Systemic Risk and the Macroeconomy: An Empirical Evaluation*

Stefano Giglio† Bryan Kelly† Seth Pruitt‡ Xiao Qiao†

†University of Chicago Booth School of Business
‡Federal Reserve Board

This Draft: March 2013
First Draft: September 2012

Abstract

We propose a criterion to evaluate the empirical relevance of systemic risk measures based on their ability to predict low quantiles of real macroeconomic aggregates. We also propose and evaluate methodologies for constructing systemic risk indices that capture the joint information content of a large cross-section of systemic risk measures. We construct over 20 measures of systemic risk in the US and Europe extending across several decades. We show that, taken individually, these measures reveal low predictive ability for macroeconomic downturns. However, an index that parsimoniously aggregates individual measures consistently performs well in forecasting downturns both in-sample and out-of-sample.

*We thank Lars Hansen for many helpful conversations about this project. We also thank John Cochrane, Doug Diamond, Amir Sufi, Amit Seru, and seminar participants at University of Chicago (Booth) for helpful comments. We thank Christian Brownlees, Mark Flood, and Andrew Lo for sharing Matlab code.

1 Introduction

The financial crisis of 2007-2009 prompted a profusion of newly proposed measures of systemic risk. Individual measures have been explored in separate papers, but there has been no empirical analysis comparing them as a collection.[1] In this paper we have three complementary objectives for establishing an empirical understanding of the compendium of systemic risk measures.

Our first goal is to provide a basic quantitative description of systemic risk measures. We examine 18 previously proposed measures of systemic risk that we are able to construct from US data, and 12 measures constructed from an international sample. In building these measures, we use the longest possible data history, which in most cases allows us to extend the measures to the 1960's or earlier for the US, and to the 1970's for international data. To the extent that systemically risky episodes are rarely observed phenomena, our longer time series may help provide new empirical insights over several business cycles, in contrast to the literature's emphasis on systemic risk behavior in the last five years. We study the extent to which different measures comove and evaluate which measures behave as contemporaneous indicators of distress in the financial sector and which may be viewed as leading indicators.

The empirical results show that all of these measures, besides showing a large spike during the recent financial crisis (2007-2009), give several extreme readings during the last century. For example, CoVaR (Adrian and Brunnermeier (2011)), which is available since 1926, shows large movements during the Great Depression as well as in 1945, 1973 and 1987. Looking at the time series of all measures, one notices that for all but a few episodes, their movements appear largely idiosyncratic. A broader historical context can be valuable in identifying false positives in addition to genuine crises and, indeed, there are many instances in which individual systemic risk measures spike in the absence of macroeconomic turmoil.

Our second objective is to provide a macroeconomic criterion for evaluating the policy-relevance of systemic risk measures. For a systemic risk measure to be informative for regulation or policy-making, it should be demonstrably associated with welfare. Such an association is obviously difficult to quantify, so to empirically operationalize our criterion we propose testing whether a given systemic risk measure has predictive power for real economic activity.[2]We believe this criterion improves our understanding of systemic risk in

[1]Bisias et al. (2012) provide an excellent survey of systemic risk measures. Their overview is qualitative in nature – they collect detailed definitions of their surveyed measures. Our analysis is empirical and quantitative.

[2]An association with macroeconomic variables is not a necessary characteristic of a systemic risk measure

two dimensions. First, it leads us to a much needed description of linkages between proposed measures of financial sector stress and macroeconomic outcomes. Second, it provides a new tool for evaluating policy relevance when selecting among a large pool of systemic risk measures.

Guided by theories that hypothesize a non-linear link between financial sector stress and adverse realizations of macroeconomic activity, our empirical analysis focuses on predictive quantile regression to estimate the association between systemic risk and the downside distribution of real economic outcomes. The notion that financial distress impacts real outcomes through capital, credit and/or liquidity contraction has been developed in a rich theoretical literature.[3] Exemplified by He and Krishnamurthy (2012), these theories typically predict that the effect of distress on real outcomes is highly non-linear. The advantage of quantile regression is that it avoids merely modeling conditional means, whose variation may be dominated by forces distinct from systemic risk most of the time, and instead provides a more complete picture of a target macroeconomic variable's downside conditional distribution.

In our US sample, we attempt to forecast quantiles of economic activity, measured by the Chicago Fed National Activity Index (CFNAI) and its subcomponents, using individual systemic risk measures. We find that few measures possess predictive power for adverse realizations of macro aggregates. Those that do work tend to work for some macro aggregates but not for others. Nearly all measures tend to miss the large negative downturn caused by the recent financial crisis. Exceptions include measures of financial sector equity volatility (realized volatility and turbulence) and the TED spread, both of which display moderate predictive success. We then extend our analysis to international data, evaluating how well different measures of systemic risk forecast quantiles of unemployment and industrial production growth outside the US. Consistent with results obtained for the US, we find that very few measures possess significant predictive power for large negative macro shocks.

Our third objective connects the previous two aspects of our analysis and asks whether data reduction techniques, which aggregate information about systemic risk that may be dispersed across a large number of measures, offer an improvement in the ability to detect

per se. Surely, the ability to measure financial sector stress is relevant for a wide range of economic pursuits, even if that stress were to never impact real outcomes. We argue that an association between systemic risk measures and the macroeconomy is crucial when considering its role for policy.

[3]See, for example, Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), Brunnermeier and Sannikov (2010), Gertler and Kiyotaki (2010), Mendoza (2010), He and Krishnamurthy (2012).

an association between systemic risk measures and the macroeconomy. We pose the following problem. Suppose all systemic risk measures are imperfectly measured versions of a true, unobserved systemic risk factor. Furthermore, suppose that future low quantiles of macroeconomic outcome distributions are especially dependent upon the unobserved factor. How may we identify this true latent factor that is associated with measured financial sector systemic risk and *also* predicts downside risk in the real economy?

We provide two solutions to this problem. The first approach, which we call principal components quantile regression (PCQR), is a two step procedure that first reduces the panel of systemic risk measures to a small number of factors via principal component analysis, and then uses these factors to predict macro quantiles. This first stage dimension reduction extracts common information that is spread over a number of mis-measured predictors. This effectively averages out the idiosyncratic errors of individual series when the number of predictors is large, zeroing in on the true underlying systemic risk factor. We prove that this approach generates consistent quantile forecasts when the cross section of systemic risk measures is driven by the same factors influencing low quantiles of the target macro series.[4]

Suppose, instead, that systemic risk measures are driven not only by variables associated with adverse macro outcomes, but also by other common factors that may be independent of the real economy. If these additional common factors are a dominant source of variation among predictor variables, then they will manifest in the extracted principal components and result in a misspecified quantile regression for the macroeconomic target. This situation is considered by Kelly and Pruitt (2012), who argue that PCA-based forecasts can suffer severe small sample difficulties when there are prominent "target-irrelevant" factors driving the cross section of predictors. For least squares, Kelly and Pruitt show that this difficulty can be overcome using the method of partial least squares (PLS; Wold (1975)).

We extend PLS to the quantile regression setting in a method called partial quantile regression (PQR), which is our second solution to the many-predictor quantile regression problem. PQR is a three stage estimation technique. The first stage runs separate univariate quantile regressions of the target variable on each predictor in the panel. The second stage builds a weighted average of predictors where the first-stage quantile coefficients serve as weights. This average places larger emphasis on the best quantile predictors, and

[4]The use of PCA to aggregate information among a large number of predictor variables is well-understood for least squares forecasting (see Stock and Watson (2002) and Bai and Ng (2006)), and the use of PCA factors as dependent variables in quantile regression has been proposed by Ando and Tsay (2011). To the best of our knowledge, this paper is the first to derive the asymptotic properties of quantile regressions using principal component factors.

de-emphasizes those with low univariate predictive ability. The final step runs a quantile regression of the macro target variable on the second-stage average of predictors.

We prove that PQR produces consistent quantile forecasts, even when there are pervasive, target-irrelevant factors among systemic risk measures.[5] The key difference between PQR and PCQR is their method of dimension reduction. PQR condenses the predictors according to their quantile covariation with the forecast target, thus choosing a linear combination of predictors that is optimal for quantile forecasting. On the other hand, PCA condenses the cross section according to covariance within the predictors, disregarding how closely each predictor relates to the target. Taking the forecast target into account in the dimension reduction stage allows PQR to isolate target-relevant factors and disregard any common factors that are ineffective in forecasting.[6] Both solutions, PCQR and PQR, may be viewed as procedures for condensing large numbers of measures into a leading systemic risk indicator.

