preissmann implicit scheme, etc.). The resulting simulation models are therefore suitable for scientific and time-consuming simulations but are too complex for on-line applications and control needs. Distributed parameter systems, considered as systems with a very large number of states can be approximated with a low order linear time invariant (LTI) model, in order to use classical linear control design tools, as it is usual practice in control engineering. However, simplified LTI parameter models lose important information about the spatial structure of the original system, although they can be satisfactory from an input-output point of view. On the other hand, the following LPV model with a first order differential equation with time delay is suitable to describe the canal dynamics at different operating points depending on the downstream level according to [11] [39]:

$$y_{dns}(t) + T(y_{dns}) \frac{dy_{dns}(t)}{dt} = K(y_{dns})u(t - \tau(y_{dns})) \quad (24)$$

⁵The results presented in this paper are based on a canal simulator developed by the "Modelling and Control of Hydraulic Systems" group at the UPC [10]. It solves numerically Saint-Venant's equations [13], which accurately describe the dynamics of this test-bench canal using the conservation of mass and momentum principles in a one-dimensional free surface flow. This provides a

Here $y_{dns} = y_0 + \Delta y_{dns}$ and $u = u_0 + \Delta u$ are, respectively, the absolute values of downstream level and the upstream gate position. Variables y_0 and u_0 are the initial values of these measurements, and Δy and Δu are the increments around these initial values.

The LPV model proposed has a FOPD structure:

$$G(s, \theta) = \frac{Y_{dns}(s)}{U(s)} = \frac{k(y_{dns})}{T(y_{dns})s + 1} e^{-\tau(y_{dns})s}$$

where the parameters are obtained of the following way (for a more detailed explanation, see [11] [39]):

Steady-state gain $k(y_{dns})$

The steady-state gain of model Eq. (24) is a static parameter that can be derived considering that in stationary regime, the mass conservation law states: $q_{ups} = q_{dns}$. Using the experimental Manning expression for this canal upstream, the flow downstream can be related with its level as follows

$$q_{ups}(t) = A^{5/3} \frac{\sqrt{I_0}}{n} \frac{1}{P^{2/3}} = (by_{ups})^{5/3} \frac{\sqrt{I_0}}{n} \frac{1}{(b + 2y_{ups})^{2/3}} \quad (25)$$

where the cross area of the rectangular canal is $A = b y_{ups}$ and the wet perimeter $P = (b + 2y_{ups})$.

Using the spillway equation (located at downstream end and according to the geometry of the canal described previously) the downstream flow is

$$q_{dns}(t) = C_{ds} B (y_{dns} - y_s)^{3/2} \quad (26)$$

Therefore, from expressions in Eqs. (25) and (26), the downstream level is

$$y_{dns} = (by_{ups})^{10/9} \frac{\sqrt{I_0}}{nC_{ds}B} \frac{1}{(b + 2y_{ups})^{4/9}} + y_s \quad (27)$$

that also depends on $y_{ups}(u)$. The experimental relation between the downstream level and the upstream level is $y_{ups} = e + fy_{dns}$, with $(e = -1.4720, f = 2)$.

The theoretical steady-state gain can be measured experimentally as

very good simulator model that is considered as the “real” system is this work.

$$k(u) = \frac{dy_{dns}}{du} \quad (28)$$

obtaining in this particular case the following expression:

$$k(y_{dns}) = \frac{7.11 (-0.174 + 0.47 (e + fy_{dns}))}{(e + fy_{dns})^{8/9} (2.5 + 2(e + fy_{dns}))^{13/9}} \quad (29)$$

Delay $\tau(y_{dns})$ and time constant $T(y_{dns})$

The delay associated to model Eq. (24) can also be derived by physical laws and considerations. The propagation velocity of a flow wave in a canal is equal to the sum of the water velocity and the celerity of the gravity wave. For this reason, the delay can be estimated as

$$\tau = \frac{L}{v + c} \quad (30)$$

Then, using

$$c = \sqrt{g\bar{y}} = \sqrt{\frac{y_{ups} + y_{dns}}{2}} \quad (31)$$

with a rectangular section canal and

$$v = \frac{v_{ups} + v_{dns}}{2} \quad (32)$$

assuming that the upstream level (y_{ups}) and downstream level (y_{dns}) are measured, the delay can also be estimated in the following way:

