

# Electrical Chirping induced by Frequency Controlled Chaos in 180nm CMOS

Sai Venkatesh Balasubramanian

*Sree Sai Vidhya Mandhir, Mallasandra, Bengaluru-560109, Karnataka, India.  
saivenkateshbalasubramanian@gmail.com*

---

## Abstract

A simple circuit to generate chaos is proposed, with the central principle being the coupling of two sinusoidal signals with mismatched frequencies to two terminals of an NMOS transistor. Standard chaotic characterization performed using Kolmogorov Entropy and Lyapunov exponents reveal that the output is a frequency controlled chaos. By coupling this output with another sinusoidal input to a second NMOS transistor, a clear segregation of frequencies is observed, analogous to the chirping phenomenon seen typically in optical systems. The nature of chirping is investigated using phase portraits, Lyapunov Exponents and distance maps. The extreme simplicity of the proposed circuit coupled with the new phenomenon of chirping being observed as a result of frequency controlled chaos form the main highlights of the present work.

*Keywords:* Frequency Controlled Chaos, Chaos Generation, Electrical Chirping

---

## 1. Introduction

The present era of information explosion has necessitated a dire need for data security [1, 2, 3, 4]. Of many techniques proposed to increase the security of communication systems at the physical level, the use of chaotic signals and systems stands as the forerunner [5, 6, 7, 8]. However, the design of Chaotic circuits has been known to cause power dissipation due to circuit complexity [9]. Thus, simple circuits that could generate usable chaotic signals in the typical microwave frequency range is the need of the hour.

The present work addresses precisely this need. On a deep submicron VLSI level using 180nm CMOS Technology, it is shown that a single NMOS transistor when driven by two signals with mismatching frequencies is able to generate chaos, whose presence is ascertained using standard characterization techniques such as Kolmogorov Entropy and Lyapunov Exponent. Further study of the effect of driving frequency proves that the frequencies indeed control the nature of the chaos generated, leading to the term ‘Frequency Controlled Chaos’.

Following this, the chaotic output of a single NMOS when coupled with a third driving signal as the inputs to a second NMOS, exhibits a phenomenon very analogous to chirping typically seen in optical systems. This is confirmed by the frequency-time variation graphs, as well as phase portraits and distance plots.

The extreme simplicity of chaos generation, coupled with the observation of electrical chirping in microwave frequencies form the highlights of the present work.

## 2. Frequency Controlled Chaos in 180nm CMOS

As a first step, a single Enhanced N-Channel MOSFET (NMOS) transistor is considered, without any external biasing [10]. The gate and source of the NMOS are connected to sinusoidal signals of 2V each, with frequencies of 2.34GHz and 4.57GHz respectively. The output is taken from the Drain terminal. It is well known that at high operating frequencies, the channel of an NMOS transistor exhibits a non-quasi static charge behavior, acting as a nonlinear transmission line [11]. This effect is captured by representing the channel resistance as an equivalent Elmore Resistance  $Re$ , whose dependence on the Gate Source potential  $V_{gs}$  is given as [11]:

$$Re = \frac{L_{eff}}{10\mu_{eff}W_{eff}C_{ox}(V_{gs} - V_{th})} \quad (1)$$

where  $\mu_{eff}$  is the effective carrier mobility,  $L_{eff}$  and  $W_{eff}$  denote the effective channel length and width of the NMOS transistor,  $C_{ox}$  denotes the oxide layer capacitance and  $V_{th}$  is the threshold voltage of the transistor. Thus, the Elmore Resistance induced channel nonlinearity is an inverse (hyperbolic) nonlinearity.

In order to ascertain the presence of chaos, standard characterizations are done using the following two measures:

Table 1: Effect of Gate to Source Frequency Ratio on the Nature of Chaos

Frequency Ratio	$LLE$	$K_2$
1.1	9.16	8.9815
1.2	10.405	8.9813
1.3	9.84	8.9946
1.4	9.43	8.9547
1.5	9.12	8.8827
1.6	9.2	9.0087
1.7	10.35	9.0077
1.8	9.87	8.9685
1.9	9.03	9.0123

1. The largest Lyapunov Exponent ( $LLE$ ), which is a measure of the output's sensitive dependence on initial conditions, a fundamental property of chaos [12]. Given the divergence samples  $d_j(i)$  between nearest trajectories represented by  $j$  with  $C_j$  being a normalization constant, the Lyapunov Exponent  $\lambda_i$  is found from the following relation [13]:

$$d_j(i) = C_j e^{\lambda_i(i\delta t)} \quad (2)$$

2. Kolmogorov Entropy, a statistical measure of the uncertainty of the signal. By assigning each of the  $N$  quantifiable states of the output amplitude as an event  $i$ , the Kolmogorov Entropy  $K_2$  obtained depends on their probabilities  $p_i$  according to the relation [14]

$$K_2 = - \sum_{i=1}^N p_i \log p_i \quad (3)$$

Keeping the Gate input signal frequency constant at 2.34GHz, the source frequency is varied such that the ratio of source to gate frequencies is altered from 1.1 to 1.9 in steps of 0.1. The implementation is carried out in 180nm CMOS Technology in Microwind, a deep submicron level VLSI layout tool, and the computed values of  $K_2$  and  $LLE$  are tabulated in Table 1 [15].

