18.515 Mathematical Logic Spring 2010

Meetings: Tuesday & Thursday 2:30–4:00PM in 2-142 Instructor: Cameron Freer freer@math.mit.edu Website: http://math.mit.edu/18.515 Prerequisites: Permission of instructor Units: 3-0-9 H-Level Graduate Credit



Soare, Automorphisms of the Lattice of Recursively Enumerable Sets



Kechris, Classical Descriptive Set Theory



Sacks, Saturated Model Theory

This course will provide a graduate-level introduction to mathematical logic, with a strong focus on several mathematical applications. No prior knowledge of mathematical logic is assumed, but some mathematical sophistication and knowledge of abstract algebra (at the level of 18.702) will be helpful. The main topics are

- First-order Logic and Model Theory;
- Computability Theory and Complexity Theory;
- Incompleteness and Undecidability; and
- Set Theory.

Additional topics include

- applications of model theory to graph theory (Ramsey's theorem; zero-one laws), algebraic geometry (Ax's theorem; Nullstellensatz), and real algebraic geometry (Artin-Schreier; Hilbert's 17th problem);
- the computational complexity of decision procedures (Fischer-Rabin);
- results in computability theory and set theory that have analogues in complexity theory (Kleene-Post and Ladner's theorems; Schröder-Bernstein theorem, Myhill's Isomorphism theorem, and the Isomorphism Conjecture); and
- how certain incompleteness results can be viewed as informal paradoxes that were turned into mathematical theorems (Santa Claus paradox and Löb's theorem; Berry paradox and Gödel's incompleteness via Kolmogorov complexity; Yablo's paradox and Σ_1 -soundness of PA_{ω}).