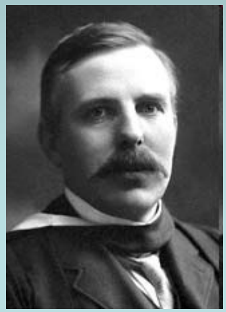
Physics of the Atom



Ernest Rutherford

The Nobel Prize in Chemistry 1908
"for his investigations into the disintegration of the elements, and the chemistry of radioactive substances"

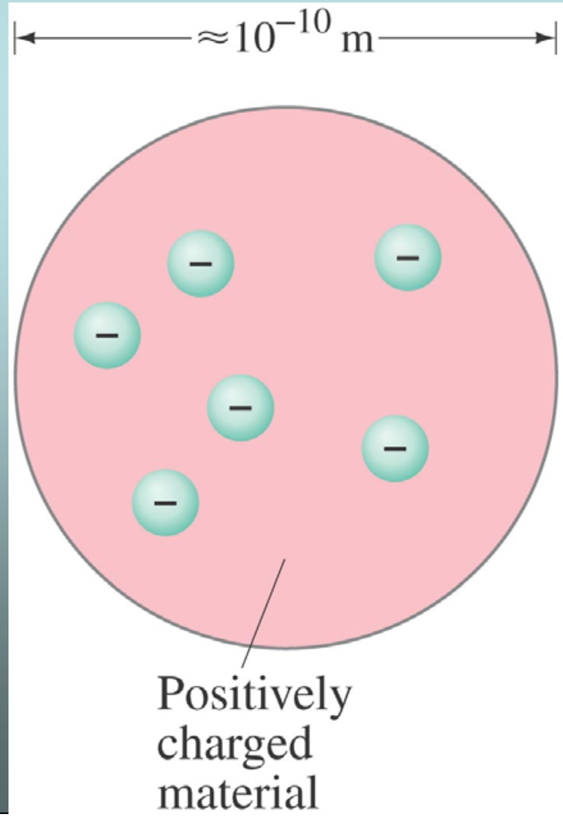
Niels Bohr

The Nobel Prize in Physics 1922
"for his services in the investigation of the structure of atoms and of the radiation emanating from them"

Early models of the atom

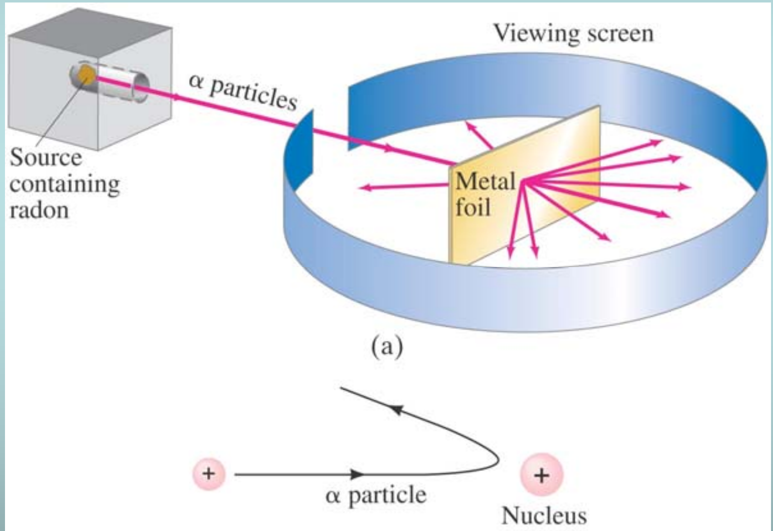
atoms : electrically neutral
they can become charged
positive and negative charges are around
and some can be removed.

popular atomic model
"plum-pudding" model:



Rutherford scattering

Rutherford did an experiment that showed that the positively charged nucleus must be extremely small compared to the rest of the atom.