Aggregating information across various measures of systemic risk significantly improves macroeconomic quantile predictions, not only in-sample but also out-of-sample, compared to univariate quantile forecasts or multiple quantile regression. The improvement is visible in the prediction of multiple post-war downturns observed, including the recent financial crisis. Based on out-of-sample forecasts for the 20th percentile of CFNAI one quarter ahead, PQR achieves an improvement of 9% relative to the historical quantile benchmark. For employment (EUH) or consumption (PH) subcomponents of the CFNAI, the gains are 10% and 13%, respectively.

Principal components forecasts also perform well when at least two factors are extracted and used to forecast the CFNAI 20th percentile. Like PQR, these improve over the benchmark forecast by 9%. A single PCQR factor also outperforms the benchmark model but insignificantly so. Interestingly, neither PCQR nor PQR consistently outperforms the benchmark in median forecasts of CFNAI. We thus conclude that systemic risk measures are specifically informative for prediction the downside distribution of real activity, though only

[5]We consider two versions of PQR. The first, which is a direct quantile regression analogue of PLS, is biased due to errors-in-variables in first-stage quantile regressions. We propose a minor modification for the first stage that uses orthogonal quantile regression (He and Liang (2000) and Liang and Li (2009)) to correct errors-in-variables bias. This version, which we refer to as cPQR ("consistent PQR"), is consistent and asymptotically normal. Our empirical tests show that macro quantile forecasts based on PQR are qualitatively identical, and numerically very similar, to those using cPQR. We continue to consider PQR throughout our analysis since it is less computationally intensive, and easily implementable using any software package that includes quantile regression. In contrast, cPQR must be programmed manually.

[6]Dodge and Whittaker (2009) propose a version of PQR, though provide no analysis of its properties.

when many systemic risk measures are aggregated using our proposed dimension reduction methods. Lastly, we run quantile predictive tests for industrial production growth in the UK and Europe. These results are consistent with the US tests. At the 20th percentile, PQR and PCQR (using two PC factors) achieve improvements of 8% and 10%, respectively, over the historical quantile benchmark.

The remainder of the paper proceeds as follows. Section 2 defines and provides a quantitative description of the set of systemic risk measures that we are able to construct for the US and Europe. In Section 3, we examine the power of these measures for predicting low quantiles of real macroeconomic aggregates using standard univariate and multivariate quantile regression. In Section 4, we define PCQR and PQR and prove their attractive asymptotic properties. We then demonstrate empirically how these methods can be used to form leading systemic risk indicators with robust predictive power for macroeconomic downside risk.

2 A Quantitative Survey of Systemic Risk Measures

This section outlines our construction of systemic risk measures that have been proposed in the literature, extending the time series as far back as possible (sometimes to the 1920's). We also construct as many measures as possible for Europe and the UK. In this section we analyze static and dynamic relationships between the systemic risk measures, including a decomposition of variance for among measures and a Granger causality analysis of their lead/lag relationships.

2.1 Data Sources for the Systemic Risk Measures

US measures are based on data for financial institutions from CRSP and COMPUSTAT. We construct measures for Europe (EU) by pooling together data on financial institutions of France, Germany, Italy and Spain, the largest continental European countries at the center of the recent financial crisis. Financial institution returns data for European countries are from Datastream.[7]

[7] Datastream data requires cleaning. We apply the following filters. 1) When a firm's data series ends with a string of zeros, the zeros are converted to missing, since this likely corresponds to a firm exiting the dataset. 2) To ensure that we use liquid securities, we require firms to have non-zero returns for at least one third of the days that they are in the sample, and we require at least three years of non-zero returns in total. 3) We winsorize positive returns at 100% to eliminate large outliers that are likely to be recording errors.

2.2 Overview of Measures

A recent survey by Bisias et al. (2012) does an excellent job categorizing and collecting systemic risk measures. We build from that survey to construct our measures of systemic risk. Below we provide a brief overview of the measures that we build, grouped by their defining features. We refer readers to Appendix A and to Bisias et al. for further details. Table 2 shows the time periods for which each measure is available in the US, the UK and Europe.

Aggregated Versions of Institution-Specific Measures These measures are constructed primarily to capture the *contribution* to systemic risk of individual institutions. In particular, they capture the relation between the distress of each individual firm and the distress of the whole system.

To produce the time series of the aggregated version of each measure, we start by constructing the institution-specific measure separately for the 20 largest financial institutions using a rolling estimation window. We then aggregate the individual measures by taking a simple cross-sectional equal-weighted average at each point in time for the twenty largest financial institutions at the time the measure is calculated. We only construct the aggregated version if we have data for at least 10 financial institutions, so that the resulting measure meaningfully captures distress in the financial system as opposed to distress of one or few individual banks for which we have data. Generally, the motivation for aggregating individual measures is to capture periods in which the largest financial institutions pose a threat to the stability of the financial system.

These measures include **Covar** and **ΔCovar** from Adrian and Brunnermeier (2011), **MES** and **SES** (or **Sysrisk**) from Acharya, Pedersen, Philippon and Richardson (2010), and **SRISK**, a version of the marginal expected shortfall proposed by Brownlees and Engle (2011).

Comovement and Contagion A second set of measures capture the degree of comovement among financial institutions' equity returns. Again, we construct them using a rolling estimation window, using the set of largest 20 financial institutions (but requiring the presence of at least 10). We consider the Absorption Ratio **(AR)** described by Kritzman et al. (2010), that captures the fraction of the variance of many returns explained by the first K principal components, the Dynamic Causality Index (**DCI)** from Billio et al. (2012), capturing the degree of interconnectedness by looking at the number of Granger causality relations between returns, and the **International Spillover Index** from Diebold and Yilmaz (2009).

Instability and volatility of the system Other measures directly look at the aggregate volatility and instability of the financial system or the stock market as a whole. We compute aggregate **Realized Volatility,** aggregate **Book Leverage** and **Market Leverage**, **Size Concentration**, the **Herfindhal** index of the size distribution of financial firms, and **Turbulence,** a measure of excess volatility in financial markets.

Liquidity and credit measures The last set of measures we consider relate to liquidity and credit conditions in financial markets: **AIM** (the Amihud 2002 liquidity measure, aggregated across financial firms), the **TED** spread, the **Default** spread (difference between BAA bond yields and the treasury), and the **Term** spread, the slope of the yield curve.

Measures Not Covered Due to data constraints we have not been able to include measures that use

1. Linkages between financial institutions (such as interbank loans or derivative positions)
2. Contingent claims analysis
3. Stress tests
4. Measures based on CDS data (since we don't have a long enough time series).

2.3 Comovement

Figure 1 plots the monthly time series of select measures from 1926 on for the US.[8] First, all measures spiked during the recent financial crisis, which is not surprising given that many of these measures were proposed following the start of the crisis. Many systemic risk measures experience abnormally high levels in earlier periods, often reaching similar levels as during the crisis. During the oil crisis and high uncertainty of the early and mid 1970's, financial sector market leverage and return turbulence spike. Generally, all the measures display substantial variability, and occasionally experience high levels during non-recessionary episodes. This

[8]For readability, the plotted measures are standardized to have the same variance (hence no y-axis labels are shown) and we only a show a subset of the series we study.

can be interpreted in two ways. One interpretation is that these measures are simply noisy. Many of the spikes that do not seem to correspond to periods of particularly high financial stress might be considered "false positives". Another interpretation is that these measures sometimes capture stress in the financial system that does not result in full-blown financial crises either because policy and regulatory responses diffused the instability or the system stabilized itself. Another interpretation is that crises develop only when many systemic risk measures are simultaneously elevated, as during the recent crisis.

Table 3 shows the correlations among the different measures for the US, and Table 4 shows the correlations for the UK and Europe. Most correlations are quite low. Only small groups of measures comove strongly. For example, turbulence, realized volatility, and the TED spread are relatively highly correlated. Similarly, CoVaR, ΔCoVaR, MES and the absorption ratio tend to comove. The other measures display low or even negative correlations with each other. This, once again, suggests that these measures are capturing different aspects of the state of the financial system or are measured with substantial noise.

To provide a further description of comovement, we perform a variance decomposition. We standardize all measures so that they have equal variance, and calculate the principal components of the standardize measures. We then report the fraction of each measure's variance attributable to each principal component. In Table 5 we report this decomposition up to the fifth principal component for the US, the UK and Europe, starting in 1984 when we have the largest number of measures available. In this sample the first PC accounts for 40% of the variance across all measures, with each of the next four PCs capturing roughly 10% of total variation.

