A Tenacious Explorer of Abstract Surfaces

Maryam Mirzakhani's monumental work draws deep connections between topology, geometry and dynamical systems.

By Erica Klarreich

As an 8-year-old, Maryam Mirzakhani used to tell herself stories about the exploits of a remarkable girl. Every night at bedtime, her heroine would become mayor, travel the world or fulfill some other grand destiny.

Today, Mirzakhani — a 37-year-old mathematics professor at Stanford University — still writes elaborate stories in her mind. The high ambitions haven't changed, but the protagonists have: They are hyperbolic surfaces, moduli spaces and dynamical systems. In a way, she said, mathematics research feels like writing a novel. "There are different characters, and you are getting to know them better," she said. "Things evolve, and then you look back at a character, and it's completely different from your first impression."

The Iranian mathematician follows her characters wherever they take her, along story lines that often take years to unfold. Petite but indomitable, Mirzakhani has a reputation among mathematicians for tackling the most difficult questions in her field with dogged persistence. "She has a fearless ambition when it comes to mathematics," said Curtis McMullen of Harvard University, who was Mirzakhani's doctoral adviser.

With her low voice and steady, gray-blue eyes, Mirzakhani projects an unwavering self-confidence. She has an equal tendency, however, toward humility. Asked to describe her contribution to a particular research problem, she laughed, hesitated and finally said: "To be honest, I don't think I've had a very huge contribution." And when an email arrived in February saying that she would receive what is widely regarded as the highest honor in mathematics — the Fields Medal, which will be awarded today at the International Congress of Mathematicians in Seoul, South Korea — she assumed that the account from which the email was sent had been hacked.

Other mathematicians, however, describe Mirzakhani's work in glowing terms. Her doctoral dissertation — about counting loops on surfaces that have "hyperbolic" geometry — was "truly spectacular," said Alex Eskin, a mathematician at the University of Chicago who has collaborated with Mirzakhani. "It's the kind of mathematics you immediately recognize belongs in a textbook."

Mirzakhani grew up in Iran and was at first more interested in reading and writing fiction than doing mathematics.

And one of Mirzakhani's more recent contributions — a monumental collaboration with Eskin about the dynamics of abstract surfaces connected to billiard tables — is "probably the theorem of the decade" in Mirzakhani's highly competitive field, said Benson Farb, also a University of Chicago mathematician.

Tehran

As a child growing up in Tehran, Mirzakhani had no intention of becoming a mathematician. Her chief goal was simply to read every book she could find. She also watched television biographies of famous women such as Marie Curie and Helen Keller, and later read "Lust for Life," a novel about Vincent van Gogh. These stories instilled in her an undefined ambition to do something great with her life — become a writer, perhaps.

Mirzakhani finished elementary school just as the Iran-Iraq war was drawing to a close and opportunities were opening up for motivated students. She took a placement test that secured her a spot at the Farzanegan middle school for girls in Tehran, which is administered by Iran's National Organization for Development of Exceptional Talents. "I think I was the lucky generation," she said. "I was a teenager when things got more stable."

In her first week at the new school, she made a lifelong friend, Roya Beheshti, who is now a mathematics professor at Washington University in St. Louis. As children, the two explored the bookstores that lined the crowded commercial street near their school. Browsing was discouraged, so they randomly chose books to buy. "Now, it sounds very strange," Mirzakhani said. "But books were very cheap, so we would just buy them."

To her dismay, Mirzakhani did poorly in her mathematics class that year. Her math teacher didn't think she was particularly talented, which undermined her confidence. At that age, "it's so important what others see in you," Mirzakhani said. "I lost my interest in math."

The following year, Mirzakhani had a more encouraging teacher, however, and her performance improved enormously. "Starting from the second year, she was a star," Beheshti said.

Mirzakhani went on to the Farzanegan high school for girls. There, she and Beheshti got hold of the questions from that year's national competition to determine which high school students would go to the International Olympiad in Informatics, an annual programming competition for high school students. Mirzakhani and Beheshti worked on the problems for several days and managed to solve three out of six. Even though students at the competition must complete the exam in three hours, Mirzakhani was excited to be able to do any problems at all.