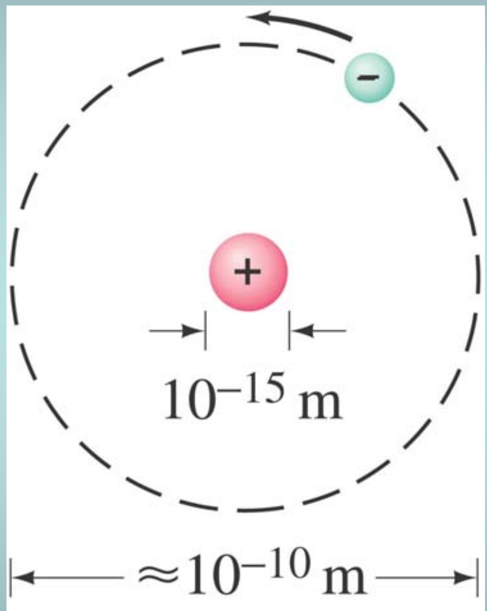
Result from Rutherford scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\varepsilon_0} \frac{Zze^2}{4K} \right)^2 \frac{1}{\sin^4(\theta/2)}$$

Applet for doing the experiment:

http://www.physics.upenn.edu/courses/gladney/phys351/classes/Scattering/Rutherford_Scattering.html

Rutherford scattering
the smallness of the nucleus



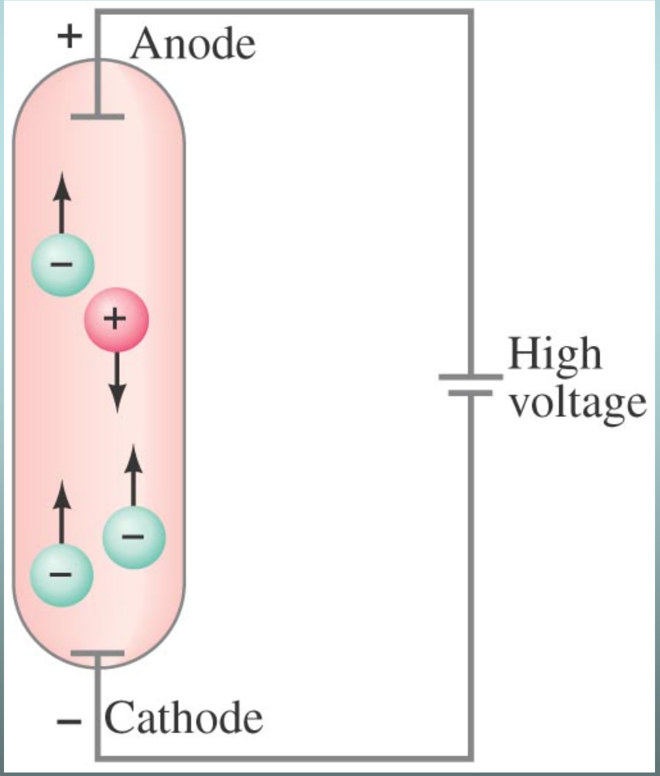
the radius of the nucleus is 1/10,000 that of the atom.