2.4 Dynamics

Given that measures of systemic risk are sometimes interpreted as *contemporaneous* measures of stress in the financial system, and sometimes as *leading indicators* of systemic risk, we turn next to explore the lead-lag relations between these variables. For each pair of variables, we conduct two-way Granger causality tests (Granger (1969)). Table 6 reports the number of other variables that each measure causes (left column) or is caused by (right column) in a Granger sense, for the US, the UK and Europe.

Two results emerge from the table. First, only about half the variables are linked by Granger causality relations. This is not a criticism of the measures, it is merely descriptive. Among the measures for which we find significant relations, some tend to Granger cause other variables more often than the reverse. These can be interpreted as leading indicators of systemic distress, and include the absorption ratio, turbulence, ΔCoVaR and realized volatility. Other variables, instead, tend to lag in measuring systemic risk, like the Term Spread, SysRisk and DCI. These variables might be better suited in capturing the actual occurrence of systemic risk during periods of distress than in forecasting incidents. These relations seem to be consistent across countries.

In sum, the empirical comparison of systemic risk measures shows that the landscape is quite heterogeneous, both for the US and internationally. Small groups of measures tend to move in a similar way, but overall the links – both contemporaneous and in the time series – between this large set of measures seem to be relatively weak. As mentioned above, this can be interpreted either as a sign of noise in these measures, or as an indication that these measures capture different *aspects* of systemic risk. Without a clear criterion to judge the effectiveness of measures for systemic risk measurement, introduced in the next section, we would not be able to disentangle these two possibilities.

3 Systemic Risk and the Real Economy

In this section, we propose a reduced form approach to modeling the relationship between financial sector distress and lower quantiles of real economic outcomes. This focuses on the downside risk associated with financial crises, which we argue is a useful criterion for determining a systemic risk measure's suitability for input into decisions for policy or regulation. The cost to our reduced form is an inability to identify "fundamental" shocks or specific mechanisms as in a structural model. The benefits of reduced forms include potentially less severe specification mistakes, the ability to identify new empirical relations to inform future theory, and to develop an understanding of systemic risk in the absence of theory. Hansen (2012) provides an excellent overview of advantages to systemic risk modeling with and without the structure of theory.

Motivated by the need to capture potentially non-linear dynamics between systemic risk and the downside distribution of macroeconomic variables, we directly model lower quantiles of real outcomes. Before describing our analysis in detail, we provide a short review of quantiles and quantile regression.

3.1 Quantile Regression

Denote the target variable by y_{t+1}, this is a scalar real macroeconomic variable whose quantiles we wish to forecast. The τ^{th} quantile of y_{t+1} is its inverse probability distribution function, denoted

$$Q_\tau(y_{t+1}) = \inf\{\mathrm{y} : P(y_{t+1} < \mathrm{y}) \geq \tau\}.$$

The quantile function may alternatively be represented as the solution to an optimization problem

$$Q_\tau(y_{t+1}) = \arg\inf_q E[\rho_\tau(y_{t+1} - q)]$$

where $\rho_\tau(x) = x(\tau - xI_{x<0})$ is known as the *quantile loss function*. The loss function ρ_τ is plotted in Figure 2 for $\tau = 0.5$ (median regression) and $\tau = 0.2$ (which places a higher penalty on downside errors).

The expectation-based quantile representation is notationally convenient for handling conditioning information sets. In particular, we use the following conditional quantile notation

$$Q_\tau(y_{t+1}|\mathcal{I}) = \arg\inf_q E[\rho_\tau(y_{t+1} - q)|\mathcal{I}].$$

In the seminal quantile regression specification of Koenker and Bassett (1978), the quantiles of y_{t+1}, conditional on all time t information (summarized by sigma algebra $\mathcal{I}_t$) are a linear function of observable conditioning variables, $\boldsymbol{X}_t$,

$$Q_\tau(y_{t+1}|\mathcal{I}_t) = \beta_{\tau,0} + \boldsymbol{\beta}'_\tau \boldsymbol{X}_t. \tag{1}$$

While OLS models the relationship between $\boldsymbol{X}_t$ and the conditional mean of y_{t+1}, quantile regression models conditional quantiles of y_{t+1} as linear functions of $\boldsymbol{X}_{t+1}$. Quantile models can provide a more complete picture of the target's distribution when conditioning information shifts more than just the distribution's location. It can be particularly useful when conditioning information changes the shape of the distribution in the tail regions. In short, quantile regression provides flexibility for modeling heterogeneous conditional distributions where simple mean regression is expected to be inadequate.

As the specification in Equation 1 suggests, we will focus on quantile *forecasting*, using information from systemic risk measures today to estimate adverse regions of conditional distributions in the future. From a policy and regulatory standpoint, this predictive formulation seems most appropriate.

3.2 Empirical Evaluation of Systemic Risk Measures

We begin by studying the performance of systemic risk measures in the US. In particular, we evaluate the ability of individual systemic risk measures to forecast the quantiles of the Chicago Fed National Activity Index (CFNAI) and its subcomponents, Production and Income (PI), Employment, Unemployment and Hours (EUH), Personal Consumption and Housing (PH) and Sales, Orders and Inventory (SOI). CFNAI corresponds to the index of economic activity outlined in Stock and Watson (1999), and its objective is to track over time current and future economic activity in the United States.

First, we compute the parameters of the univariate quantile regression of each macroeconomic variable and each measure of systemic risk individually. To take into remove the influence of predictive information in the CFNAI's own data history, we use innovations in a univariate autoregression for each CFNAI series, where the AR order chosen by the Akaike Information Criterion. AR models use use monthly data, and we forecast the sum monthly CFNAI shocks over the subsequent quarter.

We start the analysis in 1984, which is the first year in which all of our systemic risk measures are available. Quantile regression fits are evaluated via their average quantile loss, which is quantile regression analogue to average squared error in OLS. In particular, we construct for each measure a quantile version of the R^2, computed as:

$$1 - R^2 = \frac{\frac{1}{T}\sum_t [\rho_\tau(y_{t+1} - \hat{\alpha} - \hat{\beta} X_t)]}{\frac{1}{T}\sum_t [\rho_\tau(y_{t+1} - \hat{q}_\tau)]}$$

This expression captures the typical loss using conditioning information (the numerator) relative to the loss using an unconditional quantile estimate (the denominator). When conditioning information is valuable, average losses are low, and this ratio lies significantly below 1.0. As in OLS, the in-sample quantile regression R^2 always lies between zero and one. This need not be the case for out-of-sample regressions. Our in-sample measure of significance comes from from bootstrapped t-statistics, while our out-of-sample measure of significance comes tests for equality of two sequences of forecast errors (Diebold and Mariano (1995), West (1996)).

A final remark concerns the choice of the quantile of interest, τ. The following results focus attention on the 20th percentile, or $\tau = 0.2$. This choice represents a tradeoff between emphasizing extreme outcomes (very low quantiles) versus weighting a higher number of available data observations (achieved when τ is nearer to 0.5). We also report results for the median, allowing us to identify whether or not the systemic risk indicators provide information about the center of macro shock distributions, or about downside risk in particular.

To give a preliminary indication of what may be achieved with quantile regression, we show two in-sample quantile regression fits for CFNAI shocks. Figure 3 shows quantile forecasting fits using financial sector turbulence, and 4 shows fits using Δ absorption ratio. Each figure scatters the pairs (y_{t+1}, X_t) in the period 1967-2010, as well as the fitted conditional quantile lines for a range of quantiles between the 20th to the 80th percentile. These figures allow the researcher to read the conditional distribution of y_{t+1} at any given level of systemic risk, $X_t = x$. Figure 3 shows that turbulence is associated with a high degree of heterogeneity in the conditional quantiles of CFNAI shocks. As turbulence rises, the conditional distribution of CFNAI shocks fans out, especially on the downside.

An example in which conditioning on systemic risk does not produce different shapes in CFNAI's conditional distribution is shown in Figure 4. Here, quantile fits condition on Δ absorption ratio. We see that the shape of the distribution is homogeneous across different values of the predictor.

Table 7 reports the average relative losses $(1 - R^2)$ obtained from in-sample quantile forecasts of CFNAI shocks (as well as CFNAI subindices) using the full range of systemic risk measures over 1984-2010. A lower average loss means better forecasting power, and values below one mean that a measure outperforms the historical unconditional quantile benchmark. We also report significance levels for whether each predictor improves over the historical quantile. Only the term spread significantly outperforms the naive benchmark based on in-sample tests.

