Taylor series expansion of f=cos(x) about x=0.

$$f'(0) = -\sin(0) = 0, \qquad f''(0) = -\cos(0) = -1$$
$$f^{(3)}(0) = \sin(0) = 0, \qquad f^{(4)}(0) = \cos(0) = 1, \; etc.$$

So Taylor series expansion is (as given in Problem 4.10)

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \cdots$$

An m-file that calculates this approximation with n terms is

```
function apx=costaylor(x,n)
%Calculates the Maclaurin series approximaton to cos(x) using the first n
%terms in the expansion.
apx=0;
for i=0:n-1
   apx=apx+(-1)^i*x^(2*i)/factorial(2*i);
end
```

Problem 4.10 asks us to increment n from 1 until the approximate error indicates that we have accuracy to two significant digits for x=pi/3.

We start with

```
>> a1=costaylor(pi/3,1)

a1 =

    1

>> a2=costaylor(pi/3,2)

a2 =

   0.4517
```

With cos(pi/3)=0.5, the true error in a1 is 100% and in a2 it is

```
>> true2=(a2-0.5)/0.5*100
```

true2 =

-9.6623

That is about 9.7%. The approximate error, though is

```
>> aprerror=(a1-a2)/a2*100
```

aprerror =

121.3914%

With one more term we get

```
>> a3=costaylor(pi/3,3)
```

a3 =

0.5018

```
>> true3=(a3-0.5)/0.5*100
```

true3 =

0.3592

```
>> aprerror=(a2-a3)/a3*100
```

aprerror =

-9.9856

So the actual error is only 0.36% while the estimated is 10%.

Finally with a fourth term

```
>> a4=costaylor(pi/3,4)
```

a4 =

0.5000

```
>> true4=(a4-0.5)/0.5*100
```

true4 =

-0.0071

```
>> aprerror=(a3-a4)/a4*100
```

```
aprerror =

    0.3664
```

The true error is less than one percent of one percent, the approximate error is 0.37% so we have at least two significant digits and we stop.

Total error:

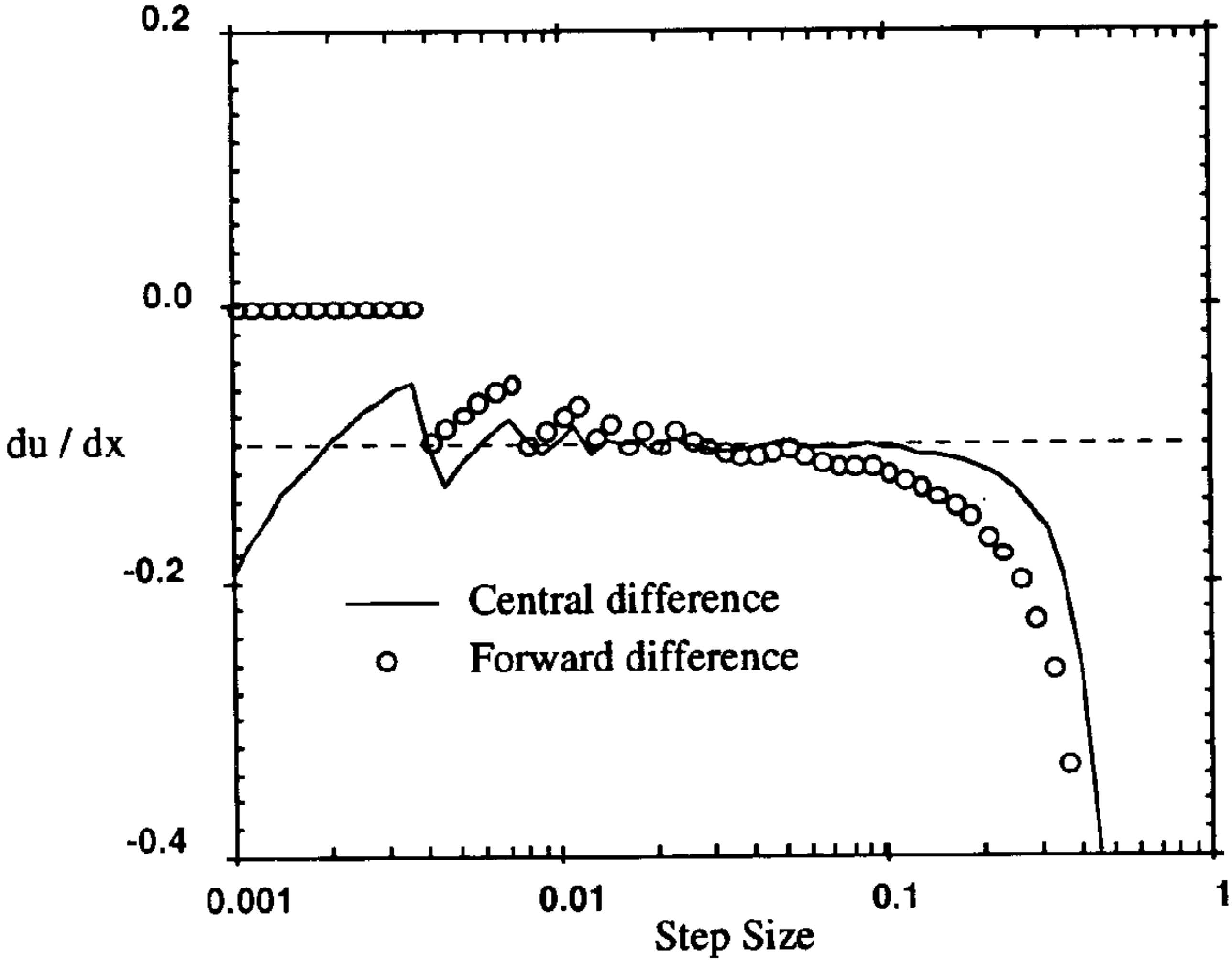
The difficulty of finding a good step size for differentiation is particularly acute when solving ill-conditioned equations. For example,