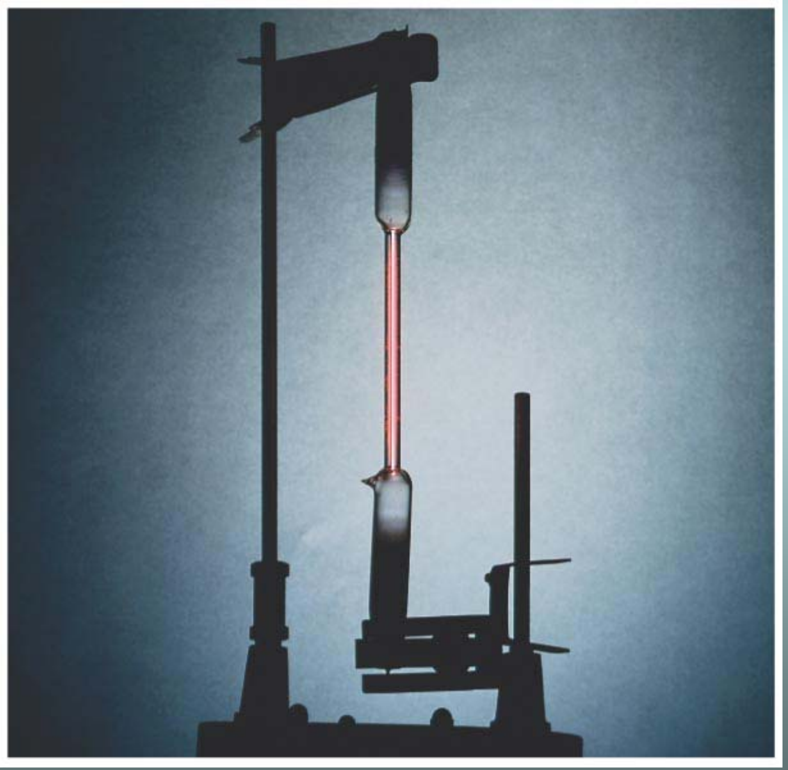
the atom is mostly empty space

Rutherford's atomic model

Atomic Spectra:
Key to the Structure of the Atom

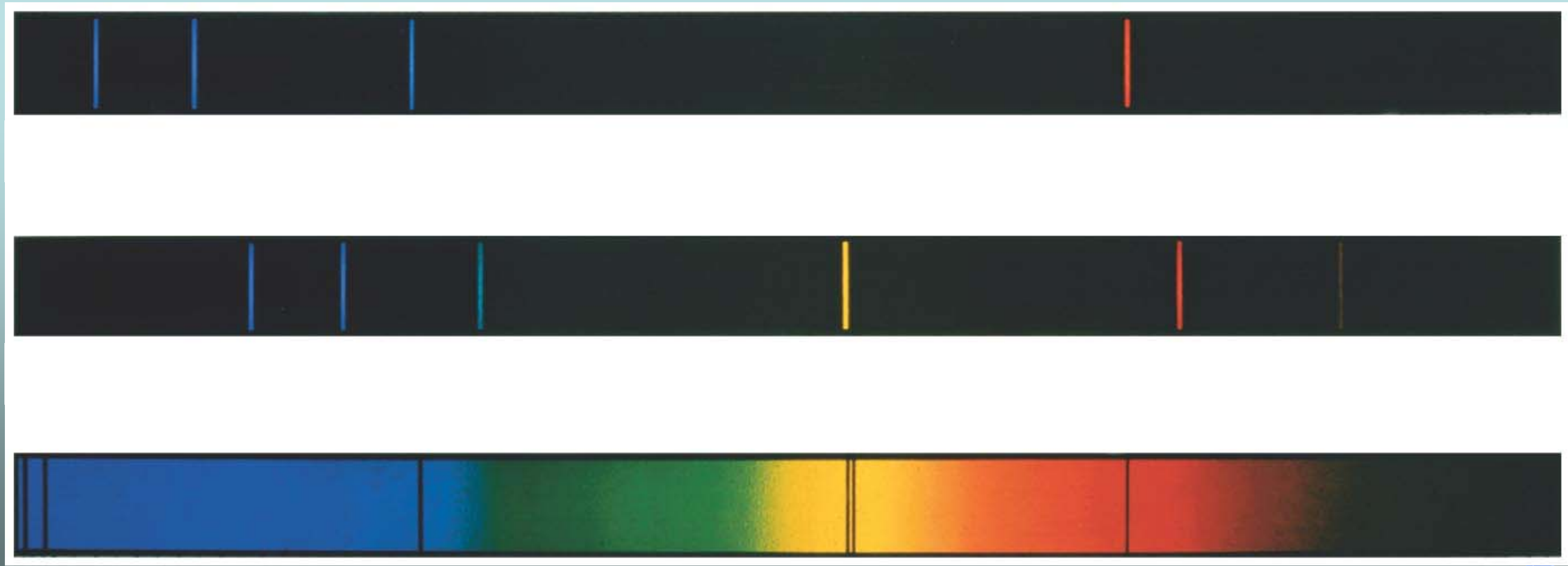
A very thin gas heated in a discharge tube emits light only at characteristic frequencies.