Table 8 reports average relative losses obtained from out-of-sample forecasts using the period 1984-1990 for training, and recursively testing throughout the 1990-2010 sample to evaluate performance.[9] Many systemic risk measures in fact perform worse than the unconditional quantile in forecasting downturns of the macro outcomes. The two exceptions are volatility measures (equity realized volatility and turbulence).

[9]This means that the out-of-sample $1 - R^2$ comes from 252 overlapping monthly forecasts, or about 84 non-overlapping quarterly forecasts.

4 Building a Systemic Risk Index

The limited success of any individual systemic risk measure in forecasting future downside macro shocks raises the question: Can forecasts be improved by considering all systemic risk measures jointly?

Motivated by this problem, we propose a simple linear factor model wherein the relevant information for y's conditional quantile is captured by an unobservable, low-dimension factor f. The factor structure is similar to well-known conditional mean factor models (e.g. Stock and Watson (2002)), which often motivates dimension reduction techniques such as principal components or partial least squares. The interesting new feature of our model is how it links latent factors to the conditional *quantiles* of target variable y.

4.1 A Latent Factor Model for Quantiles

We assume that the τ quantile of y_{t+1}, conditional on time t information, is a linear function of an unobservable univariate factor f_t:

$$Q_\tau(y_{t+1}|\mathcal{I}_t) \equiv Q_\tau(y_{t+1}|\boldsymbol{f}_t) = \alpha f_t$$

where $\mathcal{I}_t$ is the information set at time t. This formulation is identical to a standard quantile regression specification, except that f_t is a latent variable. Realizations of y_{t+1} can be written as $\alpha f_t + \eta_{t+1}$ where η_{t+1} is the quantile residual.

We assume that

$$\boldsymbol{x}_t = \boldsymbol{\phi} f_t + \boldsymbol{\Psi}\boldsymbol{g}_t + \boldsymbol{\varepsilon}_t$$

where $\boldsymbol{g}_t$ are latent factors that can drive all the risk measures and $\boldsymbol{\varepsilon}_t$ is a vector of predictor-specific idiosyncratic shocks. We denote $\boldsymbol{f}_t = (f_t, \boldsymbol{g}_t)'$ and correspondingly $\boldsymbol{\alpha} = (\alpha, \mathbf{0})'$. As is usual in this literature, we assume that f is orthogonal to $\boldsymbol{g}$, and therefore that f is the relevant information (contained in $\boldsymbol{x}$) for knowing the conditional distribution of future y, whereas $\boldsymbol{g}$ is irrelevant.

4.2 Estimators

Based on this model, we propose two factor estimation approaches to consistently estimating the quantiles of y. We provide asymptotic theory for their ability to estimate the true conditional quantile. Then we report their empirical performance, which improves upon all individual risk measures.

For the sake of exposition, we place all assumptions in the appendix. Assumption 1 formally states the factor model just discussed above. Assumption 2 is identical to Bai and Ng's (2006) assumptions A-E, imposing limited dependence between factors, idiosyncracies, and quantile residuals. Assumption 3 imposes assumptions following Engle and Manganelli (2004) and White (1994) that guarantee that quantile regression is consistent and asymptotically normal in the presence of serial correlation. We require Assumptions 1-3 for PCQR below, and cPQR needs an additional ellipticality assumption 4. Altogether, the assumptions are weak and allow for some serial and cross-correlation of the idiosyncracies, serial correlation in the factors, and GARCH effects in the idiosyncracies, factors and quantile residuals.

4.2.1 Principal Components Quantile Regression (PCQR)

Using principal components analysis to estimate factors in a linear model has been advocated by Stock and Watson (2002) and Bai (2003) among many others. Bai and Ng (2006) analyze the asymptotic behavior least squares regressions using such factor estimates. To the best of our knowledge, the asymptotic behavior of quantile regressions using principal component factor estimates (PCQR) has not been studied before. Seeing as the method of principal components is well-known, we skip a discussion of the PCQR procedure but place its algorithm in Table 1.

Generally speaking, our asymptotic analysis must analyze quantile regression on mismeasured variables, since for any finite N, T the factor estimates $\hat{\boldsymbol{f}}$ (or $\hat{f}$) are the true factors $\boldsymbol{f}$ (or f) plus error. Schennach (2008) discusses the difficulties confronting quantile regression using regressors measured with error, noting that existing econometric literature typically deals with this problem by means of instrumental variables assumed to be correlated with the true regressor and uncorrelated with the measurement error.

We instead appeal to an argument that the measurement error is vanishing as N, T get large. In order to do so, we turn to Angrist, Chernozhukov and Fernandez-Val's (2006) analysis of quantile regression under misspecification. From their analysis we obtain an expression for the errors-in-variables bias we face, for any finite N, T, in our setting vis-a-vis the pertinent factor model parameters.

PCQR provides a good introduction into the difficulties our asymptotic theory faces,

Table 1: Estimators

	Principal Components Quantile Regression (PCQR)
Factor Stage	Estimate $\hat{\boldsymbol{f}}_t$ by $(\boldsymbol{\Lambda}'\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}'\boldsymbol{x}_t$ for $\boldsymbol{\Lambda}$ the K eigenvectors associated with the K largest eigenvalues of $\sum_{t=1}^{T} \boldsymbol{x}_t\boldsymbol{x}_t'$
Predictor Stage	Time series quantile regression of y_{t+1} on a constant and $\hat{\boldsymbol{f}}_t$
	Partial Quantile Regression (PQR)
Factor Stage	1. Time series quantile regression of y_{t+1} on a constant and x_{it} to get slope estimate $\hat{\phi}_i$ 2. Cross-section OLS regression of x_{it} on a constant and $\hat{\phi}_i$ for each t to get slope estimate $\hat{f}_t$
Predictor Stage	Time series quantile regression of y_{t+1} on a constant and $\hat{f}_t$
	Consistent Partial Quantile Regression (cPQR)
Factor Stage	1. Time series orthogonal quantile regression of y_{t+1} on a constant and x_{it} to get slope estimate $\hat{\phi}_i$ 2. Cross-section OLS regression of x_{it} on a constant and $\hat{\phi}_i$ for each t to get slope estimate $\hat{f}_t$
Predictor Stage	Time series quantile regression of y_{t+1} on a constant and $\hat{f}_t$

Notes: The predictors $\boldsymbol{x}_t$ are each time-series standardized. All quantile regressions and orthogonal quantile regressions are run for quantile τ.

because principal components extracts factors based only on the covariance of the predictors $\boldsymbol{x}$ without taking account of their relationship with the ultimate object of interest, y. This means there are no quantile-regression errors-in-variables biases involved in the factor estimation stage. Nevertheless, there is such bias in the predictive quantile regression stage, and the proof of Theorem 1 deals with this.

Theorem 1 (Asymptotic Normality of PCQR)**.** *Make assumptions 1, 2 and 3 and suppose* $\frac{\sqrt{N}}{T} \to 0$*. The principal components quantile regression predictor of* $Q_\tau(y_{t+1}|\mathcal{I}_t) = \boldsymbol{\alpha}'\boldsymbol{f}_t = \alpha f_t$

is given by $\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{f}}_t$ *and is such that*

$$\frac{\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{f}}_t - \boldsymbol{\alpha}'\boldsymbol{f}_t}{\frac{1}{N}\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{\Sigma}}_{PCQR,1}\hat{\boldsymbol{\alpha}} + \frac{1}{T}\hat{\boldsymbol{f}}_t'\hat{\boldsymbol{\Sigma}}_{PCQR,2}\hat{\boldsymbol{f}}_t} \xrightarrow[N,T\to\infty]{d} N(0,1).$$

for $\hat{\boldsymbol{\Sigma}}_{PCQR,1} = \tilde{\boldsymbol{V}}^{-1}\tilde{\boldsymbol{\Gamma}}\tilde{\boldsymbol{V}}^{-1}$ *where* $\tilde{\boldsymbol{V}}$ *is the* $K \times K$ *diagonal matrix of the* K *largest eigenvalues of* $\boldsymbol{X}\boldsymbol{X}'/(TN)$ *in decreasing order,*

$$\tilde{\boldsymbol{\Gamma}} = \frac{1}{N}\sum_{i,j=1}^{N}\tilde{\boldsymbol{\lambda}}_i\tilde{\boldsymbol{\lambda}}_j'\frac{1}{T}\sum_{t=1}^{T}\tilde{\varepsilon}_{it}\tilde{\varepsilon}_{jt}$$

$\tilde{\boldsymbol{\lambda}}_i$ *the vector of the* i^{th} *elements of the* K *eigenvectors of* $\boldsymbol{X}\boldsymbol{X}'/(TN)$ *associated with the* K *largest eigenvalues,* $\tilde{\varepsilon}$ *are the estimated idiosyncracies; and for* $\hat{\boldsymbol{\Sigma}}_{PCQR,2} = \tau(1-\tau)\hat{\boldsymbol{D}}^{-1}\hat{\boldsymbol{\Sigma}}_f\hat{\boldsymbol{D}}^{-1}$ *where* $\hat{\boldsymbol{D}} = \frac{1}{T}\sum_{t=1}^{T}\hat{h}_{t\eta}(0)\boldsymbol{f}_t\boldsymbol{f}_t'$ *and* $\hat{\boldsymbol{\Sigma}}_f = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{f}_t\boldsymbol{f}_t'$.