$$\tau(y_{dns}) = \frac{f_1(y_{dns})}{1 + f_2(y_{dns})} \quad (33)$$

$$\text{where: } f_1(y_{dns}) = \frac{2L}{\left(\frac{C_{ds}(y_{dns} - y_s)^{3/2}}{y_{dns}} + 2c \right)}, \quad f_2(y_{dns}) = \frac{\frac{(b(e + fy_{dns}))^{2/3} \sqrt{I_0}}{n((b + 2(e + fy_{dns}))^{2/3})}}{\frac{C_{ds}(y_{dns} - y_s)^{3/2}}{y_{dns}} + 2c} \quad (34)$$

using Eq.(27) and the experimental relation between y_{ups} and y_{dns}

The time constant is obtained experimentally and it is a multiple of delay

$$T(y_{dns}) = \kappa \tau(y_{dns}) \quad \kappa \in [1.32, 1.58]. \quad (35)$$

This system presents a variable large delay because the time the wave takes to arrive at the downstream end after a gate opening depends on the canal downstream flow. In fact, this delay can be computed by Eqs. (30)-(34), by using the measured output canal level. Hence, the “*delay scheduled*” Smith predictor presented in *Section III.A* is used to compensate this effect. In this case, the error in the delay estimation is $\Delta \hat{\tau}(y_{dns}) \in [0.37 \ 10]$ (s), which is modelled by an LTI unstructured multiplicative uncertainty using

$$\text{the function } W_{\Delta}(s, \Delta \hat{\tau}) = \frac{375s}{25s + 1}.$$

Following the proposed approach, an LPV PID controller is designed guaranteeing closed loop stability and the following (step response) performance specifications:

- tracking error of 0.1,
- different operating points corresponding to several gate opening variation in the range [0, 0.4] (m) that deliver flows in the range [1.13, 7.21] (m³/s),
- closed-loop damping of $\xi \geq 0.4$ and settling time $3.5 \leq t_{ss} \leq 17$ (min),

for any arbitrarily fast parameter variation. The tracking error and the bounded control signal are represented by performance weights $W_e = 10$ and $W_u = 2.5$, respectively. Furthermore, to achieve this desired transient behaviour and prevent controller fast dynamics, a pole clustering constraint is added. To this end, the LMI region $S(h_1, h_2, \alpha)$ represented in Fig. 5 is required, that is a combination of three subregions as in the previous example, except that now we have different numerical values, i.e. $\xi \geq 0.4$, $h_1 = -0.004$ and $h_2 = -0.02$.

Again as in the case of the artificial simulation example, a robust LTI H_∞ PID controller with a standard LTI Smith Predictor is designed for the worst case set of parameters (k_{\max} , T_{\min} and τ_{\max}) to compare it with the performance of its LPV version. The parameters of both controllers are presented in Table 2.

The results of simulation using the LPV controller in an arbitrary scenario (see Fig. 12) through the trajectory of the canal model parameters of Fig. 13 verifies all the above desired time specifications inside the admissible control operating range. On the other hand, using the robust LTI controller, the time response is slower than the one obtained by its LPV counterpart. In addition, when the reference signal suffers a large and abrupt change from 1.1125 to 1.4125 the time response does not fulfil the desired settling time.

V. CONCLUSIONS

The main contribution of this paper is the development of a new approach to design a gain-scheduled Smith PID controller for FOPD LPV systems with time varying delays solving a MSP problem with closed loop pole placement constraints. The time varying delay is handled by a “delay-scheduling” Smith predictor and the estimated delay error is treated as an unstructured dynamic uncertainty. Thanks to the FOPD system structure, the PID controller design can be viewed as an state-feedback controller whose design can be transformed to a convex optimisation problem involving LMI’s. This approach has successfully been applied to an artificial simulation example and a simulated open flow canal system. Both are LPV systems which suffer from large varying changes in the time delay as well as in the dynamics. The obtained results have shown that the stability and performance are guaranteed while this is not the case when a robust LTI PID controller is used.

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FIGURES

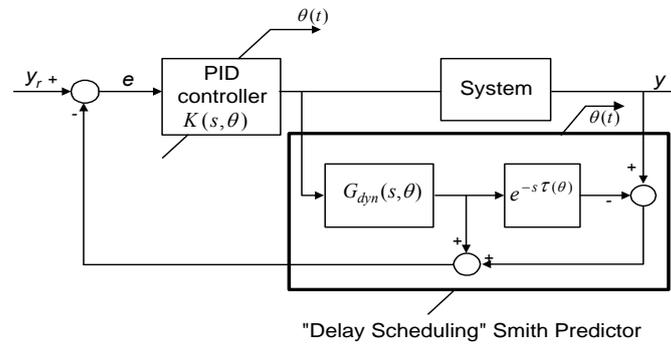


Fig.1. "Delay scheduled" Smith Predictor scheme.