Consider the system:

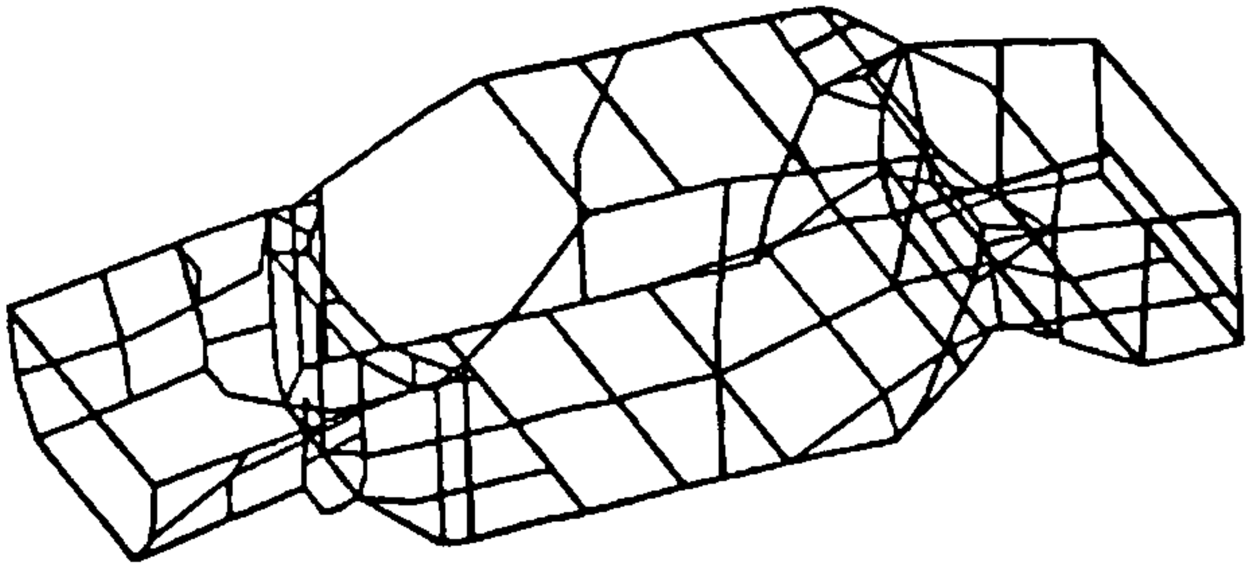
$10001u + xv = 1000$,

$xu + 10000v = 1000$.