Eager to discover what they were capable of in similar competitions, Mirzakhani and Beheshti went to the principal of their school and demanded that she arrange for math problem-solving classes like the ones being taught at the comparable high school for boys. "The principal of the school was a very strong character," Mirzakhani recalled. "If we really wanted something, she would make it happen." The principal was undeterred by the fact that Iran's International Mathematical Olympiad team had never fielded a girl, Mirzakhani said. "Her mindset was very positive and upbeat — that 'you can do it, even though you'll be the first one,' " Mirzakhani said. "I think that has influenced my life quite a lot."

In 1994, when Mirzakhani was 17, she and Beheshti made the Iranian math Olympiad team. Mirzakhani's score on the Olympiad test earned her a gold medal. The following year, she returned and achieved a perfect score. Having entered the competitions to discover what she could do, Mirzakhani emerged with a deep love of mathematics. "You have to spend some energy and effort to see the beauty of math," she said.

Mirzakhani with her parents during a visit to Isfahan, Iran.

Even today, said Anton Zorich of the Université Paris Diderot-Paris 7 in France, Mirzakhani gives "the impression of a 17-year-old girl who is absolutely excited by all the mathematics that happens around her."

Harvard

Gold medals at the mathematical Olympiad don't always translate into success in mathematics research, McMullen observed. "In these contests, someone has carefully crafted a problem with a clever solution, but in research, maybe the problem doesn't have a solution at all." Unlike many Olympiad high-scorers, he said, Mirzakhani "has the ability to generate her own vision."

After completing an undergraduate degree in mathematics at Sharif University in Tehran in 1999, Mirzakhani went to graduate school at Harvard University, where she started attending McMullen's seminar. At first, she didn't understand much of what he was talking about but was captivated by the beauty of the subject, hyperbolic geometry. She started going to McMullen's office and peppering him with questions, scribbling down notes in Farsi.

“She had a sort of daring imagination,” recalled McMullen, a 1998 Fields medalist. “She would formulate in her mind an imaginary picture of what must be going on, then come to my office and describe it. At the end, she would turn to me and say, ‘Is it right?’ I was always very flattered that she thought I would know.”

Mirzakhani became fascinated with hyperbolic surfaces — doughnut-shaped surfaces with two or more holes that have a non-standard geometry which, roughly speaking, gives each point on the surface a saddle shape. Hyperbolic doughnuts can’t be constructed in ordinary space; they exist in an abstract sense, in which distances and angles are measured according to a particular set of equations. An imaginary creature living on a surface governed by such equations would experience each point as a saddle point.

It turns out that each many-holed doughnut can be given a hyperbolic structure in infinitely many ways — with fat doughnut rings, narrow ones, or any combination of the two. In the century and a half since such hyperbolic surfaces were discovered, they have become some of the central objects in geometry, with connections to many branches of mathematics and even physics.

But when Mirzakhani started graduate school, some of the simplest questions about such surfaces were unanswered. One concerned straight lines, or “geodesics,” on a hyperbolic surface. Even a curved surface can have a notion of a “straight” line segment: it’s simply the shortest path between two points. On a hyperbolic surface, some geodesics are infinitely long, like straight lines in the plane, but others close up into a loop, like the great circles on a sphere.

The number of closed geodesics of a given length on a hyperbolic surface grows exponentially as the length of the geodesics grows. Most of these geodesics cut across themselves many times before closing up smoothly, but a tiny proportion of them, called “simple” geodesics, never intersect themselves. Simple geodesics are “the key object to unlocking the structure and geometry of the whole surface,” Farb said.

Yet mathematicians couldn’t pin down just how many simple closed geodesics of a given length a hyperbolic surface can have. Among closed geodesic loops, the simple ones are “miracles that [effectively] happen zero percent of the time,” Farb said. For that reason, counting them accurately is incredibly difficult: “If you have a little bit of an error, you’ve missed it,” he said.

In her doctoral thesis, completed in 2004, Mirzakhani answered this question, developing a formula for how the number of simple geodesics of length L grows as L gets larger. Along the way, she built connections to two other major research questions, solving both. One concerned a formula for the volume of the so-called “moduli” space — the set of all possible hyperbolic structures on a given surface. The other was a surprising new proof of an old conjecture proposed by the physicist Edward Witten of the Institute for Advanced Study in Princeton, N.J., about certain topological measurements of moduli spaces related to string theory. Witten’s conjecture is so difficult that the first mathematician to prove it — Maxim Kontsevich of the Institut des Hautes Études Scientifiques, near Paris — was awarded a Fields Medal in 1998 in part for that work.