Atomic Spectra:
Key to the Structure of the Atom

Line spectra: *absorption and emission*



The Balmer series in atomic hydrogen

Johann Jakob Balmer

The wavelengths of electrons emitted from hydrogen have a regular pattern:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \qquad n = 3, 4, \cdots.$$

Lyman, Paschen and Rydberg series

the Lyman series:

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \qquad n = 2, 3, \cdots.$$

the Paschen series:

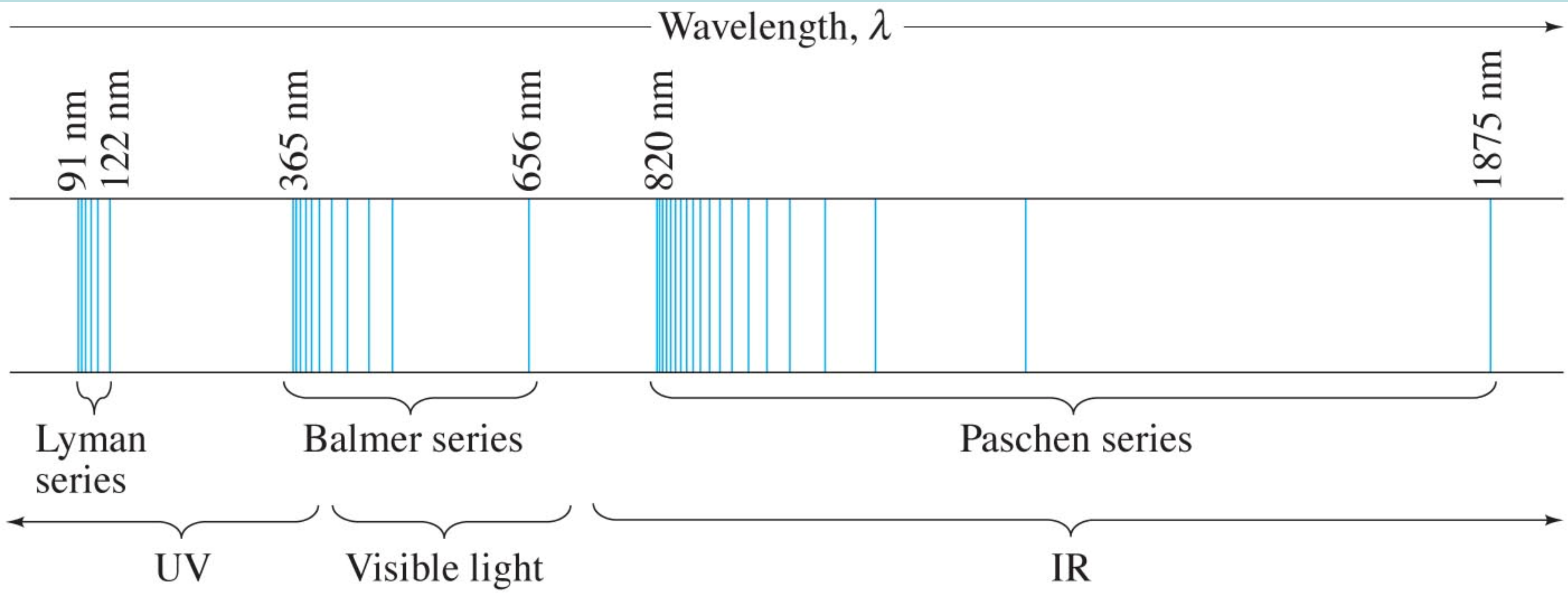
$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \qquad n = 4, 5, \cdots.$$

Janne Rydberg

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The Spectrum of the hydrogen Atom

A portion of the complete spectrum of hydrogen is shown here. The lines cannot be explained by classical atomic theory.