The proof of Theorem 1 is relegated to the appendix and looks similar to Bai and Ng's (2006) Theorem 3. The first term in the asymptotic covariance of $\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{f}}_t - \boldsymbol{\alpha}'\boldsymbol{f}_t$ is found just as Bai and Ng (2006) find it, building from Bai's (2003) result which requires $\frac{\sqrt{N}}{T} \to 0$. The second term is differs from Bai and Ng's (2006) only insofar as the asymptotic covariance of a quantile regression coefficient differs from that of the OLS regression coefficient. As is typically the case with quantile regression, the asymptotic covariance matrix estimator requires a density estimator $\hat{h}_{t\eta}$ which could be obtained from Powell (1991) as suggested in Engle and Manganelli (2004), or else could be bootstrapped. Theorem 1 differs from existing proofs of quantile regression's consistency by explicit consideration of the N limit and the behavior of the vanishing measurement error contained in $\hat{\boldsymbol{f}}$.

Our factor model assumes that a scalar f contains the relevant information, but PCQR and Theorem 1 uses the vector $\hat{\boldsymbol{f}}$. This is because PCQR is only consistent for the true quantile forecast if the entire factor space $(f, \boldsymbol{g})$ is estimated. This is analogous to the distinction between principal components OLS regression and a method like partial least squares – the former produces an asymptotically-efficient forecast only when the entire factor space is covered, whereas the latter can produce an asymptotically-efficient forecast when only the relevant factor space is covered (see Kelly and Pruitt (2012)). The root of this distinction is that, as mentioned above, principal components extract factors using only information in $\boldsymbol{x}$. This leads to the possibility that PCQR could be inefficient in finite samples owing to the irrelevant information it retains in its factor estimates. We now turn to two new procedures that attempt to extract only the relevant factor information.

4.2.2 Partial Quantile Regression (PQR and cPQR)

Partial quantile regression (PQR) forecasts can be constructed from a series of quantile and OLS regressions as summarized in Table 1 and works as follows. In the first pass we calculate the quantile slope coefficient of y_{t+1} on each individual predictor x_{it}, $i = 1, ..., N$ using univariate quantile regression.[10] The second pass consists of T cross-sectional OLS regressions. In each period t, the predictors x_{it}, $(i = 1, ..., N)$ are regressed on their respective quantile slopes with y_{t+1} estimated in the first pass. The OLS slope estimate in each period serves as an estimate of the time t value of (an affine transformation of) the latent factor f_t. In effect, this step forms a weighted average of individual predictors where the weights are given by first stage slope estimates. The stronger the univariate predictor (the larger its slope estimate), the more weight it gets when constructing a univariate predictor. The third and final pass estimates a quantile regression of y_{t+1} on the time series of latent factor estimates from the second stage.[11]

Compared to PCQR, PQR involves the additional difficulty that quantile regression has been used in the factor estimation stage. Similar to Kelly and Pruitt's (2012) argument for partial least squares, this has been done in order to try and extract from $\boldsymbol{x}$ only the relevant information f while leaving out the irrelevant $\boldsymbol{g}$. Roughly speaking, this will work if the first-stage quantile regression slopes $\hat{\phi}_i$ are functions of factor model parameters which only vary across i due to $\boldsymbol{\phi}$ – in other words, $\boldsymbol{\Psi}$ must not enter into $\hat{\phi}_i$. However, in general it appears the $\boldsymbol{\Psi}$ enters into the first-stage quantile regression slopes $\hat{\phi}_i$ via the errors-in-variables bias obtained by Angrist, Chernozhukov and Fernandez-Val's (2006) results.[12] Therefore, at the current time a proof of PQR's consistency is beyond reach.

Analysis of PQR leads to another procedure we describe next, for which a proof of consistency appears possible. Nevertheless, we continue to include PQR estimates because it is a simpler algorithm (numerically) than the following procedure and because its empirical

[10] It is important that, in a preliminary step, all predictors are standardized to have equal variance. This is typically done in other dimension reduction techniques as well, such as principal components regression and partial least squares.

[11] This procedure is similar to one of the same name proposed in Dodge and Whittaker (2009), who analyze neither its asymptotic behavior nor its relationship to factor models described here. The name is meant to connect to the method of partial least squares.

[12] It is possible that our expression from ACF (2006) is not the most expedient for this purpose, because we have used their results to represent an errors-in-variables bias as an omitted variables bias.

results are quite competitive.

He and Liang (2000) propose a solution to quantile regression under errors-in-variables by appealing to a quantile version of orthogonal regression. Recently, Liang and Li (2009) show that this method is consistent for elliptically-distributed data. Therefore, we build on these insights to propose the method of consistent partial quantile regression (cPQR). This method differs from PQR only in that the first-stage estimates $\hat{\phi}_i$ come from the *orthogonal* quantile regression

$$(\alpha_0, \alpha) = arg \min_{\alpha_0, \alpha} \frac{1}{T} \sum_{t=1}^{T} \rho_\tau \left(\frac{y_{t+1} - \alpha_0 - \alpha x_{it}}{\sqrt{1 + |\alpha|^2}} \right).$$

The orthogonal quantile regression problem can be more computationally involved than standard quantile regression because it is a nonlinear optimization problem whereas quantile regression solves a linear programming problem. However, we have found that the additional time taken is rather minimal.

By additionally assuming that our data are from a elliptical condtional distribution, we can appeal to Liang and Li's (2009) Theorem 3 to show consistency of the first-stage coefficients to a function that does not involve $\mathbf{\Psi}$. We conjecture the following

Theorem 2 (CONJECTURE – Asymptotic Normality of cPQR)**.** *Make assumptions 1, 2, 3 and 4 and suppose* $\frac{\sqrt{N}}{T} \rightarrow 0$. *The consistent partial quantile regression predictor of* $Q_\tau(y_{t+1}|\mathcal{I}_t) = \alpha f_t$ *is given by* $\hat{\alpha}\hat{f}_t$ *and is such that*

$$\frac{\hat{\alpha}\hat{f}_t - \alpha f_t}{\frac{1}{N}\hat{\alpha}^2 \hat{\sigma}_{cPQR,1} + \frac{1}{T}\hat{f}_t^2 \hat{\sigma}_{cPQR,2}} \xrightarrow[N,T\rightarrow\infty]{d} N(0,1).$$

A sketch of the conjectured proof of Theorem 2 is relegated to the appendix.

4.3 Empirical Results

The last four rows of Tables 7 and 8 report the results of PCQR, PQR and cPQR forecasts of the lower tail (20th percentile) of shocks to the CFNAI and its subindices. The in-sample fit is significantly better than the unconditional quantile for PCQR2 (PCQR using two principal components), PQR and cPQR. None of these procedures or any one systemic risk indicator alone can beat the in-sample fit of multiple quantile regression, but we suspect this is due to the latter's overfitting. Table 8 supports our hunch, as the out-of-sample performance of multiple quantile regression is far worse than its in-sample fit.

The PCQR, PQR and cPQR produce good out-of-sample forecasts. The forecasts are as good as the best individual forecast, whichever individual risk indicator happens to produce the best forecast for that macro variable. This implies that our dimension reduction techniques provide a stable way of extracting from a host of systemic risk indicators the most important information about future macro shocks. Furthermore, PQR and cPQR perform very similarly: Since it is conjectured that the latter can be shown to be asymptotically normal for the true conditional quantile, this provides some comfort that the former may indeed be close to consistent.

Table 10 reports the out-of-sample results of forecasting the median of shocks to the CFNAI and its subindices.[13] We see that very few risk measures tell us anything meaningful about the central tendency of future macro shocks. When one or a few do contain some information, the PQR or cPQR methods effectively identify the factor and produce significantly better out-of-sample forecasts. Nevertheless, our interpretation of these median results is that systemic risk indicators tell us about the lower tail of the future macroeconomic shock distribution and not its central tendency.

