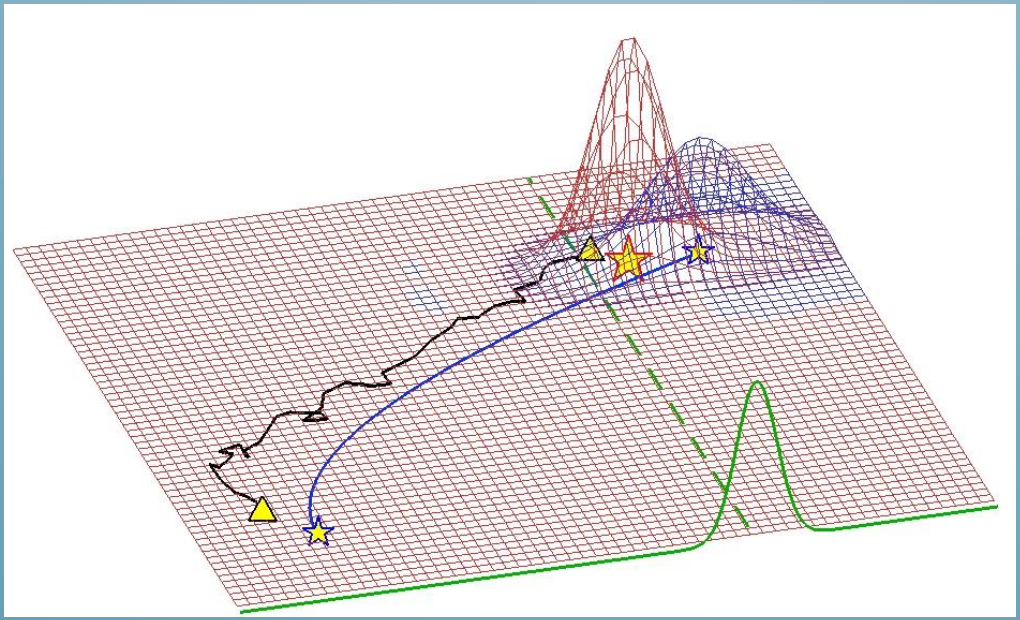
A TUTORIAL ON ASSIMILATING LAGRANGIAN DATA INTO OCEAN MODELS



DATA ASSIMILATION

Objectives of DA:

- Combine ocean ***data*** and ***models***
- obtain best ***state estimate***
- reflect ***uncertainty*** in estimates
  - typically as ***pdf***, probability density function

Primary Tool: ***Bayes Theorem***

$x =$ state

$y =$ obs

$$P^{\text{posterior}}(x|y) = P^{\text{obs}}(y|x)\,P^{\text{prior}}(x)$$

MODELS AND DATA

Ocean model:

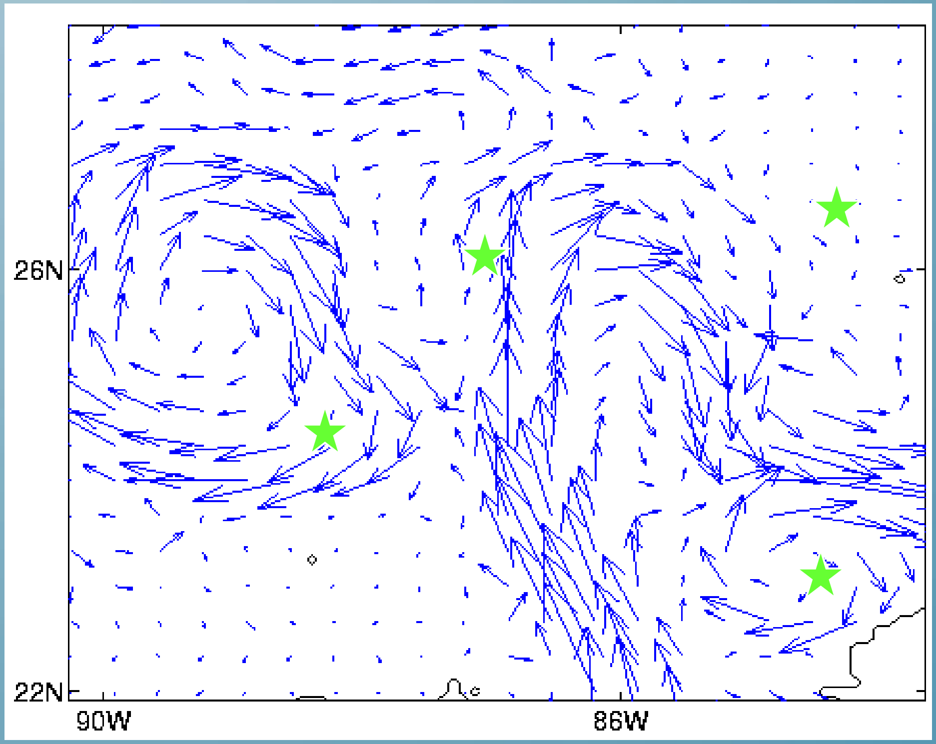
$\mathbf{x} \in \mathbf{R}^N$ – state vector comprising all relevant dynamical variables