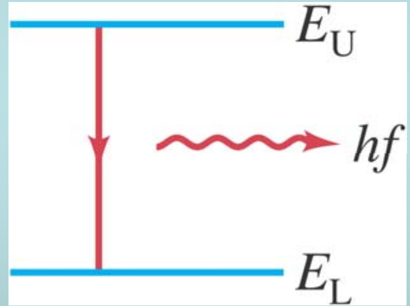
The Bohr Model

Solution to radiative instability of the atom:

1. atom exists in a discrete set of stationary states
no radiation when atom is in such state

2. radiative transitions → quantum jumps between levels

$$h\nu = \frac{hc}{\lambda} = E_i - E_f$$



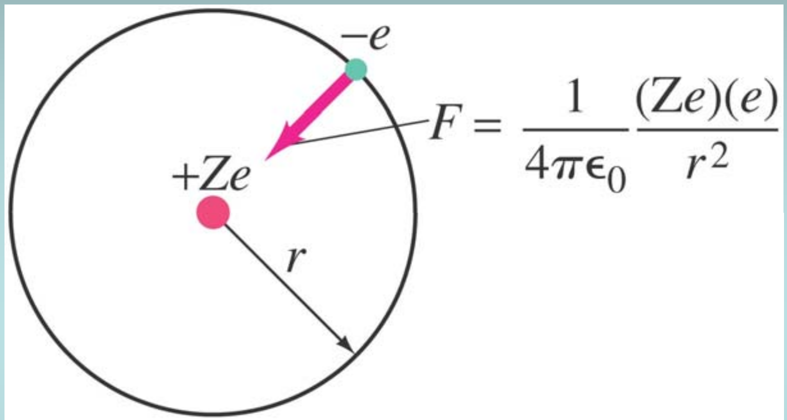
3. angular momentum is quantized

$$L = mvr_n = n\frac{h}{2\pi}, \qquad n = 1, 2, 3, \cdots.$$

These are *ad hoc* hypotheses by Bohr, against intuitions of classical physics

The Bohr Model: derivation

An electron is held in orbit by the Coulomb force: (equals centripetal force)



$$F_{centr} = F_{Coulomb}$$

$$\frac{mv^2}{r_n} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2}$$

Solve this equations for *r:*

$$r_n = \frac{Ze^2}{4\pi\varepsilon_0 mv^2}$$

Impose the quantisation condition:

$$L = mvr_n = n\hbar$$

The Bohr Model: derivation

Solve: the size of the orbit is quantized
(and we know the size of an atom !)

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m Z e^2} = \frac{n^2}{Z} r_1.$$

$$r_1 = \frac{h^2 \epsilon_0}{\pi m e^2}$$

$$= 0.529 \times 10^{-10}\,\text{m}.$$

Size of an atom: **0.5 Å**

The Bohr Model: derivation

The energy of a particle in orbit

$$E_n = E_{kin} + E_{pot} = \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\varepsilon_0 r_n} = \left(\frac{1}{2} - 1\right)\frac{Ze^2}{4\pi\varepsilon_0 r_n}$$

So

$$E_n = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 r_n} = -\left(\frac{Z^2}{n^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{m}{2\hbar^2}$$