As seen from the table, it is clear that the ratio of the gate to source driving signal frequencies does indeed have an effect on the nature of the chaos generated and for this reason, it is termed as 'Frequency Controlled Chaos'.

### 3. Microwave Chirping using two Transistors

The chaotic output 's2' of the single NMOS mentioned above is fed to the Gate of a Second NMOS Transistor, with its source connected to a sinusoidal generator of frequency 9.3GHz. The final output 's1' is taken from the drain of the second transistor. The schematic and waveforms of the two outputs 's1' and 's2' are shown in Fig. (1) and Fig. (2) respectively.

As seen from the plots, the second transistor output 's1' exhibits a clear segregation of low and high frequency components respectively into the rise and fall portions of a cycle, an indication of chirping, as typically seen in optical systems[16].

In order to study the nature of chaos in the chirped and unchirped signals, the corresponding phase portraits are plotted in Fig. (3). The phase portrait plots the time derivative of a signal ( $dV/dt$ ) as a function of the signal ( $V$ ) illustrating the dynamics of the signal in the phase space and describing the stability aspects of the chaotic system behavior, qualitatively serving as a tool to assess various chaotic parameters such as sensitivity and ergodicity [17, 18]. As seen from the obtained plots, the chirped signal phase portrait shows much more ornamental pattern and thus ergodicity than the unchirped counterpart. This fact is quantitatively validated by computing the  $LLE$  for the unchirped and chirped signals, which are obtained as 9.03 and 9.67 respectively.

Another useful tool to explore the chaotic nature of the signals 's1' and 's2' is the distance map. For a time series  $x$ , the distance map is a visualization of the two dimensional matrix  $d_{ij}$  where for every pair of samples ( $i, j$ ) in  $x$ , the distance  $d_{ij} = |x_i - x_j|$  [18]. The distance maps for the signals 's1' and 's2' are plotted in Fig. (4).

As seen from the plots, the distance map of the unchirped signal shows a smooth gradient along the off-diagonal axis, with a uniform grid spacing, whereas the chirped signal distance map shows gradients in two directions, corresponding to both main and off diagonals, with heavily distorted grid spacings. These differences are a result of the new asymmetry created due to the segregation of frequencies in the chirped signal.

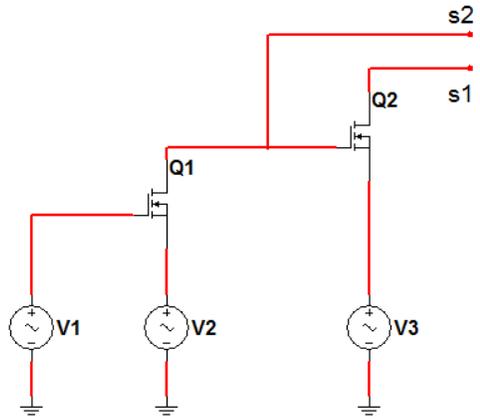


Figure 1: Schematic of two transistor Microwave Chirp Generator

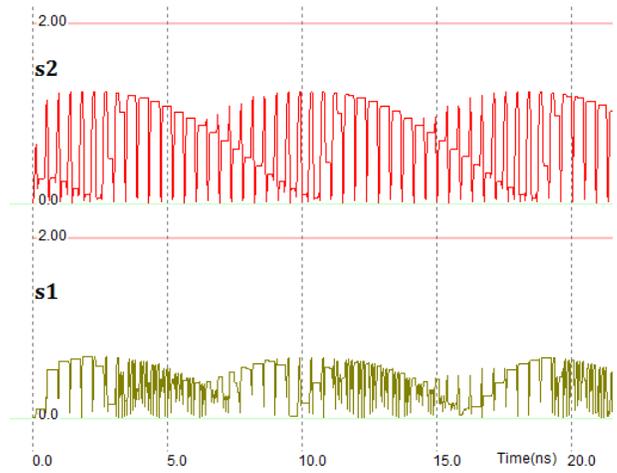


Figure 2: Chirp Generator waveforms of outputs 's2' (Unchirped) and 's1' (Chirped)

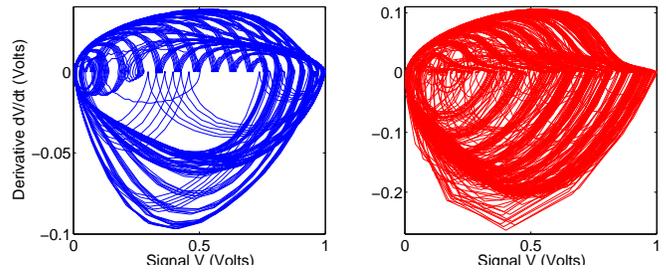


Figure 3: Phase portraits of Unchirped 's2' (left) and Chirped 's1' (right) Signals