(e.g. flow velocity, temperature, salinity, etc. at each grid point)

$d\mathbf{x}^{\mathrm{f}} = M(\mathbf{x}^{\mathrm{f}}, t)dt$ prognostic model

$d\mathbf{x}^{\mathrm{t}} = M(\mathbf{x}^{\mathrm{t}}, t)dt + \boldsymbol{\eta}(t)dt$ actual evolution

$E[\boldsymbol{\eta}(t)\boldsymbol{\eta}^T(t')] = \delta(t-t')\mathbf{Q}(t)$ covariance of the model residual



Observations:

$\mathrm{y}_i^{\mathrm{o}} = H_i[\mathbf{x}_i^{\mathrm{t}}] + \boldsymbol{\varepsilon}_i$

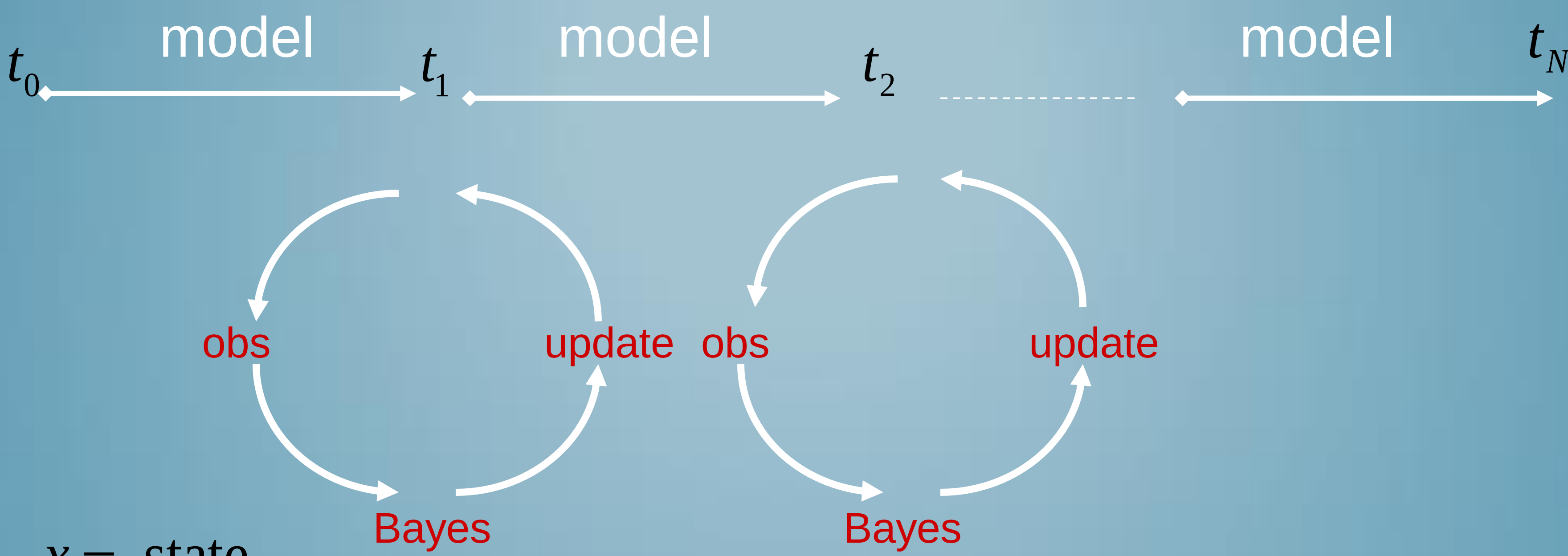
H_i – observation operator

$\boldsymbol{\varepsilon}_i$ – observation error

$\mathrm{y}_i^{\mathrm{o}} \in \mathbf{R}^L$, typically $L \ll N$