Define

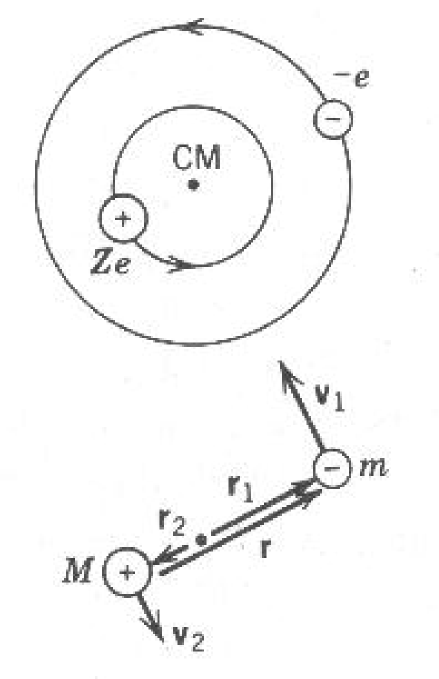
$$E_n = -\left(\frac{Z^2}{n^2}\right) R_\infty$$

With the Rydberg constant

$$R_\infty = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{m_e}{2\hbar^2}$$

EXTRA

Reduced mass in the old Bohr model → the one particle problem



Relative coordinates:

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

Centre of Mass

$$m\vec{r}_1 + M\vec{r}_2 = 0$$

Position vectors:

$$\vec{r}_1 = \frac{M}{m+M}\vec{r}$$

$$\vec{r}_2 = -\frac{m}{m+M}\vec{r}$$

Velocity vectors:

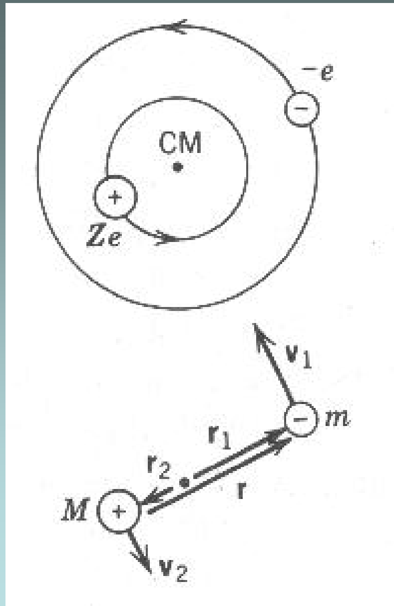
$$\vec{v}_1 = \frac{M}{m+M}\vec{v}$$

$$\vec{v}_2 = -\frac{M}{m+M}\vec{v}$$

Relative velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

EXTRA



Reduced mass

$$\mu = \frac{mM}{m+M}$$

Centripetal force

$$F = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{\mu v^2}{r}$$

Kinetic energy

$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mu v^2$$

Quantisation of angular momentum:

$$L = \mu v r = n \frac{h}{2\pi} = n\hbar$$

Angular momentum

$$L = m_1 v_1 r_1 + m_2 v_2 r_2 = \mu v r$$

These are the equations for a single (reduced) particle

EXTRA

Result for the one-particle problem

Quantisation of radius in orbit:

$$r_n = \frac{n^2}{Z}\frac{4\pi\varepsilon_0\hbar^2}{e^2\mu} = \frac{n^2}{Z}\frac{m_e}{\mu}a_0$$

Energy levels in the Bohr model:

$$E_n = -\frac{Z^2}{n^2}\frac{\mu}{m_e}R_\infty$$

Isotope effect

Rewrite:

$$E_n = -\frac{Z^2}{n^2}\left(\frac{\mu}{m_e}\right)\frac{1}{2}\alpha^2 m_e c^2$$

