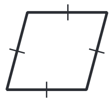
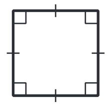
Name ______________________________ Date ____________ Class ____________

LESSON 6-4

Practice A
Properties of Special Parallelograms

Match each figure with the letter of one of the vocabulary terms. Use each term once.

1.  ________________

2.  ________________

3.  ________________

A. rectangle
B. rhombus
C. square

Fill in the blanks to complete each theorem.

4. If a parallelogram is a rhombus, then its diagonals are ________________________.

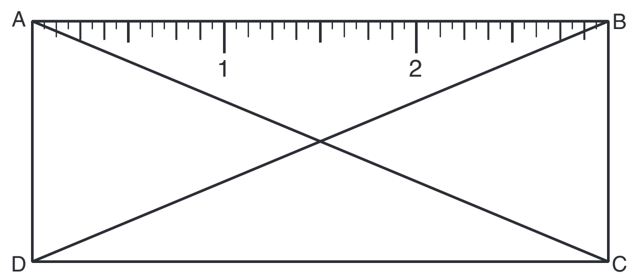
5. If a parallelogram is a rectangle, then its diagonals are ________________________.

6. If a quadrilateral is a rectangle, then it is a ________________________.

7. If a parallelogram is a rhombus, then each diagonal ________________________ a pair of opposite angles.

8. If a quadrilateral is a rhombus, then it is a ________________________.

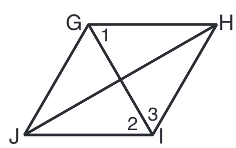
The part of a ruler shown is a rectangle with *AB* = 3 inches and *BD* = $3\frac{1}{4}$ inches. Find each length.



9. $DC =$ ________________________

10. $AC =$ ________________________

Use the phrases and theorems from the Word Bank to complete this two-column proof.



Alternate Interior $\angle$s Thm.
GHIJ is a parallelogram.
Trans. Prop. of $\cong$
$\angle 2 \cong \angle 3$

11. Given: *GHIJ* is a rhombus.
Prove: $\angle 1 \cong \angle 3$

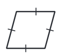
Statements	Reasons
1. *GHIJ* is a rhombus.	1. Given
2. **a.** ________________________	2. rhomb. → ▱
3. $\overline{GH} \parallel \overline{JI}$	3. ▱ → opp. sides $\parallel$
4. $\angle 1 \cong \angle 2$	4. **b.** ________________________
5. **c.** ________________________	5. rhomb. → each diag. bisects opp. $\angle$s
6. $\angle 1 \cong \angle 3$	6. **d.** ________________________

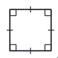
LESSON 6-4

Practice A

Properties of Special Parallelograms

Match each figure with the letter of one of the vocabulary terms. Use each term once.

1.  B

2.  C

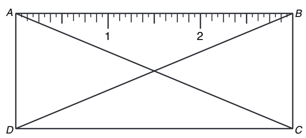
3.  A

A. rectangle
B. rhombus
C. square

Fill in the blanks to complete each theorem.

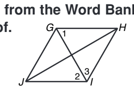
4. If a parallelogram is a rhombus, then its diagonals are **perpendicular**.
5. If a parallelogram is a rectangle, then its diagonals are **congruent**.
6. If a quadrilateral is a rectangle, then it is a **parallelogram**.
7. If a parallelogram is a rhombus, then each diagonal **bisects** a pair of opposite angles.
8. If a quadrilateral is a rhombus, then it is a **parallelogram**.

The part of a ruler shown is a rectangle with $AB = 3$ inches and $BD = 3\frac{1}{4}$ inches. Find each length.



9. $DC =$ **3 in.**
10. $AC =$ $3\frac{1}{4}$ **in.**

Use the phrases and theorems from the Word Bank to complete this two-column proof.



Word Bank: Alternate Interior ∠s Thm.; $GHIJ$ is a parallelogram.; Trans. Prop. of ≅; $\angle 2 \cong \angle 3$

11. **Given:** $GHIJ$ is a rhombus.
Prove: $\angle 1 \cong \angle 3$

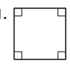
Statements	Reasons
1. $GHIJ$ is a rhombus.	1. Given
2. a. ***GHIJ* is a parallelogram.**	2. rhomb. → ▱
3. $\overline{GH} \parallel \overline{JI}$	3. ▱ → opp. sides ∥
4. $\angle 1 \cong \angle 2$	4. b. **Alternate Interior ∠s Thm.**
5. c. $\angle 2 \cong \angle 3$	5. rhomb. → each diag. bisects opp. ∠s
6. $\angle 1 \cong \angle 3$	6. d. **Trans. Prop. of ≅**

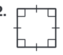
LESSON 6-4

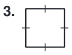
Practice B

Properties of Special Parallelograms

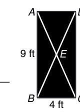
Tell whether each figure must be a rectangle, rhombus, or square based on the information given. Use the most specific name possible.

1.  rectangle

2.  square

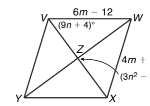
3.  rhombus

A modern artist's sculpture has rectangular faces. The face shown here is 9 feet long and 4 feet wide. Find each measure in simplest radical form. (*Hint:* Use the Pythagorean Theorem.)