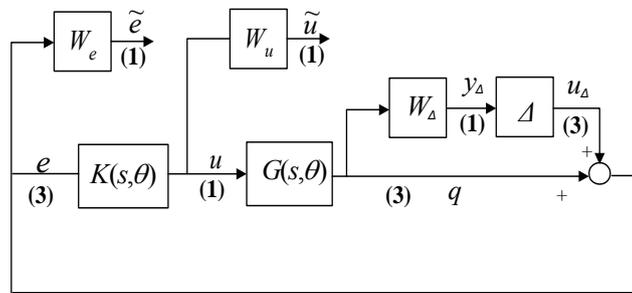


Fig.2. Proposed LPV feedback system scheme (MSP scheme).

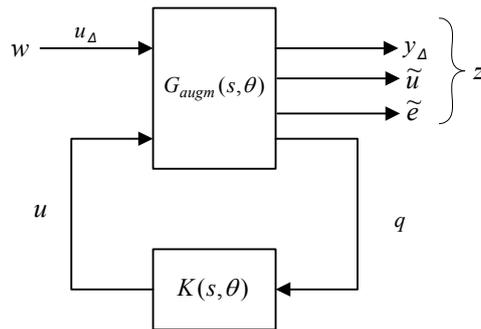


Fig.3. Interconnection of the LPV generalised plant and the LPV controller (LFT representation).

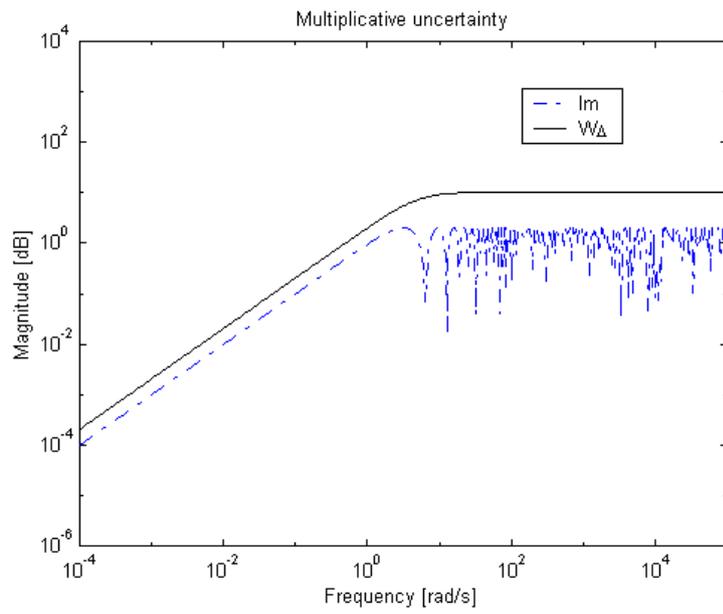


Fig. 4. Frequency response of the uncertainty of the tank delay estimation error and its upper bound.

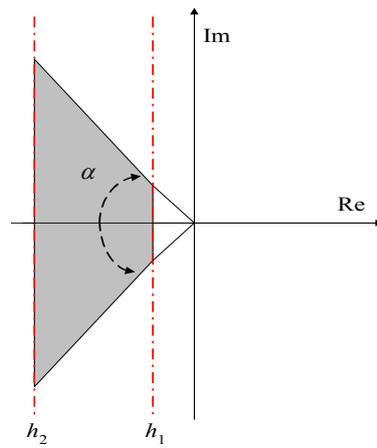


Fig. 5. LMI design region.

