

Maryam Mirzakhani (1977-2017)

A profound thinker on the mathematics of abstract surfaces

On 14 July, Maryam Mirzakhani, a luminary in pure mathematics, died of cancer at the age of 40. Her achievements had been most recently honored in 2014 by the Fields Medal, the most prestigious award in mathematics.

Born in Tehran, Mirzakhani set a record as the first Iranian to earn two gold medals in the International Mathematical Olympiad, before studying mathematics at the Sharif University of Technology. Her early success earned her a spot in Harvard University's PhD program, where she wrote a thesis so exceptional that it yielded publications in three of mathematics' most prestigious journals. She joined Stanford University's faculty of mathematics in 2009, after working as a Clay Research Fellow and professor at Princeton University.

Mirzakhani studied the geometry and dynamics of surfaces. Imagine a surface with more than one hole, such as the surface of a pretzel. Looking closely, you will see that the curvature varies: At some points, the surface bulges outward, and at others it is shaped like a saddle. In the same way that one might shape a lump of clay into a perfect sphere, it is possible to bend a surface with holes so that it is uniformly curved at every point. However, this bending cannot be accomplished so easily. The result is a puzzle that can be understood mathematically, but the saddle-shaped pieces cannot be put together in three-dimensional reality.

There are many possible uniformly curved shapes into which the surface can be bent. These shapes are called hyperbolic metrics, and they exhibit the non-Euclidean geometry discovered in the 1800s after 2000 years of attempts to prove its nonexistence. The plethora of all possible hyperbolic metrics on a surface populate a universe called a moduli space, which is itself a higher-dimensional version of a surface, and is one of the focal points of modern mathematics and theoretical physics.

Mirzakhani's work illuminates the geometry of moduli spaces. In her thesis, supervised by Curtis McMullen, she calculated their volume--that is, she could understand the size of the universe of possible geometries. Building on that calculation, she gave a new proof of Witten's celebrated conjecture relating to the theory of quantum gravity, and leveraged her insights into the space of all possible metrics to give precise information on each individual metric. For example, she calculated the number of loops on each surface that have been pulled taut, like a rubber band, but do not cross over themselves.

More recently, with Alex Eskin and Amir Mohammadi, Mirzakhani considered surfaces that can be built from flat puzzle pieces, instead of saddle-shaped ones. Again, these surfaces do not fit into our reality, but they appear uninvited in the general theory of surfaces and also in the study of physical problems such as the path of a laser beam in a mirrored room. An explorer braving such a surface would find their compass works perfectly well (except at a handful of perplexing locations). Adding to the mystery and power of these surfaces, the mathematician can help the explorer traveling north by warping the geometry so that north-south distances are decreased and east-west distances are proportionately increased. For decades, mathematicians had tried to understand the geometries resulting from such stretching and shearing, fearing that perhaps the answer might be so intricate and complicated as to be beyond human understanding. In a stunning achievement spanning hundreds of pages, Eskin, Mirzakhani, and Mohammadi showed that instead the answers are so tame that

mathematicians can hope to detail every possibility. Some call the result the “magic wand theorem” because of its seemingly endless applicability to more concrete problems, such as the path of a billiard ball moving in a polygonal table.

In defiance of the (sound) advice given to most young researchers, Maryam declared that she wished to “avoid the low hanging fruit,” instead finding ambitious problems she could ponder for years at a time. Her unflinching eagerness to take on the most daunting problems, and her great success, was all the more notable given her unfailingly good-natured and humble personality. In casual conversation she came across at first as any other friendly and perhaps slightly shy mathematician would, and professionally she seemed to neither expect nor desire any special recognition.

Mathematicians working in pairs often fall into “good cop, bad cop” roles, in which the more optimistic one proposes daring plans and dreams of success, and the more grounded one tempers this optimism by looking for pitfalls and providing an ongoing reality check. Mirzakhani was always the good cop. During lunch breaks separating multi-hour sessions at the blackboard, she would joke that the true time to complete a project could be obtained by doubling the estimate and then changing the units, from days to weeks, weeks to months, or even months to years. Happy to put mathematics aside and speak as a friend, she asked about the lives of her dining companions, and often spoke of the challenges and joys of being a parent.

To a young researcher such as myself, Mirzakhani had a calming influence, rising above the churn of academic politics and career pressures. Although sympathetic to issues involving women in mathematics and generous in her encouragement of aspiring mathematicians, she resisted calls to play a greater political role as a role model and international figure. She wanted nothing more than to think deeply about mathematics. In so doing, she inspired countless others. Her colleagues will miss her dearly.