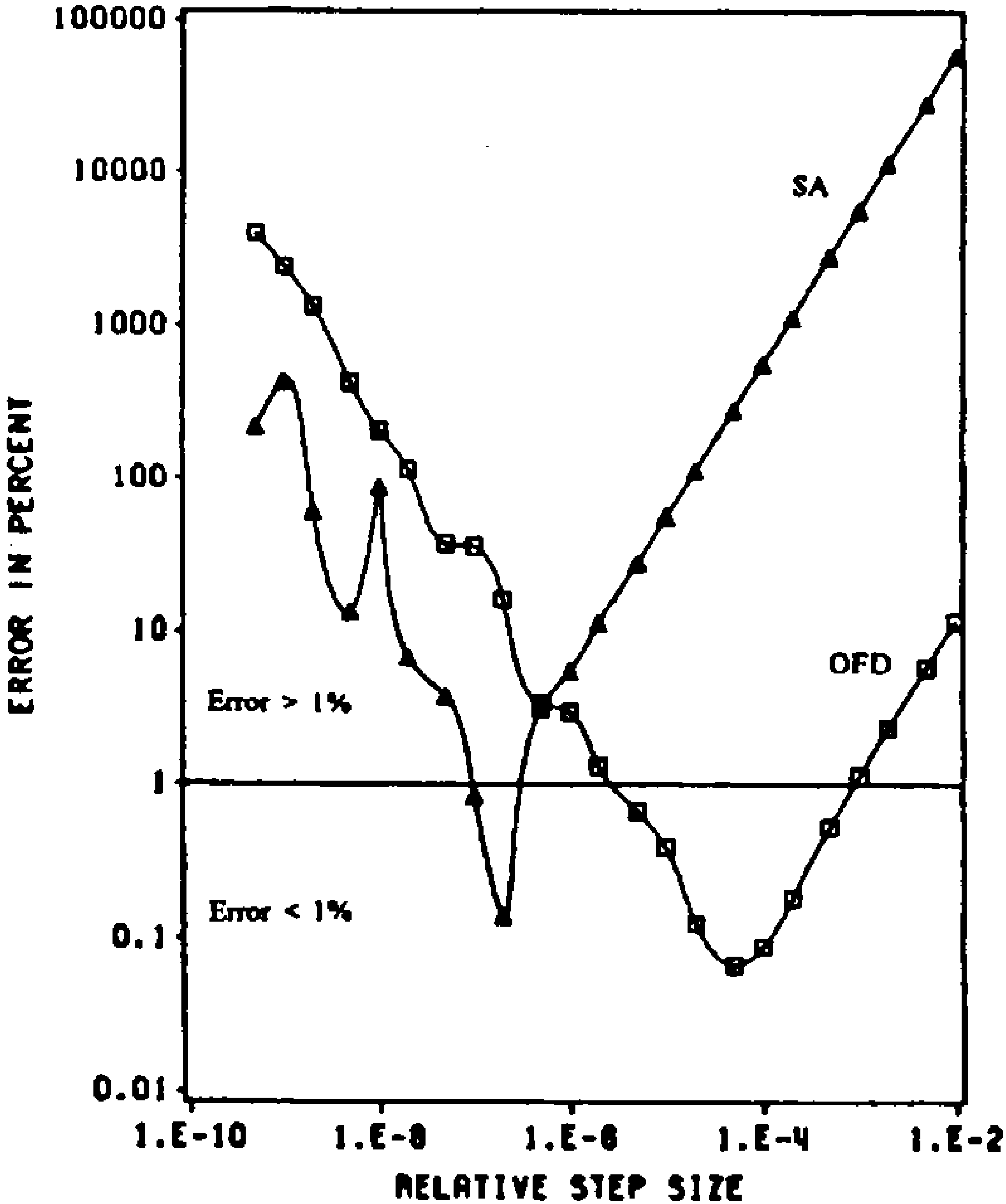
For x=10,000 the determinant is almost zero. The system is ill conditioned, and it is hard to find a good step size for calculating derivatives with respect to x.



One of my graduate students ran into difficulties calculating derivatives of the deformation of a car model seen below



With respect to variables that define the dimensions of the car.


100000
10000
1000
100
10
1
0.1
0.01
ERROR IN PERCENT
1.E-10
1.E-8
1.E-6
1.E-4
1.E-2
RELATIVE STEP SIZE
SA
OFD
Error > 1%
Error < 1%