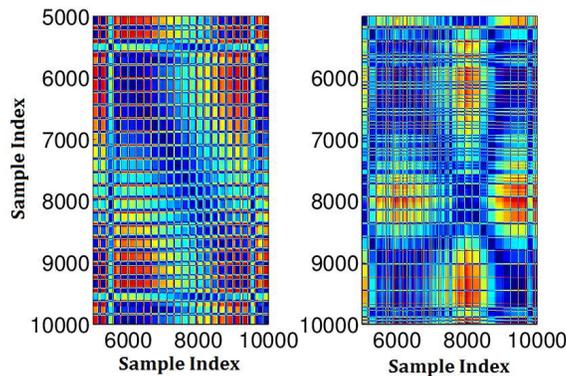


Figure 4: Distance Plots of Unchirped 's2' (left) and Chirped 's1' (right) Signals

#### 4. Conclusion

A simple circuit for the generation of chaos, by driving the two terminals of an NMOS with two signals of mismatched frequencies is demonstrated in 180nm CMOS Technology. The effect of the driving signals on the nature of chaos generated is studied using Kolmogorov Entropy and Lyapunov Exponent. Following this, the generated chaos is fed as an input, along with a third sinusoidal source, to a second NMOS transistor. The segregation of frequencies, leading to the phenomenon of chirping is observed, and this is studied using phase portraits and distance maps. The extreme simplicity of the proposed design coupled with the observation of microwave based chirping as a result of frequency controlled chaos form the highlights of the present work.

#### References

- [1] M. Hilbert, *How much of the global information and communication explosion is driven by more, and how much by better technology?*, Wiley Journal of the Association for Information Science and Technology, **65**, 856-861 (2014).
- [2] G. B. Giannakis, F. Bach, R. Cendrillon, M. Mahoney, J. Neville, *Signal Processing for Big Data*, IEEE Signal Processing Magazine, **31**, 15-16 (2014).
- [3] X.Wu, X.Zhu, G.Q.Wu and W.Ding, *Data mining with big data*, IEEE Trans. on Knowledge and Data Engineering **26**,97-107 (2014).
- [4] A.McEwan and H.Cassimally, *Designing the Internet of Things*, (Wiley, 2013).
- [5] P. H. Mahajan, P. B. Bhalerao, *A Review of Digital Watermarking Strategies*, International Journal of Advanced Research in Computer Science And Management Studies, **7** (2014).
- [6] L. Shujun, M. Xuanqin and C. Yuanlong, *Pseudo-random Bit Generator Based on Couple Chaotic Systems and Its Applications in Stream-Cipher Cryptography*, Springer, **2247**, 316-329 (2001).
- [7] A.Uchida, K. Amano, M. Inoue, K. Hirano, S. Naito, H. Someya, I. Oowada, T. Kurashige, M. Shiki, S. Yoshimori, K. Yoshimura, and P. Davis, *Fast physical random bit generation with chaotic semiconductor lasers*, Nature Photonics, **2**, 728-732 (2008).
- [8] V. Patidar , K. K. Sud and N. K. Pareek, *A Pseudo Random Bit Generator Based on Chaotic Logistic Map and its Statistical Testing*, Informatica, **33**, 441-452 (2009).
- [9] E.Bilotta and P.Pantano, *A gallery of Chua attractors*, (World Scientific, Singapore, 2008).
- [10] B. Razavi, *RF Microelectronics*, (Prentice Hall, US, 2011).
- [11] M.Chan, K.Hui, C.Hu and P.K.Ko. *A robust and physical BSIM3 non quasi static transient and AC small signal model for circuit simulation*. IEEE Transactions on Electron Devices. **45**, 834 (1998).
- [12] James, R.G., Burke, K., Crutchfield, J.P.: 'Chaos forgets and remembers: Measuring information creation, destruction, and storage', *Int. J Bifurcation Chaos.*, 2014, **378**, pp.2124-2127.
- [13] Rosenstein, M.T., Collins, J.J., De Luca, C.J.: 'A practical method for calculating largest Lyapunov exponents from small data sets', *Physica D.*, 1993, **65**, pp.117-134.
- [14] Maragos, P., Maragos, F.K.Sun., Petros., Fang-Kuo Sun.: 'Measuring the fractal dimension of signals: morphological covers and iterative optimization', *IEEE Trans. Signal Processing.*, 1993, **41**, pp.108-121.
- [15] J.P.Uyemura. *Chip Design for Submicron VLSI: CMOS Layout and Simulation*. USA: Thomson/Nelson, (2006).
- [16] Y.S.Kivshar and G.Agrawal. *Optical Solitons: From Fibers to Photonic Crystals*. USA: Academic Press (2013).
- [17] S. H. Strogatz, *Nonlinear Dynamics and Chaos*,(Westview Press, 2014).
- [18] J. M. T. Thompson and H. B. Stewart, *Nonlinear Dynamics and Chaos* (Wiley,UK, [2002]).