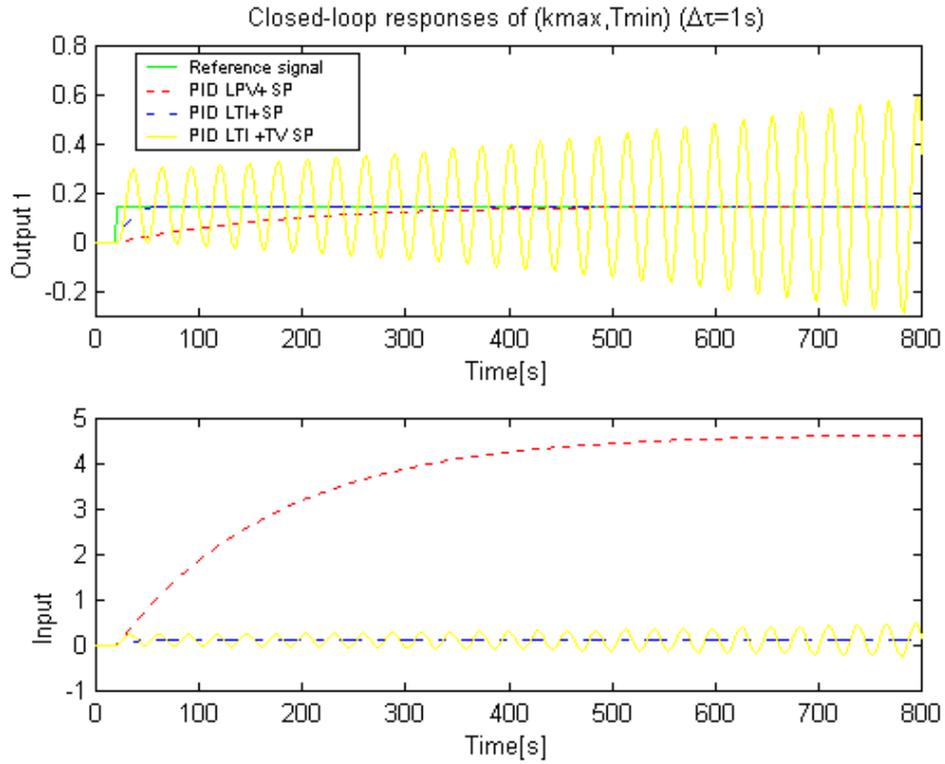


Fig. 6. Closed-loop responses using LPV PID+SP, LTI PID+SP and LTI PID+TV SP for the (k_{max}, T_{min}) ($\Delta\tau = 1s$) vertex of the artificial simulated example.

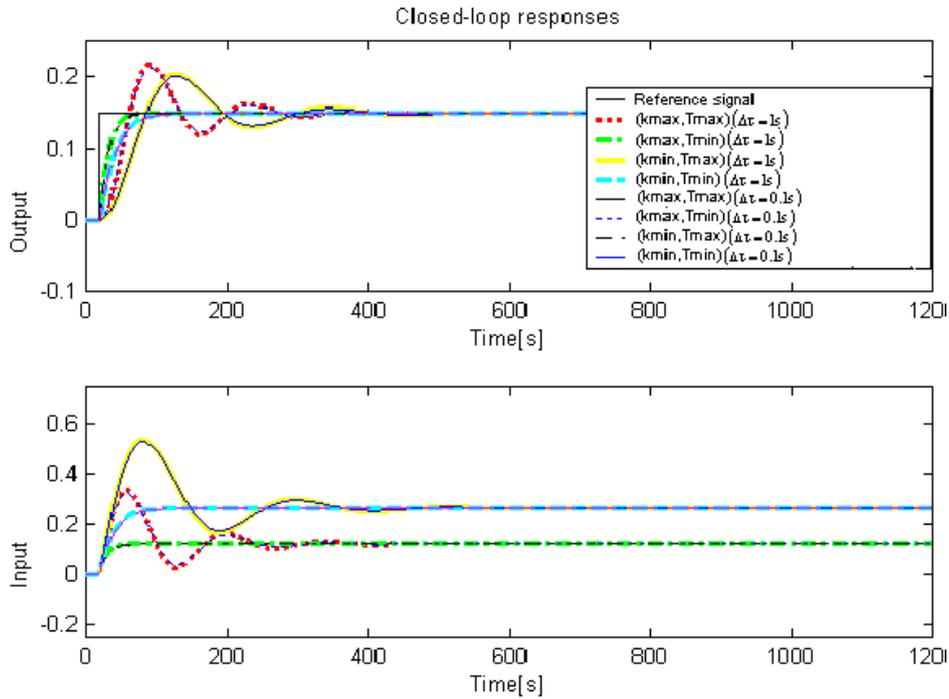


Fig. 7. Closed-loop responses (LPV PID+SP) for the vertices of the artificial simulated example.