Table 12 turns to international data, forecasting future shocks to industrial production in the US, UK and Europe.[14] Once again, the dimension reduction techniques are effective at extracting useful information from the host of risk measures, producing out-of-sample forecasts for these countries that are significantly better than using the historical quantile alone.[15]

Summarizing our findings, using a method of dimension reduction to collapse various systemic risk indicators to one or two informative factors yields impressive out-of-sample forecasting success. This yields forecasts that perform significantly better than the historical quantile alone, and is more robust than forecasts based on any single risk indicator alone. The systemic risk predictor holds information about the lower tail of future macroeconomic shocks, but not its central tendency.

[13] Table 9 provides the in-sample results.

[14] Table 11 provides the in-sample results.

[15] Results for the median are also provided in Tables 13 and 14.

5 Conclusion

In this paper we quantitatively examine a large collection of recently proposed systemic risk measures. We construct a time series for more than 20 measures dating back to 1967 and, in some cases, to 1926, for the US, and more than 10 measures for other countries. Based on this panel, we study whether these measures agree in signaling periods of financial sector distress. We find that, outside the recent financial crisis, the correlations of these measures are low, and each measure displays idiosyncratic peaks at different times during the last 100 years, often as high as the peaks observed during the financial crisis.

Next, we propose a criterion to evaluate the empirical success of these measures in predicting systemic risk. We argue that, for a systemic risk measure to be useful as an input for decisions regarding policy and regulation geared towards welfare improvement, it should be observably associated with real macroeconomic outcomes. Motivated by macroeconomic theories with financial frictions, we evaluate the importance of each candidate measure by testing whether it predicts low quantiles of future macroeconomic realizations.

Finally, we propose two methodologies for aggregating systemic risk information over a large number of mis-measured individual series. We motivate this with a framework in which each measure contains useful information about future economic downturns, but the information is obscured by noise or other factors that are irrelevant to the macroeconomy. Dimension reduction based on principal components and partial quantile regression forecast macro quantiles using a small number of factors that are each a linear combination of individual systemic risk measures. We prove consistency and asymptotic normality of both estimators. Systemic risk indexes constructed using these dimension reduction techniques have significant power to forecast economic downturns in the US and Europe.

References

Acharya, V., L. Pedersen, T. Philippon, and M. Richardson (2012): "Measuring Systemic Risk," *Working Paper, NYU.*

Adrian, T., and M. Brunnermeier (2011): "CoVaR," Discussion paper, National Bureau of Economic Research.

Amihud, Y. (2002): "Illiquidity and stock returns: cross-section and time-series effects," *Journal of Financial Markets*, 5(1), 31–56.

Bernanke, B., and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 79(1), 14–31.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, 1.

Billio, M., A. Lo, M. Getmansky, and L. Pelizzon (2012): "Econometric Measures of Connectedness and Systemic Risk in the Finance and Insurance Sectors," *Journal of Financial Economics*, 104, 535–559.

Bisias, D., M. Flood, A. W. Lo, and S. Valavanis (2012): "A Survey of Systemic Risk Analytics," *Working Paper, Office of Financial Research.*

Brownlees, C., and R. Engle (2011): "Volatility, correlation and tails for systemic risk measurement," *Working Paper, NYU.*

Brunnermeier, M. K., and Y. Sannikov (2012): "A Macroeconomic Model with a Financial Sector," *Working Paper, Princeton University.*

De Nicolò, G., and M. Lucchetta (2011): "Systemic Risks and the Macroeconomy," Discussion paper, National Bureau of Economic Research.

Dodge, Y., and J. Whittaker (2009): "Partial quantile regression," *Metrika*, 70, 35–57.

Gertler, M., and N. Kiyotaki (2010): "Financial intermediation and credit policy in business cycle analysis," *Handbook of Monetary Economics*, 3, 547.

Granger, C. (1969): "Investigating causal relations by econometric models and cross-spectral methods," *Econometrica: Journal of the Econometric Society*, pp. 424–438.

Hansen, L. P. (2012): "Challenges in Identifying and Measuring Systemic Risk," *Working Paper, University of Chicago*.

He, Z., and A. Krishnamurthy (2012): "A Macroeconomic Framework for Quantifying Systemic Risk," *Working Paper, Chicago Booth*.

Hu, X., J. Pan, and J. Wang (2010): "Noise as information for illiquidity," Discussion paper, National Bureau of Economic Research.

Kelly, B. T., and S. Pruitt (2012): "The Three-Pass Regression Filter: A New Approach to Forecasting with Many Predictors," *Working Paper, Chicago Booth*.

Kiyotaki, N., and J. Moore (1997): "Credit chains," *Journal of Political Economy*, 105(21), 211–248.

Kritzman, M., and Y. Li (2010): "Skulls, Financial Turbulence, and Risk Management," *Working Paper*.

Kritzman, M., Y. Li, S. Page, and R. Rigobon (2010): "Principal components as a measure of systemic risk," *Working Paper, MIT*.

Mendoza, E. (2010): "Sudden stops, financial crises, and leverage," *The American Economic Review*, 100(5), 1941–1966.

Stock, J. H., and M. W. Watson (2002): "Forecasting Using Principal Components from a Large Number of Predictors," *Journal of the American Statistical Association*, 97(460), 1167–1179.

A Appendix

A.1 Systemic Risk Measures

$CoVaR$ and $\Delta CoVaR$ (Adrian and Brunnermeier 2011) CoVaR is defined as the value-at-risk (VaR) of the financial system as a whole conditional on an institution being in distress. The distress of the institution, in turn, is captured by the institution being at its own individual VaR (computed at quantile q):

$$Pr(X^i < Var^i) = q$$

CoVar for institution i is then defined as:

$$Pr(X^{syst} < CoVar_t | X^i = VaR^i) = q$$

which we estimate using conditional linear quantile regression after estimating VaR^i. $\Delta CoVaR_t^i$ is defined as the VaR of the financial system when institution i is at quantile q (in distress) *relative* to the VaR when institution i is at the median of its distribution

$$\Delta CoVaR_t^i = CoVaR_t^i(q) - CoVaR_t^i(0.5).$$

In estimating CoVar, we set q to the 5th percentile. Note that Adrian and Brunnermeier (2011) propose the use of a *conditional* version of CoVaR as well, called *forward CoVaR*, in which the relation between the value-at-risk of the system and the individual one is modeled as conditional on an additional set of covariates X_t. Here we use the alternative approach of rolling windows CoVaR estimates (in particular, we use a window of 252 days).

MES, SysRisk (Acharya, Pedersen, Philippon and Richardson (2010)) These measures capture the exposure of each individual firm to shocks to the aggregate system. MES captures the expected return of a firm conditional on the system being in its lower tail:

$$MES_t^i = E[R_t^i | R_m^t < q]$$

where q is a low quantile of the distribution of R_m^t. Together with leverage (LVG, defined as the ratio of assets to market equity) this comprises a measure of *expected capital shortfall*, or SysRisk, when the system is in its lower tail:

$$SysRisk_t^i = \alpha MES_t^i + (1-\alpha) LVG_t^i.$$

We construct MES and SES using a rolling window of 252 days.

SRISK (Brownlees and Engle (2011)) We construct the SRISK version of MES, which employs dynamic volatility models (GARCH/DCC for $\sigma_{\cdot,t}, \rho_t$) to estimate the components of MES:

$$SRISK_{i,t-1} = \sigma_{i,t}\rho_t E\left[\epsilon_{m,t}|\epsilon_{m,t} < \frac{k}{\sigma_{m,t}}\right] + \sigma_{i,t}\sqrt{1-\rho_t^2} E\left[\epsilon_{i,t}|\epsilon_{m,t} < \frac{k}{\sigma_{m,t}}\right].$$

Absorption Ratio (Kritzman et al. (2010)) This measure computes the fraction of return variance of a set of N financial institutions explained by the first $K < N$ principal components:

$$AR(K) = \frac{\sum_{i=1}^{K} Var(PC_i)}{\sum_{i=1}^{N} Var(PC_i)}.$$

A leading distress indicator is then constructed as the difference between absorption ratios calculated long (one year) and short (one month) estimation windows

$$\Delta AR(K) = AR(K)_{\text{short}} - AR(K)_{\text{long}}.$$

In our empirical analysis we construct the $AR(3)$ measure, and construct $\Delta AR(K)$ using respectively 22 and 252 days for the short and the long windows.

