

A Novel Feedback PD Compensator Used with Underdamped Second-order Processes

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Abstract

Compensators are used in place of classical PID controllers for possible achievement of better performance. Unsatisfactory dynamics of some industrial processes represent an engineering problem that has to be solved. Highly oscillating processes and very slow processes are examples.

In this paper a novel feedback PD compensator based is proposed and applied to control second-order like processes having oscillating characteristics (damping ratio ≤ 0.8). The compensator proposed is capable of controlling the steady-state characteristics of the closed-loop control system and its dynamic characteristics. The advantage of the proposed compensator is its simple structure and tuning using an ITAE objective function. It was possible with the proposed compensator to satisfy a system performance with very low overshoot and settling time. The compensator tuning is reduced to simple relation with the process parameters with correlation coefficient as high as 0.9999.

Keywords : Underdamped Second-order Processes , Compensators , Feedback PD Compensator , Control System Performance..

1 Introduction

Compensators find wide application in both linear and nonlinear dynamic systems. The design of classical compensators such as lag, lead , lag-lead, PID and pre-filter are investigated in automatic control textbooks [1-5].

Kawada and Sogo (2001) proposed a variable gain PD controller design scheme. The controller parameters are tuned corresponding to the jib length, rope length and jib angle [6]. Patel (2002) derived analytical structures of fuzzy PD controllers. He studied the properties of such PD

controllers showing the influence of variable cross-point level on controller performance [7]. Gao (2003) used a set of tools to standardize controller tuning. He compared using a PD and LADRC controllers use with a motion control test bed [8]. Wong and Kapila (2004) presented an approach to perform position control of a DC motor experimental setup via the internet. They used a PD controller structure to control the DC motor test bed from a remote web-client PC [9]. De Luca, Siciliano and Zollo (2005) proposed a PD control with online gravity compensation for regulation tasks of robot manipulators with elastic joints [10].

Wang, Tao, Hong and Cho (2006) presented the use of a PD visual controller for microassembly system to acquire better dynamic response. They applied the fuzzy logic to tune the controller which is a model free method [11]. Zhao and others (2007) used a conventional PD controller with 100 % gain and a STR controller to control a low cost linear switched reluctance motor. Both controllers give good dynamic performance and accurate position tracking [12]. Sato and Kameoka (2008) proposed an adaptive control method of a weigh feeder. They used three different controllers: 1DOF PID, 1DOF PD and 2DOF PD controllers. They designed the controllers on the basis of generalized minimum variance control (GMVC) [13]. Aphiratsakun and Parnichkun (2009) studied the design of a fuzzy based gains tuning of a PD controller for joints positions control of the Asian Institute of Technology's leg Exoskeleton-I. They compared the performance with that of the conventional PD controller [14]. Rahimian and Tavazoei (2010) proposed a scheme for computing the stable regions for fractional-order PI and PD controllers. They studied the effectiveness of the proposed method through simulation results for two example systems [15].

Allen, Neff and Faloutsos (2011) proposed critically damped PD control strategies to precisely obtain target position and velocity constraints for arbitrary initial conditions [16]. Singh and Yadav (2012) presented comparison of the time specification performance between two type of control (LQR and PD) for a double inverted pendulum system. They determined the control strategy for better performance with respect to pendulum angles and cart position. The use of a simple multi PD controller designed by the theory of pole placement [17]. Kadam and Tiwari (2013) presented a simple method for tuning a PD controller for controlling the depth of an autonomous underwater vehicle. They used the gain margin specification to tune the PD controller [18]. Moraes, Castelan and Moren (2013) proposed a full-order dynamic output feedback compensator for time-stamped network control system. They synthesized compensator gains in terms of linear matrix inequalities [19]. Rao, Raghu and Rajasekaran (2013) designed a feedback controller for a DC-DC boost converter to obtain a constant output values of the feedback controller [20]. Liu and Akasaka (2014) addressed the stabilization problem of linear systems subject to input saturation. They revealed that any linear observer can be used to realize the output feedback stabilization [21]. Zhang, Lam and Xia (2014) studied the design and analysis of output feedback delay compensation controller for network control systems. They used an output feedback strategy to generate the control input packet [22].

2 Analysis

Process:

The process considered in this analysis has the transfer function, $G_p(s)$:

$$G_p(s) = \omega_{np}^2 / (s^2 + 2\zeta_p\omega_{np}s + \omega_{np}^2) \quad (1)$$

Where:

ω_{np} = process natural frequency

ζ_p = process damping ratio

The Proposed Compensator:

The proposed compensator is a feedback PD compensator. The PD controller is well known as one of the first generation of PID controllers. However, using the PD-structure as a feedback compensator needs inversion which is the purpose of this work. The block diagram of the control system in this case is shown in Fig.1.