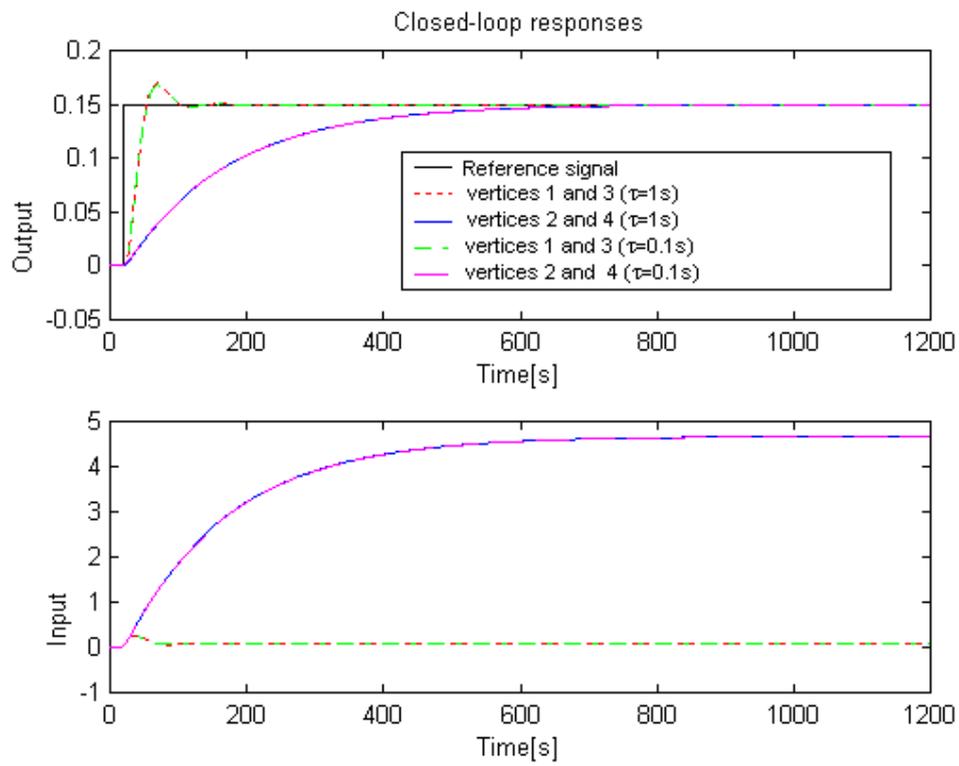


Fig. 8. Closed-loop responses (LTI PID+SP) for the vertices of the artificial simulated example.

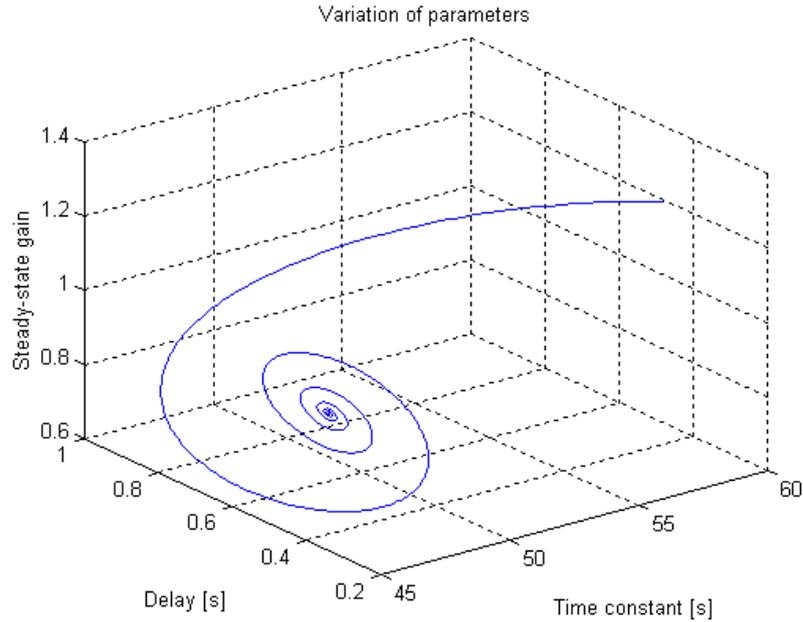


Fig. 9. Parameter trajectory.