$E[\boldsymbol{\varepsilon}_i \boldsymbol{\varepsilon}_m^T] = \delta_{im}\mathbf{R}_i$ – observation error covariance

SEQUENTIAL DATA ASSIMILATION (METHODS: KALMAN FILTERS, PARTICLE FILTERS)



$x =$ state

$y =$ obs

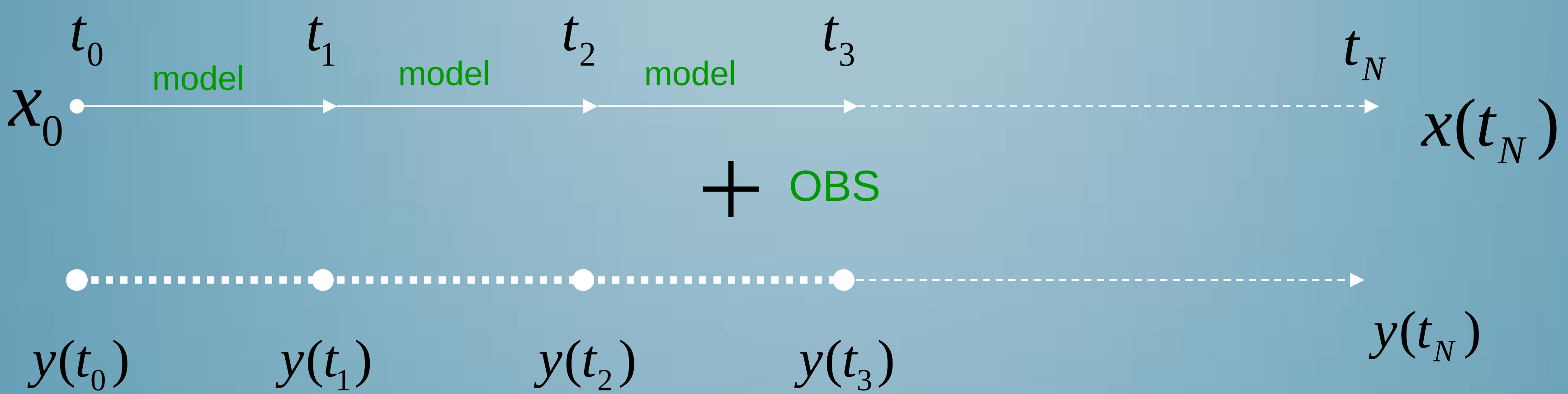
$$P^{\text{posterior}}(x|y) = P^{\text{obs}}(y|x)\,P^{\text{prior}}(x)$$

Key: difference in methods is how we obtain the distributions on RHS

- Kalman: approximate pdfs with Gaussians
- Particle: approximate posterior with discrete samples

SMOOTHING – COMBINING ALL DATA AT ONCE (METHODS: 4DVAR, MCMC SAMPLING)

Model runs + observations → state estimate



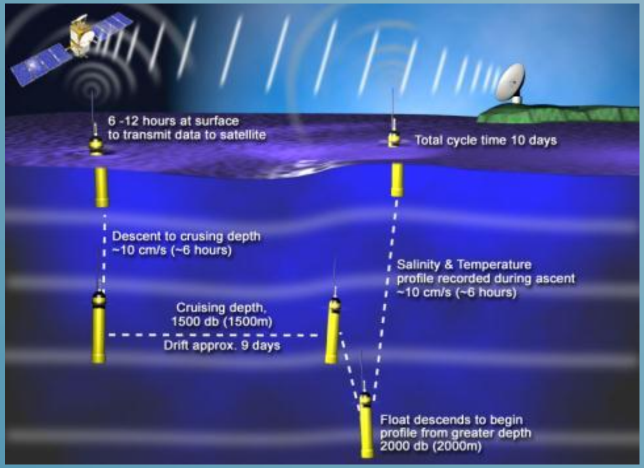
Bayes: $$P^{\text{posterior}}(x|y) = P^{\text{obs}}(y|x)\,P^{\text{prior}}(x)$$