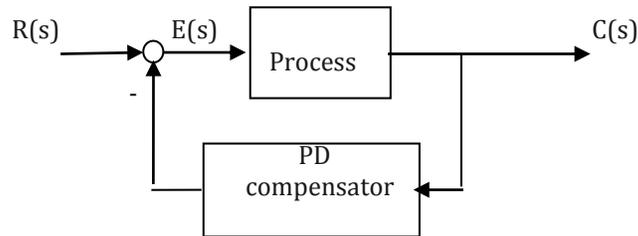


Fig.1 Block diagram of a feedback compensated control system.

The feedback compensator has a transfer function, $G_c(s)$ given by:

$$G_c(s) = K_{pc} + K_{ds} \quad (2)$$

It has the 2 parameters:

- Proportional gain, K_{pc} .

- Derivative gain, K_d .

Control System Transfer Function:

Using the block diagram of Fig.1, the transfer function of the closed-loop control system is:

$$M(s) = \omega_n^2 / \{s^2 + (2\zeta\omega_n + \omega_n^2 K_d)s + \omega_n^2(1 + K_{pc})\} \quad (3)$$

System Step Response and Performance:

A unit step response is generated by MATLAB using the numerator and denominator of Eq. 3 providing the system response $c(t)$ as function of time for a set of compensator parameters.

The characteristics of the compensated control system quantifying its performance are:

- Steady-state error, e_{ss} :

Using Eq.3, the system steady-state error for a unit step input is:

$$e_{ss} = K_{pc} / (1 + K_{pc}) \quad (4)$$

- Maximum percentage overshoot, OS_{max} :

Using the time response of the control system to a unit step input, the maximum percentage overshoot is:

$$OS_{max} = 100 (C_{mas} - C_{ss}) / C_{ss} \quad (5)$$

Where: C_{max} = maximum time response to a step input.

C_{ss} = steady state response of the control system to the unit step input

- Settling time, T_s :

The time response of the system enters a band of $\pm 5\%$ of the steady-state response and remains inside this band.

2 Compensator Tuning

The compensator proposed in this work is tuned using the ITAE objective function given by:

$$F = \int t |c_{ss} - c| dt \quad (6)$$

Three functional constraints c_1 , c_2 and c_3 are used to control the performance of the control system:

$$C_1 = e_{ss} - e_{ssdes} \quad (7)$$

$$c_2 = OS - OS_{des} \tag{8}$$

$$c_3 = T_s - T_{sdes} \tag{9}$$

- Eqs.7-9 are functions of the compensator parameters K_{pc} and K_d .
- The objective function given by Eq.6 is minimized using the MATLAB optimization toolbox subjected to bounds on the compensator parameters and the functional constraints of Eqs.7, 8 and 9.

3 Tuning Results

MATLAB optimization is used for desired performance parameters of:

$$\begin{aligned} e_{ssdes} &= 0.005 \\ OS_{des} &= 0.100 \quad \% \\ T_{sdes} &= 1 \quad s \end{aligned}$$

The proportional gain K_{pc} of the compensator for different values of process natural frequency ($5 \leq \omega_n \leq 20$ rad/s) and damping ratio ($0.05 \leq \zeta \leq 0.8$) is constant at:

$$K_{pc} = 0.005009$$

The value of K_{pc} is defined only by the desired steady-state error. This is why it did not change with the process parameters.

The other parameter of the compensator depends on the process parameters. Table 1 gives K_d , settling time and maximum percentage overshoot for $\omega_n = 10$ rad/s as function of the process damping ratio.

Table 1: Tuned K_d , settling time and maximum overshoot using the feedback compensator for $\omega_n = 10$ rad/s.

Z	K_d	T_s (s)	OS (%)
0.05	0.17251	0.4076	0.1
0.1	0.162512	0.4076	0.1
0.2	0.14251	0.4076	0.1
0.3	0.12251	0.4076	0.1
0.4	0.10251	0.4076	0.1
0.5	0.08251	0.4076	0.1
0.6	0.06251	0.4076	0.1
0.7	0.04251	0.4076	0.1
0.8	0.02251	0.4076	0.1

The tuned derivative gain is dependent of the process natural frequency and damping ratio. To facilitate computer-aided control of industrial processes for adaptive application of compensator tuning, the tuning results of K_d against ω_n and ζ of the process are fitted in an exponential model of very high multiple correlation coefficient. That is:

$$K_d = a_0 (a_1^{\omega_n}) (a_2^{\zeta}) \quad (10)$$

Where:

$$a_0 = 0.62075573206$$

$$a_1 = 0.91502088308$$

$$a_2 = 0.08485309035$$

The multiple correlation coefficient of this curve fitting is 0.99995.