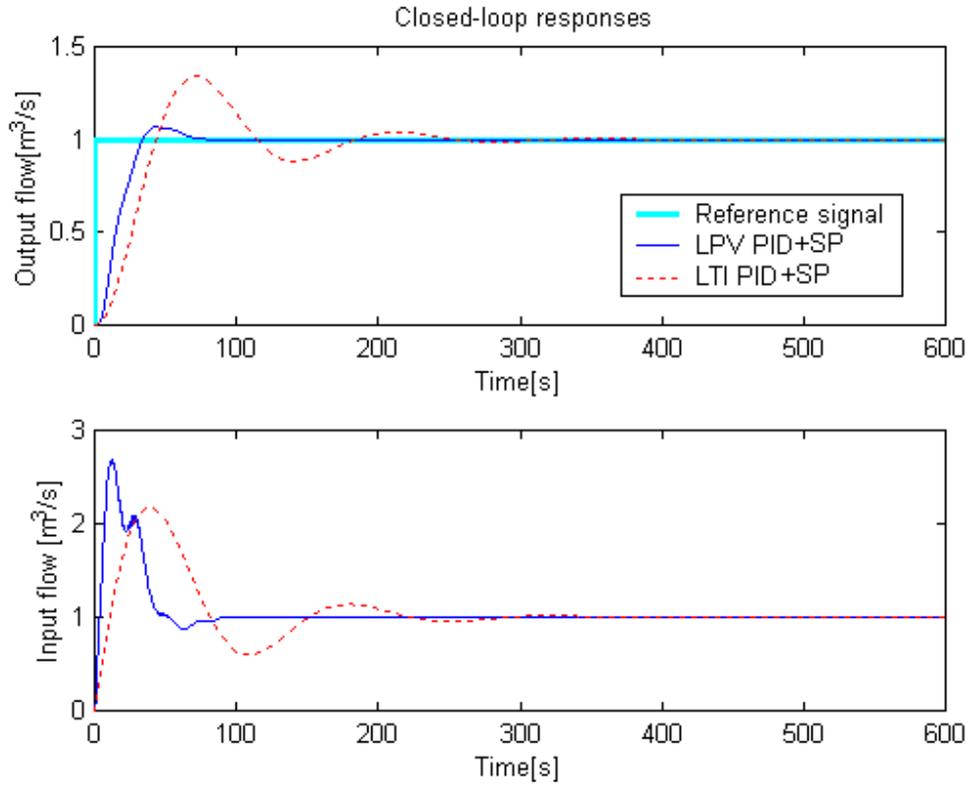


Fig. 10. Step response for trajectory of Fig. 9.