Key: difference in methods is how we obtain the distributions on RHS

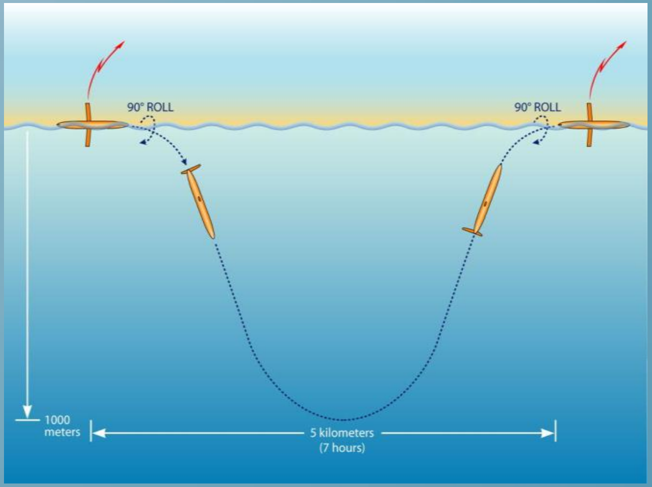
- MCMC SAMPLING: sample exact posterior distribution
- 4DVAR: obtains mode of posterior

LAGRANGIAN OCEAN DATA

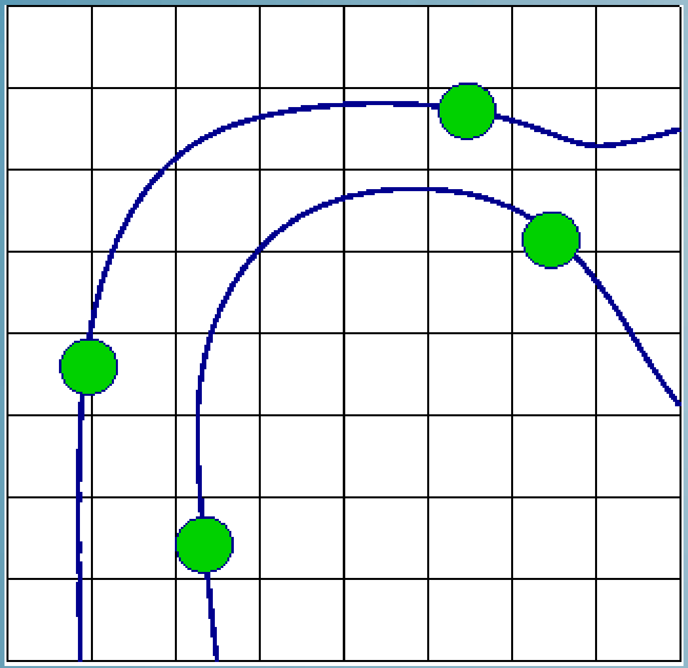
Argo floats
(Lagrangian at depth)



Ocean Gliders
(semi-Lagrangian)



LAGRANGIAN DATA ASSIMILATION



Lagrangian observations from drifters, gliders, and floats do not give the data in terms of model variables.