Farb said that solving each of these problems “would have been an event, and connecting them would have been an event.” Mirzakhani did both.

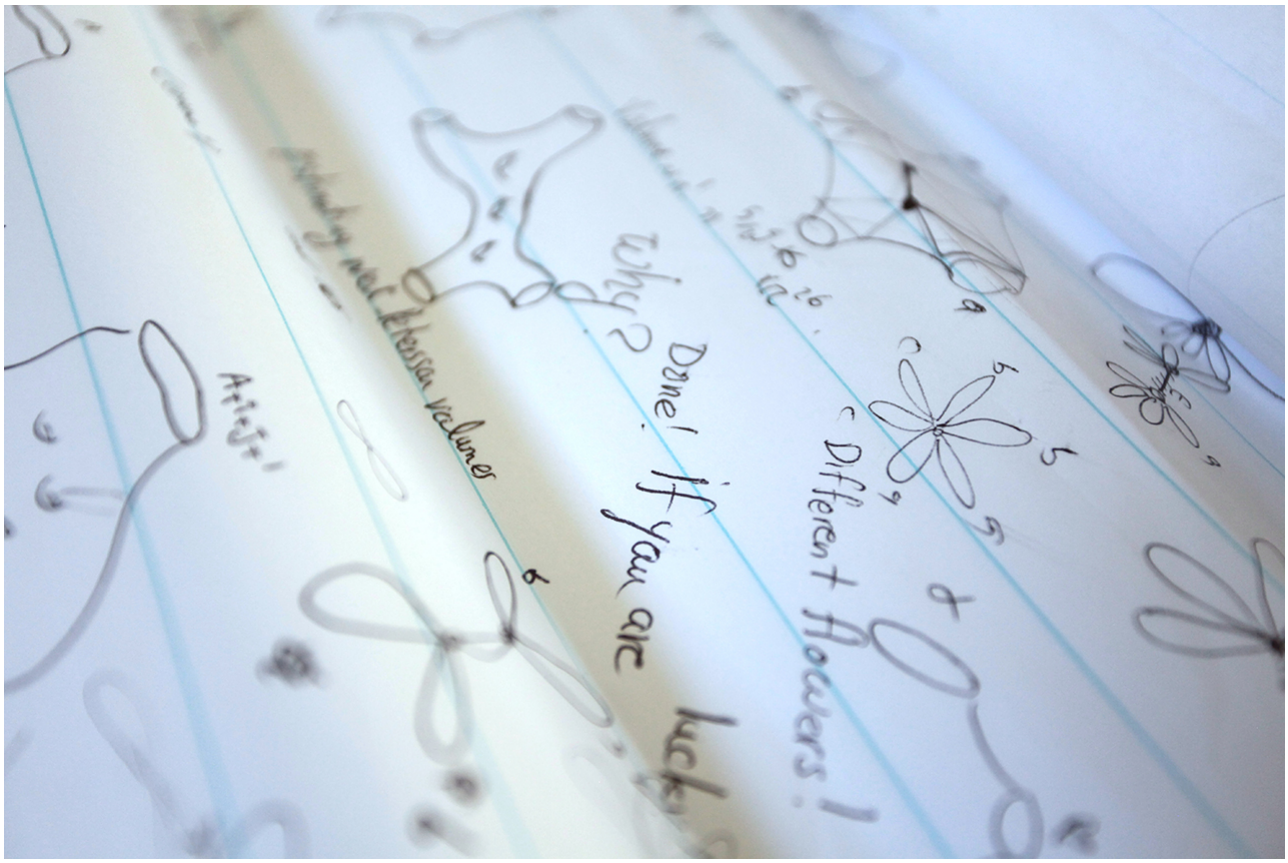
Mirzakhani’s thesis resulted in three papers published in the three top journals of mathematics: Annals of Mathematics, Inventiones Mathematicae and Journal of the American Mathematical Society. The majority of mathematicians will never produce something as good, Farb said — “and that’s what she did in her thesis.”

‘A Titanic Work’

Mirzakhani likes to describe herself as slow. Unlike some mathematicians who solve problems with quicksilver brilliance, she gravitates toward deep problems that she can chew on for years. “Months or years later, you see very different aspects” of a problem, she said. There are problems she has been thinking about for more than a decade. “And still there’s not much I can do about them,” she said.

Mirzakhani doesn’t feel intimidated by mathematicians who knock down one problem after another. “I don’t get easily disappointed,” she said. “I’m quite confident, in some sense.”