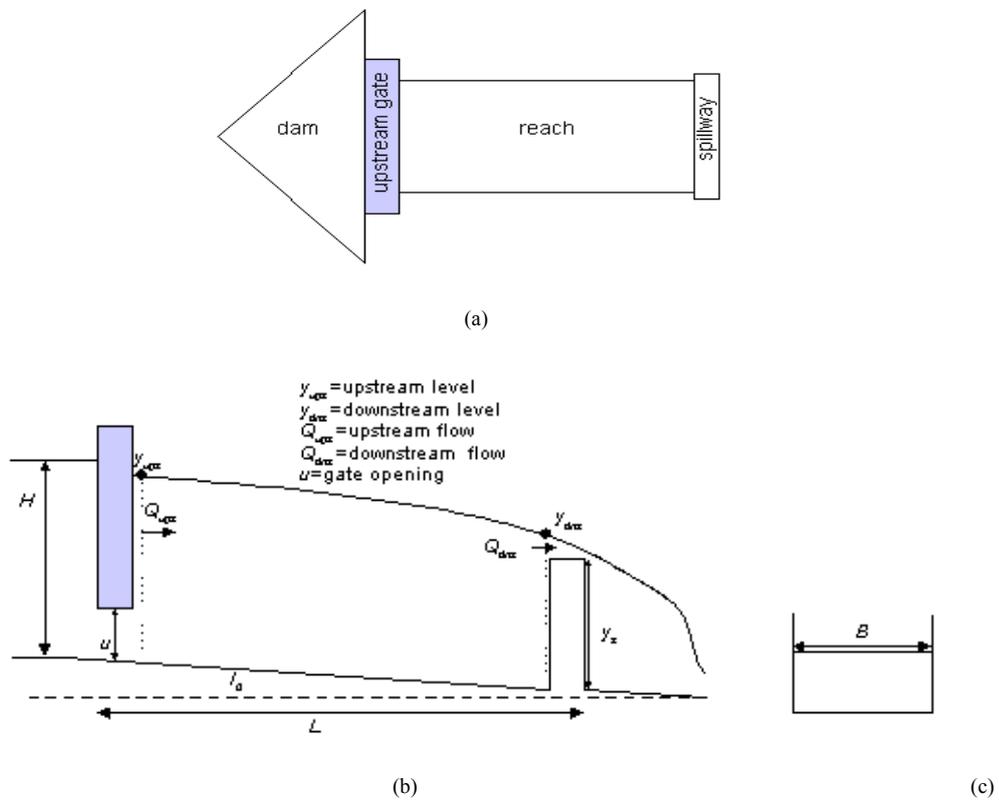
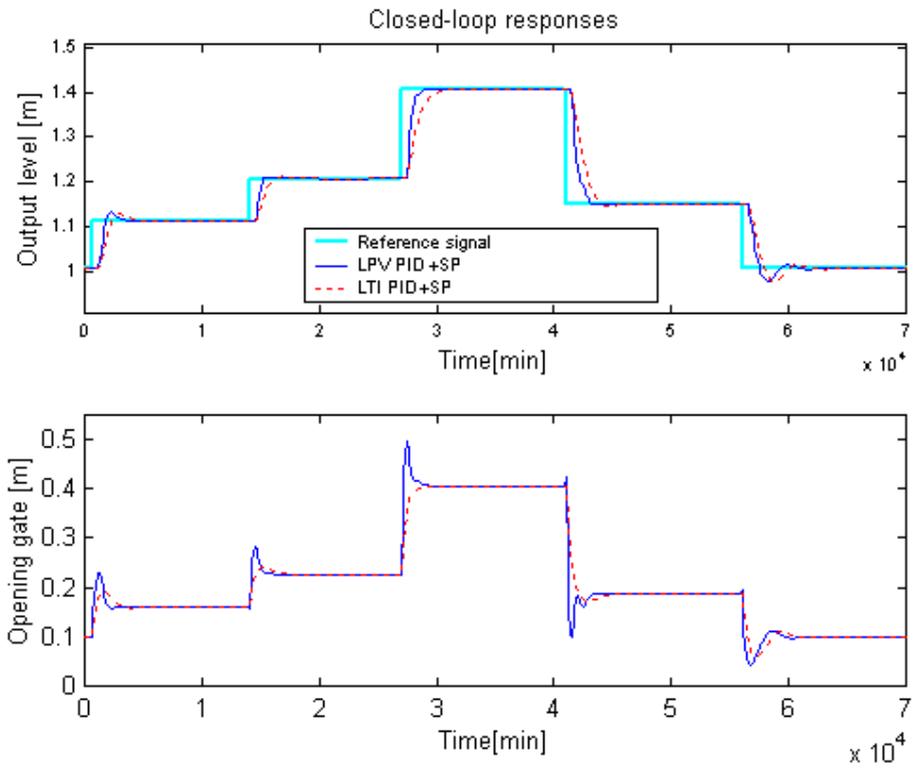


Fig.11. Canal scheme. (a) Up, (b) longitudinal and (c) cross section.



(a)