4. $DC =$ **9 feet**
5. $AD =$ **4 ft**
6. $DB =$ $\sqrt{97}$ **feet**
7. $AE =$ $\frac{\sqrt{97}}{2}$ **ft**

***VWXY* is a rhombus. Find each measure.**



8. $XY =$ **36**
9. $m\angle YVW =$ **107°**
10. $m\angle VYX =$ **73°**
11. $m\angle XYZ =$ **36.5°**

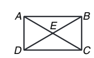
12. The vertices of square $JKLM$ are $J(-2, 4)$, $K(-3, -1)$, $L(2, -2)$, and $M(3, 3)$. Find each of the following to show that the diagonals of square $JKLM$ are congruent perpendicular bisectors of each other.

$JL =$ $2\sqrt{13}$ $KM =$ $2\sqrt{13}$

slope of $\overline{JL} =$ $-\frac{3}{2}$ slope of $\overline{KM} =$ $\frac{2}{3}$

midpoint of $\overline{JL} =$ (0, 1) midpoint of $\overline{KM} =$ (0, 1)

Write a paragraph proof.



13. **Given:** $ABCD$ is a rectangle.
Prove: $\angle EDC \cong \angle ECD$

Possible answer: *ABCD* is a rectangle, so $\overline{AC}$ is congruent to $\overline{BD}$. Because *ABCD* is a rectangle, it is also a parallelogram. Because *ABCD* is a parallelogram, its diagonals bisect each other. By the definition of bisector, $EC = \frac{1}{2}AC$ and $ED = \frac{1}{2}BD$. But by the definition of congruent segments, $AC = BD$. So substitution and the Transitive Property of Equality show that $EC = ED$. Because $\overline{EC} \cong \overline{ED}$, $\triangle ECD$ is an isosceles triangle. The base angles of an isosceles triangle are congruent, so $\angle EDC \cong \angle ECD$.

LESSON 6-4

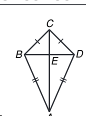
Practice C

Properties of Special Parallelograms

For Exercises 1–5, give your answers in simplest radical form.

1. Find the length of the diagonals of a rectangle with sides of lengths a and b. $\sqrt{a^2 + b^2}$
2. Find the length of the diagonals of a square with sides of length a. $\sqrt{2}a$
3. Find the length of the sides of a square with diagonals of length a. $\frac{\sqrt{2}}{2}a$
4. Find the length of the sides of a rhombus with diagonals of lengths a and b. $\frac{\sqrt{a^2 + b^2}}{2}$
5. Find the length of a rectangle with width x and a diagonal of length $2x$. $\sqrt{3}x$
6. Find the measures of the angles in the triangles formed by one diagonal of the rectangle in Exercise 5. **30°-60°-90°**

The figure shows a kind of quadrilateral called a *kite*. A kite is a quadrilateral with exactly two pairs of congruent consecutive sides. Use the figure to write paragraph proofs for Exercises 7 and 8.



7. **Prove:** $\angle CBA \cong \angle CDA$
Possible answer: It is given that $\overline{CB} \cong \overline{CD}$ and $\overline{AB} \cong \overline{AD}$. $\overline{CA}$ is congruent to $\overline{CA}$ by the Reflexive Property of Congruence. Thus $\triangle ABC$ is congruent to $\triangle ADC$ by SSS. By CPCTC, $\angle CBA \cong \angle CDA$.

8. **Prove:** $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.
Possible answer: It is given that $\overline{CB} \cong \overline{CD}$ and $\overline{AB} \cong \overline{AD}$. So *C* and *A* are on the perpendicular bisector of $\overline{BD}$ by the Conv. of the Perpendicular Bisector Thm. So, since two points determine a line, $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.

For Exercises 9–11, name all the types of quadrilaterals (kite, parallelogram, rectangle, rhombus, or square) that satisfy the given conditions.

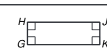
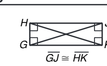
9. The diagonals bisect each other.
parallelogram, rectangle, rhombus, square
10. The diagonals are perpendicular.
kite, rhombus, square
11. The diagonals are congruent.
rectangle, square

LESSON 6-4

Review for Mastery

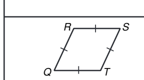
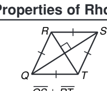
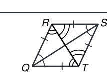
Properties of Special Parallelograms

A **rectangle** is a quadrilateral with four right angles. A rectangle has the following properties.

Properties of Rectangles	
 $GHJK$ is a parallelogram.	 $\overline{GJ} \cong \overline{HK}$
If a quadrilateral is a rectangle, then it is a parallelogram.	If a parallelogram is a rectangle, then its diagonals are congruent.

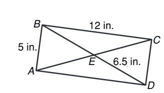
Since a rectangle is a parallelogram, a rectangle also has all the properties of parallelograms.

A **rhombus** is a quadrilateral with four congruent sides. A rhombus has the following properties.

Properties of Rhombuses		
 $QRST$ is a parallelogram.	 $\overline{QS} \perp \overline{RT}$	 $\angle RQS \cong \angle SQT$
If a quadrilateral is a rhombus, then it is a parallelogram.	If a parallelogram is a rhombus, then its diagonals are perpendicular.	If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

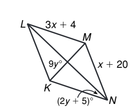
Since a rhombus is a parallelogram, a rhombus also has all the properties of parallelograms.

***ABCD* is a rectangle. Find each length.**



1. BD **13 in.**
2. CD **5 in.**
3. AC **13 in.**
4. AE **6.5 in.**

***KLMN* is a rhombus. Find each measure.**



5. KL **28**
6. $m\angle MNK$ **50°**