Her slow and steady approach also applies to other areas of her life. One day while she was a graduate student at Harvard, her future husband, then a graduate student at the Massachusetts Institute of Technology, learned this lesson about Mirzakhani when the two went for a run. “She’s very petite, and I was in good shape, so I thought I’d do well, and at first, I was ahead,” recalled Jan Vondrak, who is now a theoretical computer scientist at IBM Almaden Research Center in San Jose, Calif. “But she never slows down. After half an hour, I was done, but she was still running at the same pace.”



Mirzakhani, who says she thinks about mathematics in pictures, often doodles her ideas on giant sheets of paper.

As she thinks about mathematics, Mirzakhani constantly doodles, drawing surfaces and other images related to her research. “She has these huge pieces of paper on the floor and spends hours and hours drawing what look to me like the same picture over and over,” Vondrak said, adding that papers and books are scattered haphazardly about her home office. “I have no idea how she can work like this, but it works out in the end,” he said. Perhaps, he speculates, that is because “the problems she is working on are so abstract and complicated, she can’t afford to make logical steps one by one but has to make big jumps.”

Doodling helps her focus, Mirzakhani said. When thinking about a difficult math problem, “you don’t want to write down all the details,” she said. “But the process of drawing something helps you somehow to stay connected.” Mirzakhani said that her 3-year-old daughter, Anahita, often exclaims, “Oh, Mommy is painting again!” when she sees the mathematician drawing. “Maybe she thinks I’m a painter,” Mirzakhani said.

Mirzakhani’s research connects to many areas of mathematics, including differential geometry, complex analysis and dynamical systems. “I like crossing the imaginary boundaries people set up between different fields — it’s very refreshing,” she said. In her area of research, “there are lots of tools, and you don’t know which one would work,” she said. “It’s about being optimistic and trying to connect things.”

Sometimes, the connections that Mirzakhani makes are mind-blowing, McMullen said. In 2006, for example, she tackled the problem of what happens to a hyperbolic surface when its geometry is deformed using a mechanism akin to a strike-slip earthquake. Before Mirzakhani’s work, “this problem was completely unapproachable,” McMullen said. But with a one-line proof, he said, “she constructed a bridge between this completely opaque theory and another theory that’s completely transparent.”

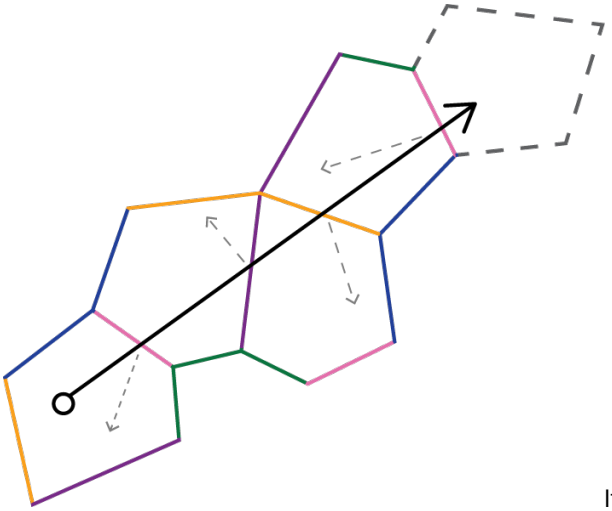
In 2006, Mirzakhani began her fruitful collaboration with Eskin, who considers her one of his favorite collaborators. “She is very optimistic, and that’s infectious,” he said. “When you work with her, you feel you have a much better chance of solving problems that at first seem hopeless.”

After several projects together, Mirzakhani and Eskin decided to tackle one of the largest open problems in their field. It concerned the range of behaviors of a ball that is bouncing around a billiard table shaped like any polygon, provided the angles are a rational number of degrees. Billiards provides some of the simplest examples of dynamical systems — systems that evolve over time according to a given set of rules — but the behavior of the ball has proven unexpectedly hard to pin down.

“Rational billiards got started a century ago, when some physicists were sitting around saying, ‘Let’s understand a billiard ball bouncing in a triangle,’ ” said Alex Wright, a postdoctoral researcher at Stanford. “Presumably, they thought they would be done in a week, but 100 years later, we’re still thinking about it.”