Dynamic Causality Index – or DCI (Billio et al. 2012) The index aims to capture how interconnected a set of financial institutions is by computing the fraction of significant Granger-causality relationships among their returns:

$$DCI_t = \frac{\#\, significant\, GC\, relations}{\#\, relations}$$

We consider significant Granger-causality relations with a p-value below 0.05. We construct the measure using a rolling window of 36 months.

International Spillover (Diebold and Yilmaz 2009) The index, kindly shared by Professors Diebold and Yilmaz, aggregates the contribution of each variable to the forecast error variance of other variables across multiple return series. It captures the total extent of spillover across the series considered (a measure of interdependence).

Realized Volatility We construct individual volatility series of financial institutions by computing the within-month standard deviation of daily returns. We construct the aggregated series of realized volatility by averaging the individual volatility across the 20 largest institutions. We exclude firm-month observations in which the realized volatility is 0.

Insolvency We start by constructing individual series of the inverse of the within-month standard deviation of daily returns. We construct the aggregated series of insolvency by averaging them across the 20 largest institutions. We exclude firm-month observations in which the realized volatility is 0, and for UK and EU we additionally exclude observations in which the realized volatility is below the 1st percentile of the overall empirical distribution of realized monthly volatilities (in these cases, the inverse of volatility of an individual firm becomes extremely large and dominates the insolvency measure).

Book and Market Leverage We construct a measure of aggregate book leverage (debt/assets) and aggregate market leverage (debt/market equity) for the largest 20 financial institutions to capture the potential for instability and shock propagation that occurs when large intermediaries are highly levered.

Size concentration We construct the Herfindal index of the size distribution among financial firms:

$$Herf_t = N\frac{\Sigma_{i=1}^{N} ME_i^2}{(\Sigma_{i=1}^{N} ME_i)^2}$$

The concentration index also captures potential instability due to the threat of default of the largest firms. Note that we construct this measure using all available data (i.e., not restricting ourselves to the top 20 institutions only).

Turbulence (Kritzman and Li (2010)) Turbulence is a measure of "excess volatility", that compares in each period the squared returns of financial institutions with their historical volatility:

$$Turb_t = (r_t - \mu)'\Sigma^{-1}(r_t - \mu)$$

where r_t is the vector of returns of financial institutions, and μ and Σ are the historical mean and variance-covariance matrix. We compute it using data for the largest 20 financial institutions and a rolling window of 60 months.

AIM (Amihud 2002) AIM captures a weighted average of stock-level illiquidity AIM_t^i, defined as:

$$AIM_t^i = \frac{1}{K}\sum_{\tau=t-K}^{t}\frac{|r_{i,\tau}|}{turnover_{i,\tau}}$$

We construct an aggregated measure by averaging the measure across the top 20 financial institutions.

TED Spread The difference between three-month LIBOR and three-month T-bill interest rates.

Default Yield Spread The difference between yields on BAA corporate bonds and Treasuries.

Term Spread The difference between yields on ten year and one month US Treasury bonds.

A.2 Proofs

A.2.1 Assumptions

Assumption 1. *Let $\mathcal{I}_t$ denote the information set at time t and $Q_\tau(y_{t+1}|\mathcal{I}_t)$ denote the time-t conditional $\tau-$quantile of y_{t+1}. Let $f+1$ be 1×1 and $\boldsymbol{g}_t$ be $K_g \times 1$ with $K = 1 + K_g$ and f independent of $\boldsymbol{g}$, the loadings $\boldsymbol{\phi}$ and $\boldsymbol{\Psi}$ are orthogonal to each other, $\boldsymbol{f}_t \equiv (f_t, \boldsymbol{g}_t')'$, and $\boldsymbol{x}_t$ be $N \times 1$, for $t = 1, \ldots, T$. Then*

- $Q_\tau(y_{t+1}|\mathcal{I}_t) = Q_\tau(y_{t+1}|\boldsymbol{f}_t) = \boldsymbol{\alpha}'\boldsymbol{f}_t = \alpha f_t$
- $y_{t+1} = \alpha f_t + \eta_{t+1}$
- $\boldsymbol{x}_t = \boldsymbol{\phi} f_t + \boldsymbol{\Psi}\boldsymbol{g}_t + \boldsymbol{\varepsilon}_t = \boldsymbol{\Lambda}\boldsymbol{f}_t + \boldsymbol{\varepsilon}_t$

Assumption 2. *Let $||\boldsymbol{A}|| = (tr(\boldsymbol{A}'\boldsymbol{A}))^{1/2}$ denote the norm of matrix $\boldsymbol{A}$, and M be some positive finite scalar.*

- $\mathbb{E}||\boldsymbol{f}_t||^4 \leq M < \infty$ *and* $\frac{1}{T}\sum_{t=1}^{T} \boldsymbol{f}_t\boldsymbol{f}_t' \to \boldsymbol{\Sigma}_f$ *or some $K \times K$ positive definite matrix* $\boldsymbol{\Sigma}_f$
- $||\lambda_i|| \leq \bar{\lambda} < \infty$ *and* $||\boldsymbol{\Lambda}'\boldsymbol{\Lambda}/N - \boldsymbol{\Sigma}_\Lambda|| \to 0$ *for some $K \times K$ positive definite matrix* $\boldsymbol{\Sigma}_\Lambda$.
- *For all* (i,t), $\mathbb{E}(\varepsilon_{it}) = 0, \mathbb{E}|\varepsilon_{it}|^8 \leq M$
- *There exist* $\mathbb{E}(\varepsilon_{it}\varepsilon_{js}) = \sigma_{ij,ts}$ *and* $|\sigma_{ij,ts}| < \bar{\sigma}_{ij}$ *for all* (t,s), *and* $|\sigma_{ij,ts} \leq \tau_{ts}$ *for all* (i,j) *such that* $\frac{1}{N}\sum_{i,j=1}^{N} \bar{\sigma}_{ij} \leq M$, $\frac{1}{T}\sum_{t,s=1}^{T} \tau_{ts} \leq M$, *and* $\frac{1}{NT}\sum_{i,j,s,t=1} |\sigma_{ij,ts}| \leq M$
- *For every* (t,s), $\mathbb{E}|\frac{1}{\sqrt{N}}\sum_{i=1}^{N}[\varepsilon_{is}\varepsilon_{it} - \mathbb{E}(\varepsilon_{is}\varepsilon_{it})||^4 \leq M$
- *For each* t, $\frac{1}{\sqrt{N}}\sum_{i=1}^{N} \boldsymbol{\lambda}_i\varepsilon_{it} \xrightarrow[N\to\infty]{d} N(\boldsymbol{0}, \boldsymbol{\Gamma}_t)$ *for* $\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} \boldsymbol{\lambda}_i\boldsymbol{\lambda}_j'\mathbb{E}(\varepsilon_{it}\varepsilon_{jt}) \xrightarrow[N\to\infty]{p} \boldsymbol{\Gamma}_t$
- *The variables* $\{\boldsymbol{\lambda}_i\}$, $\{f_t\}$, $\{\boldsymbol{g}_t\}$ *and* $\{\varepsilon_{it}\}$ *are four mutually independent groups. Dependence within each group is allowed.*
- *Let* $\boldsymbol{z}_t = (\boldsymbol{f}_t', \boldsymbol{w}_t')'$, $\mathbb{E}||\boldsymbol{z}_t||^4 \leq M$. *Then* $\mathbb{E}(\eta_{t+h}|y_t, \boldsymbol{z}_t, y_{t-1}, \boldsymbol{z}_{t-1}, \ldots) = 0$ *for any* $h > 0$, *and* $\boldsymbol{z}_t, \eta_t$ *are independent of the idiosyncratic errors* ε_{is} *for all* i, s, *and*
  - $\frac{1}{T}\sum_{t=1}^{T} \boldsymbol{z}_t\boldsymbol{z}_t \xrightarrow[T\to\infty]{p} \boldsymbol{\Sigma}_{zz} > 0$
  - $\frac{1}{\sqrt{T}}\sum_{t=1}^{T} \boldsymbol{z}_t\eta_{t+h} \xrightarrow[T\to\infty]{d} N(\boldsymbol{0}, \boldsymbol{\Sigma}_{zz,\eta})$ *where* $\frac{1}{T}\sum_{t=1}^{T} \boldsymbol{z}_t\boldsymbol{z}_t\eta_{t+h}^2 \xrightarrow[T\to\infty]{p} \boldsymbol{\Sigma}_{zz,\eta}$

Assumption 3. *Let M, m be positive finite scalars. The $\tau-$quantile shock η_{t+1} has conditional density $h_{t\eta\tau}(\cdot|\mathcal{I}_t) \equiv h_{t\eta}(\cdot)$ and is such that*