The mathematical model of Eq.10 is used for second-order like processes having equivalent natural frequency and damping ratio in the range:

$$5 \leq \omega_n \leq 20 \quad \text{rad/s}$$

and $0.05 \leq \zeta \leq 0.8$

4 Application

To examine the validity of the tuning approach and the compensator effectiveness consider the following 2 different levels of process parameters:

Level 1: $\omega_n = 5$ rad/s and $\zeta = 0.2$

Level 2: $\omega_n = 15$ rad/s and $\zeta = 0.8$

Compensator parameters:

$$K_{pc} = 0.005025 \quad \text{independent of process parameters levels.}$$

$$\text{Level 1: } K_d = 0.28502 \quad (\text{exact})$$

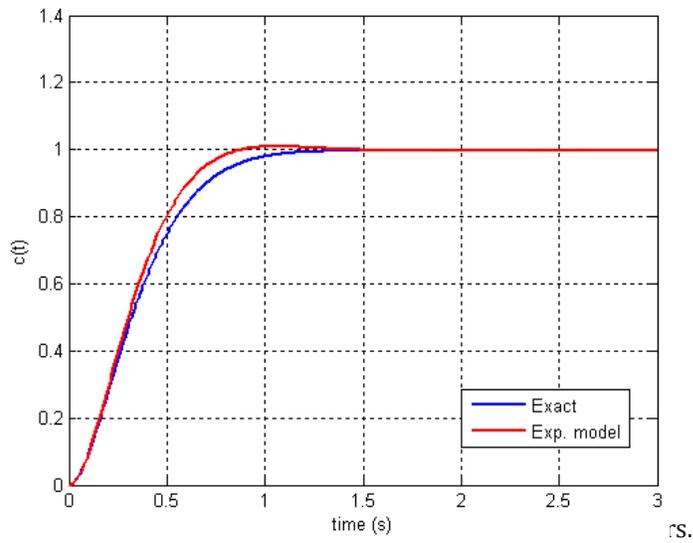
$$K_d = 0.24310 \quad (\text{model})$$

$$\text{Level 2: } K_d = 0.015009 \quad (\text{exact})$$

$$K_d = 0.022800 \quad (\text{model})$$

Control system step response:

Level 1: Fig.2 shown the unit step response of the control system.



Level 2: Fig.3 shown the unit step response of the control system.

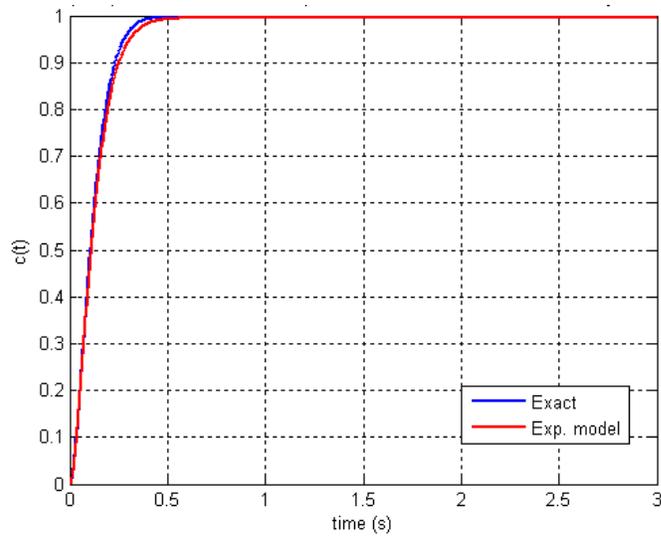


Fig.3 Unit step response with level 2 process parameters.

Performance parameters:

Level 1:

Using exact tuning parameters:

$$T_s = 0.8152 \text{ s}$$

$$OS_{\max} = 0.1 \quad \%$$

Using model tuning parameter:

$$T_s = 0.6819 \text{ s}$$

$$OS_{\max} = 1.39 \quad \%$$

Level 2:

Using exact tuning parameters:

$$T_s = 0.2717 \text{ s}$$

$$OS_{\max} = 0.1 \quad \%$$

Using model tuning parameter:

$$T_s = 0.2998 \text{ s}$$

$$OS_{\max} = 0 \quad \%$$

5 Conclusions

- The suggested suggested feedback PD compensator is suitable for controlling second-order underdamped processes.
- Through using the proposed tuning technique, it was possible to reduce the tuning process of the compensator to a constant value for the proportional gain and a derivative gain function of the process parameters and defined by an exponential model.
- The compensator was capable of eliminating the oscillating characteristic of underdamped second-order like processes.
- The fitted model of the derivative gain had a large multiple correlation coefficient measuring the reliability of the model without need to run an optimization process online during process control.

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