dimensionless energy

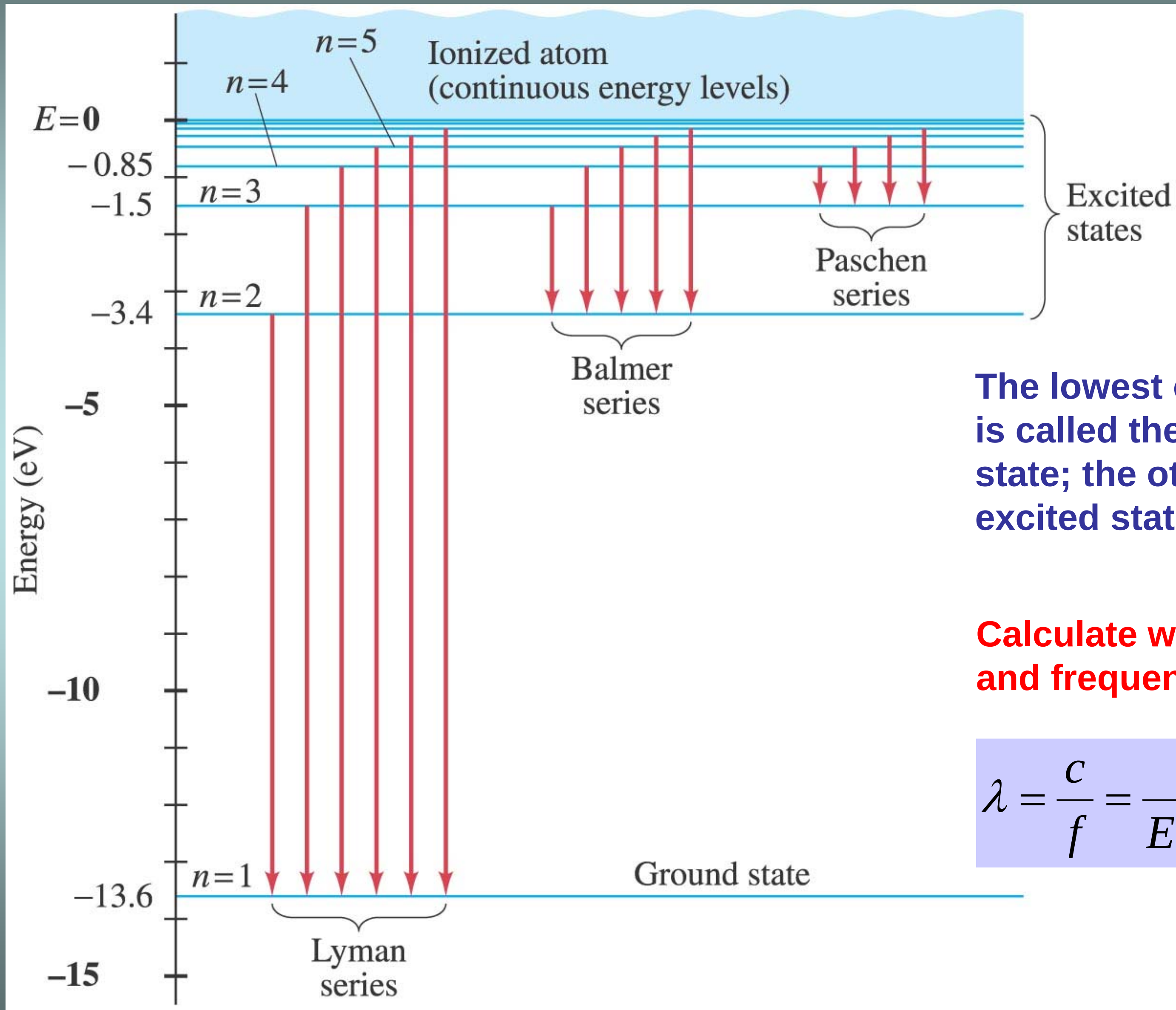
$$\alpha \approx \frac{1}{137}$$

$$\mu = \frac{mM}{m+M}$$

For all atoms: $\mu \approx m_e$

Calculate the isotope shift on Lyman-α:

Optical transitions in The Bohr Model



The lowest energy level is called the ground state; the others are excited states.

Calculate wavelengths and frequencies

$$\lambda = \frac{c}{f} = \frac{hc}{E_2 - E_1}$$

Quantum states and kinetics

Hydrogen at 20°C.