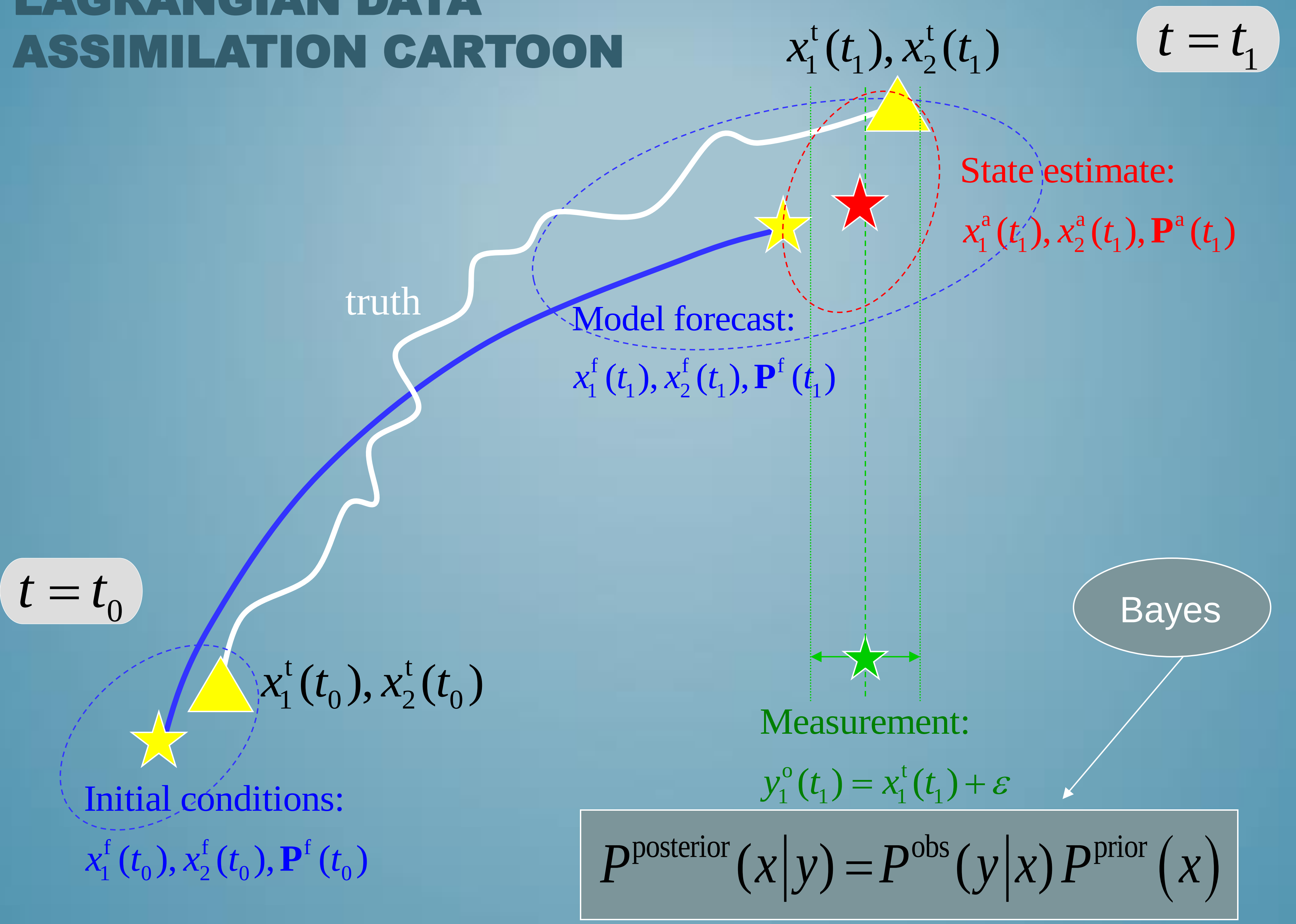
Solution: Include drifter coordinates into the model

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_F \\ \mathbf{x}_D \end{pmatrix} \quad - \text{augmented state vector}$$

$$\frac{d\mathbf{x}_F^f}{dt} = M_F(\mathbf{x}_F^f, t) \quad - \text{flow equations}$$

$$\frac{d\mathbf{x}_D^f}{dt} = M_D(\mathbf{x}_D^f, \mathbf{x}_F^f, t) \quad - \text{tracer advection equations}$$

LAGRANGIAN DATA ASSIMILATION CARTOON



LAGRANGIAN DATA ASSIMILATION EXAMPLE

- Model initialized with no eddy present
- Eddy is "discovered" and tracked with incorporation of Lagrangian data
- Below: analysis snapshots of eddy shedding
  (*Vernieres, Jones, Ide* Phys. D, 240, 2011)