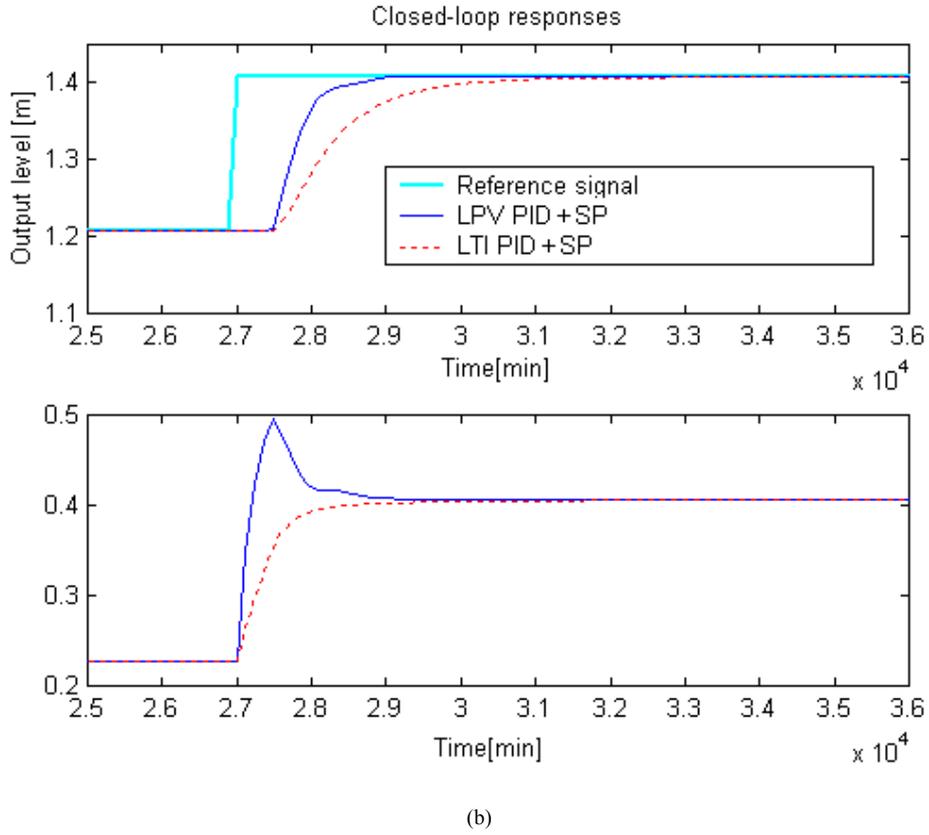


Fig.12. Step responses for LPV and LTI PID+SP. (a) Output and control signals, (b) in detail.

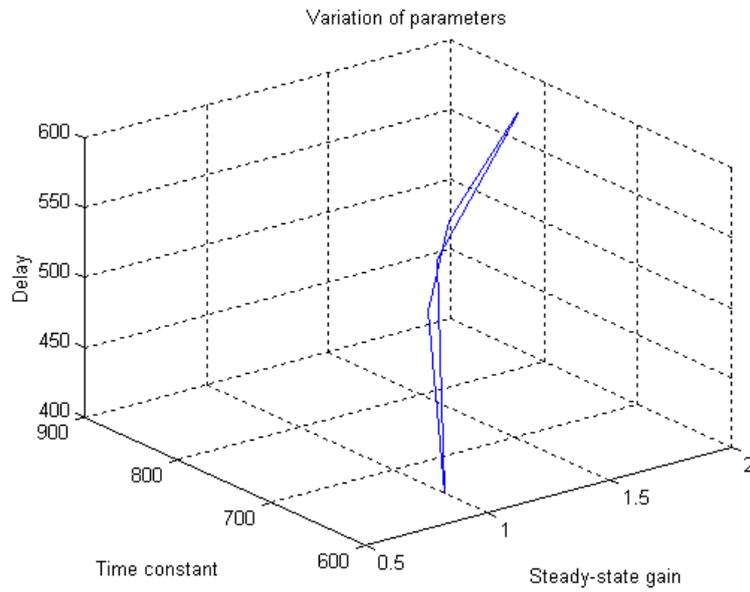


Fig.13. Variation of canal parameters in the scenario of Fig.12.

TABLES

Vertex	LPV PID+SP	LTI PID+SP	LTI PID+ TV SP
1 (k_{\max}, T_{\max})	$K_{P1}=-3.4859$ $K_{D1}=0.6411$ $K_{I1}=-0.2907$	$K_P = -0.4548$ $K_D = 0.4213$ $K_I = -0.1027$	$K_P = 0.7309$ $K_D = 0.8540$ $K_I = -0.0404$
2 (k_{\min}, T_{\max})	$K_{P2} = -7.7058$ $K_{D2} = 1.4171$ $K_{I2} = -0.6427$		
3 (k_{\max}, T_{\min})	$K_{P3}=0.0228$ $K_{D3}=0.0213$ $K_{I3}=-0.0052$		
4 (k_{\min}, T_{\min})	$K_{P4}=0.0504$ $K_{D4}=0.0471$ $K_{I4}=-0.0115$		

Table 1. Parameter values of the LPV PID+SP, the LTI PID+SP and the LTI PID+TV SP.

Vertex	LPV PID+SP	LTI PID+SP
1 (k_{\max}, T_{\max})	$K_{P1} = 0.4297$ $K_{D1} = 0.2813$ $K_{I1} = 0.0022$	$K_P = 0.3051$ $K_D=0.0533$ $K_I = 0.0016$
2 (k_{\min}, T_{\max})	$K_{P2} = 1.7188$ $K_{D2} = 1.1252$ $K_{I2} = 0.0088$	
3 (k_{\max}, T_{\min})	$K_{P3} = 0.2380$ $K_{D3} = 0.0489$ $K_{I3} = 0.0013$	
4 (k_{\min}, T_{\min})	$K_{P4} = 0.9519$ $K_{D4} = 0.1942$ $K_{I4} = 0.0050$	

Table 2. Parameter values of the LPV PID+SP and the LTI PID+SP.