To study a long billiard ball trajectory, a useful approach is to imagine gradually deforming the billiard table by squishing it along the direction of the trajectory so that more of the ball’s path can be seen in a given amount of time. This transforms the original billiard table into a succession of new ones, moving the table around in what mathematicians call the “moduli” space consisting of all possible billiard tables with a given number of sides. By transforming each billiard table into an abstract surface called a “translation surface,” mathematicians can analyze billiard dynamics by understanding the larger moduli space consisting of all translation surfaces. Researchers have shown that understanding the “orbit” of a particular translation surface as the squishing action moves it around in the moduli space helps in answering a host of questions about the original billiard table.

Billiard Ball Trajectories



If you place mirrors on the walls of a billiard table, a ball bouncing off a wall looks as if it is continuing to roll in a straight line in the looking-glass world. Follow this straight path through one looking glass after another as the ball hits more walls, and after a finite number of reflections, you'll be back in a billiard table world that has the exact same orientation as the original table.

If you glue together the sides of this finite succession of billiard table worlds, you end up with a surface — a doughnut with two or more holes — that inherits a flat geometry from the billiard table (except at the handful of points that correspond to the corners of the table). Paths on the original billiard table correspond to straight lines on this surface, called a "translation" surface. Mathematicians have shown that understanding the "moduli space" of all translation surfaces is the key to understanding billiards.

On the face of it, this orbit might be an extremely complicated object — a fractal, for instance. In 2003, however, McMullen showed that this isn't the case when the translation surface is a two-holed ("genus two") doughnut: Every single orbit fills up either the entire space or some simple subset of the space called a submanifold.

McMullen's result was hailed as a huge advance. He recalled that before his paper was published, however, Mirzakhani — then still a graduate student — came to his office and asked, "Why did you just do genus two?"

"That's the kind of person she is," he said. "What she sees hints of, she wants to understand more clearly."

After years of work, in 2012 and 2013, Mirzakhani and Eskin, partly in collaboration with Amir Mohammadi of the University of Texas at Austin, succeeded in generalizing McMullen's result to all doughnut surfaces with more than two holes. Their analysis is "a titanic work," Zorich said, adding that its implications go far beyond billiards. The moduli space "has been intensively studied for the last 30 years," he said, "but there's still so much we don't know about its geometry."

Mirzakhani and Eskin's work is "the beginning of a new era," said Wright, who spent months studying their 172-page paper. "It's as if we were trying to log a redwood forest with a hatchet before, but now they've invented a chain saw," he said. Their work has already been applied — for example, to the problem of understanding the sightlines of a security guard in a complex of mirrored rooms.

In Mirzakhani and Eskin's paper, "under every layer of difficulty and ideas lay another, hidden beneath," Wright wrote in an email. "By the time I got to the center, I was amazed at the machine they had built."

It was Mirzakhani's optimism and tenacity that kept the pair going, Eskin said. "Sometimes there

were setbacks, but she never panicked," he said.

Even Mirzakhani herself is amazed, in retrospect, that the two stuck with it. "If we knew things would be so complicated, I think we would have given up," she said. Then she paused. "I don't know; actually, I don't know," she said. "I don't give up easily."

Next Chapter

Mirzakhani is the first woman to win a Fields Medal. The gender imbalance in mathematics is long-standing and pervasive, and the Fields Medal, in particular, is ill-suited to the career arcs of many female mathematicians. It is restricted to mathematicians younger than 40, focusing on the very years during which many women dial back their careers to raise children.

Mirzakhani feels certain, however, that there will be many more female Fields medalists in the future. "There are really many great female mathematicians doing great things," she said.

In the meantime, while she feels greatly honored to have been awarded a Fields Medal, she has no desire to be the face of women in mathematics, she said. Her ambitious teenage self would have been overjoyed by the award, she said, but today, she is eager to deflect attention from her achievements so she can focus on research.

Mirzakhani has big plans for the next chapters of her mathematical story. She has started working with Wright to try to develop a complete list of the kinds of sets that translation surface orbits can fill up. Such a classification would be a "magic wand" for understanding billiards and translation surfaces, Zorich has written.

It's no small task, but Mirzakhani has learned over the years to think big. "You have to ignore low-hanging fruit, which is a little tricky," she said. "I'm not sure if it's the best way of doing things, actually — you're torturing yourself along the way." But she enjoys it, she said. "Life isn't supposed to be easy."

Thomas Lin contributed reporting from Stanford, Calif.

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