Estimate the average kinetic energy of whole hydrogen atoms (not just the electrons) at room temperature, and use the result to explain why nearly all H atoms are in the ground state at room temperature, and hence emit no light.

$$\overline{K} = \frac{3}{2} k_B T = 6.2 \times 10^{-21} J = 0.04 eV$$

Gas at elevated temperature – probability exited state is populated:

$$P_n(T) = e^{-E_n / kT}$$

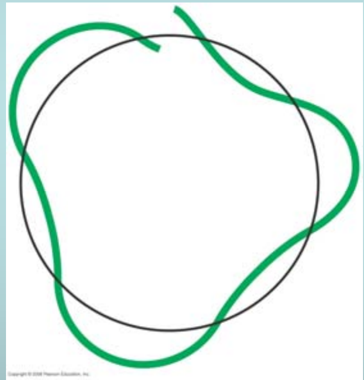
Energy in gas:

$$\langle E \rangle = \frac{\sum_n E_n e^{-E_n / kT}}{\sum_n e^{-E_n / kT}}$$

Another way of looking at the quantisation in the atom

de Broglie’s Hypothesis Applied to Atoms

Electron of mass m has a wave nature



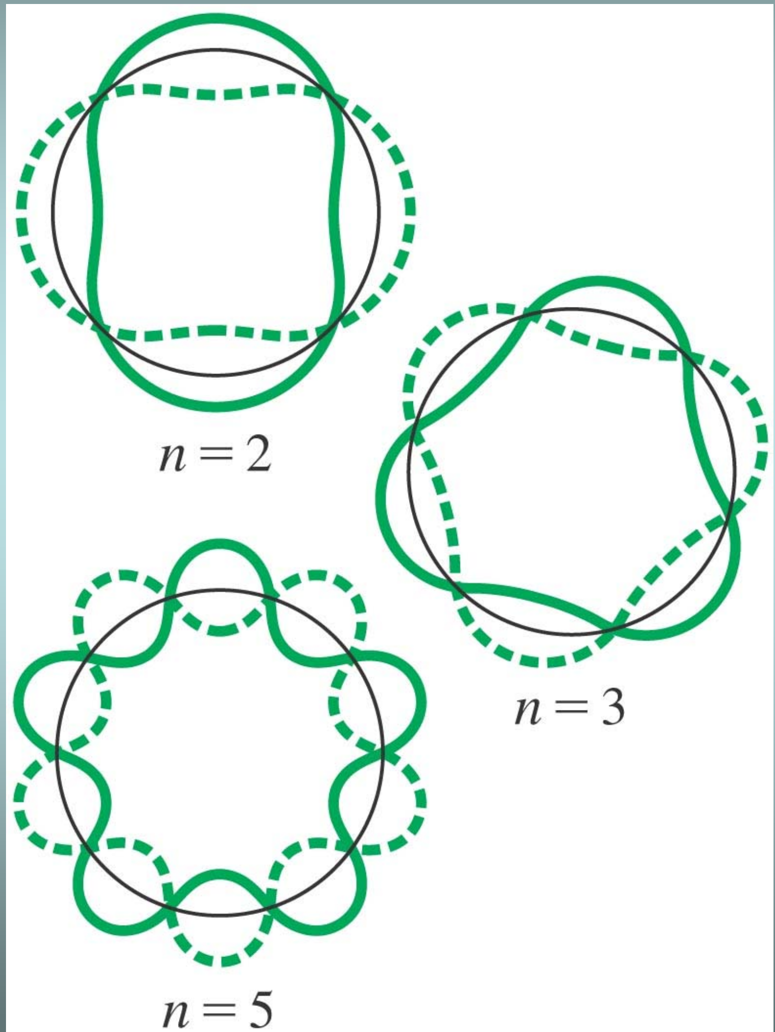
Electron in orbit: a standing wave

$$2\pi r_n = n\lambda$$

Substitution gives the quantum condition

$$L = mvr_n = \frac{nh}{2\pi}$$

de Broglie's Hypothesis Applied to Atoms



These are circular standing waves for n = 2, 3, and 5.

Standing waves do not radiate;
Interpretation:
electron does not move
(no acceleration)