- *$h_{t\eta}$ is everywhere continuous*
- *$m \leq h_{t\eta} \leq M$ for all t*
- *$h_{t\eta}$ satisfies the Lipschitz condition $|h_{t\eta}(\kappa_1) - h_{t\eta}(\kappa_2)| \leq M|\kappa_1 - \kappa_2|$ for all t*
- *$\tau \in (0, 1)$*
- *$\boldsymbol{D} = \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} h_{t\eta}(0)\boldsymbol{f}_t\boldsymbol{f}_t'\right] \geq m$ and $\frac{1}{T}\sum_{t=1}^{T} h_{t\eta}(0)\boldsymbol{f}_t\boldsymbol{f}_t' \xrightarrow[T\to\infty]{p} \boldsymbol{D}$*
- *The sequence $\left\{\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\rho_\tau(\eta_{t+1})\boldsymbol{f}_t\right\}$ obeys a central limit theorem.*

Assumption 4. *Let the sequences $\{\boldsymbol{f}_t\}$, $\{\boldsymbol{\varepsilon}_t\}$ and $\{\eta_{t+1}\}$ follow elliptical distributions.*

A.2.2 Proof of Theorem 1

Proof. The predictive quantile regression coefficient is given by

$$\hat{\boldsymbol{\alpha}} = arg\min_{\boldsymbol{\alpha}} \frac{1}{T}\sum_{t=1}^{T}\rho_\tau(y_{t+1} - \boldsymbol{\alpha}'\hat{\boldsymbol{f}}_t).$$

Note that $\boldsymbol{f}_t$ linearly depends on $(\hat{\boldsymbol{f}}_t, \hat{\boldsymbol{f}}_t - \boldsymbol{H}\boldsymbol{f}_t)$ where following Bai (2003) we have $\boldsymbol{H} = \tilde{\boldsymbol{V}}^{-1}(\tilde{\boldsymbol{F}}'\boldsymbol{F}/T)(\boldsymbol{\Lambda}'\boldsymbol{\Lambda}/N)$ where $\tilde{\boldsymbol{F}} \equiv (\tilde{\boldsymbol{f}}_1, \ldots, \tilde{\boldsymbol{f}}_T)$ is the matrix of r eigenvectors (multiplied by $\sqrt{T}$) associated with the r largest eigenvalues of $\boldsymbol{X}\boldsymbol{X}'/(TN)$ in decreasing order and $\tilde{\boldsymbol{V}}$ is the $r \times r$ diagonal matrix of these r largest eigenvalues.

By White (1994) Corollary 5.12 and the conditional quantile factor model assumption

$$(\dot{\boldsymbol{\alpha}}_1, \dot{\boldsymbol{\alpha}}) = arg\min_{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}} \frac{1}{T}\sum_{t=1}^{T}\rho_\tau(y_{t+1} - \boldsymbol{\alpha}'\hat{\boldsymbol{f}}_t - \boldsymbol{\alpha}_1'(\hat{\boldsymbol{f}}_t - \boldsymbol{H}\boldsymbol{f}_t)) \xrightarrow[T\to\infty]{p} (\boldsymbol{\alpha}'\boldsymbol{H}^{-1}, -\boldsymbol{\alpha}\boldsymbol{H}^{-1})$$

for each N. This can be verified by seeing that assumptions 1, 2 and 3 satisfy Engle and Manganelli's (2004) assumptions C0-C7 and AN1-AN4.

ACF (2006) Thm 2 implies that

$$\hat{\boldsymbol{\alpha}} = \dot{\boldsymbol{\alpha}} + \left(\frac{1}{T}\sum_{u=1}^{T} w_u\hat{\boldsymbol{f}}_u\hat{\boldsymbol{f}}_u'\right)^{-1}\left(\frac{1}{T}\sum_{u=1}^{T} w_u\hat{\boldsymbol{f}}_u\dot{\boldsymbol{\alpha}}_1'(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u)\right).$$

Note that w_u depends on $(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u)$, and this deviation is a function of the cross-sectional errors $(\epsilon_{i1}, \ldots, \epsilon_{iT})$ for $i = 1, \ldots, N$. These are independent of the true $\boldsymbol{f}_u$. The w_u is the weight defined in ACF (2006) that is the integral of the specification error (the distance $\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{f}}_t - \boldsymbol{\alpha}'\boldsymbol{f}_t$). Note that as this distance vanishes the weight sequence $\{w_u\}$ becomes, in general, a deterministic sequence for all u (and a constant should η be homoskedastic).

Following Bai and Ng (2006) we write

$$\begin{aligned}\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{f}}_t - \boldsymbol{\alpha}'\boldsymbol{f}_t &= \hat{\boldsymbol{\alpha}}'(\hat{\boldsymbol{f}}_t - \boldsymbol{H}\boldsymbol{f}_t) + (\hat{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1})\boldsymbol{H}\boldsymbol{f}_t \\ &= \frac{1}{\sqrt{N}}\hat{\boldsymbol{\alpha}}'\left[\sqrt{N}(\hat{\boldsymbol{f}}_t - \boldsymbol{H}\boldsymbol{f}_t)\right] + \left[\sqrt{T}(\hat{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1})\right]\frac{1}{\sqrt{T}}\boldsymbol{H}\boldsymbol{f}_t. \qquad \text{(A1)}\end{aligned}$$

$\sqrt{N}(\hat{\boldsymbol{f}}_t - \boldsymbol{H}\boldsymbol{f}_t)$ and $\sqrt{T}(\hat{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1})$ are asymptotically independent just as Bai and Ng (2006) argue since the limit of the former is determined by $(\epsilon_{i1}, \ldots, \epsilon_{iT})$ for $i = 1, \ldots, N$ while the limit of the latter is determined by $(\eta_2, \ldots, \eta_{T+1})$ which are independent of the cross-sectional errors. Bai (2003) says that $\sqrt{N}\hat{\boldsymbol{\Sigma}}_{PCQR,1}^{-1/2}(\hat{\boldsymbol{f}}_t - \boldsymbol{H}\boldsymbol{f}_t) \xrightarrow[N\to\infty]{d} N(\mathbf{0}, \boldsymbol{I})$ if $\frac{\sqrt{N}}{T} \to 0$ and is bounded otherwise, for $\hat{\boldsymbol{\Sigma}}_{PCQR,1}$ given by Bai and Ng's (2006) theorem 4. For simplicity we assumed that $\frac{\sqrt{N}}{T} \to 0$ so we have this normal asymptotic distribution. This means that the first term of (A1) is normal in the limit with asymptotic variance estimated by $\frac{1}{N}\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{\Sigma}}_{PCQR,1}\hat{\boldsymbol{\alpha}}$.

We can write

$$\begin{aligned}&\sqrt{T}(\hat{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1}) \\ &\quad = \sqrt{T}(\dot{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1}) + \left(\frac{1}{T}\sum_{u=1}^{T} w_u\hat{\boldsymbol{f}}_u\hat{\boldsymbol{f}}_u'\right)^{-1}\left(\frac{1}{T}\sum_{u=1}^{T} w_u\hat{\boldsymbol{f}}_u\dot{\boldsymbol{\alpha}}_1'(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u)\sqrt{T}\right) \\ &\quad = \sqrt{T}(\dot{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1}) + \left(\frac{1}{T}\sum_{u=1}^{T} w_u\hat{\boldsymbol{f}}_u\hat{\boldsymbol{f}}_u'\right)^{-1}\left(\frac{1}{T}\sum_{u=1}^{T} w_u\hat{\boldsymbol{f}}_u(\dot{\boldsymbol{\alpha}}_1' + \boldsymbol{\alpha}'\boldsymbol{H}^{-1})(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u)\sqrt{T}\right) \\ &\qquad + \left(\frac{1}{T}\sum_{u=1}^{T} w_u\hat{\boldsymbol{f}}_u\hat{\boldsymbol{f}}_u'\right)^{-1}\left(\frac{1}{T}\sum_{u=1}^{T} w_u\hat{\boldsymbol{f}}_u\boldsymbol{\alpha}'\boldsymbol{H}^{-1}(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u)\sqrt{T}\right) \qquad \text{(A2)}\end{aligned}$$

We can rewrite the third term of (A2) as

$$\left(\frac{1}{T}\sum_{u=1}^{T} w_u \hat{\boldsymbol{f}}_u \hat{\boldsymbol{f}}_u'\right)^{-1} \left(\frac{\sqrt{T}}{\sqrt{N}}\frac{1}{T}\sum_{u=1}^{T} w_u \hat{\boldsymbol{f}}_u \boldsymbol{\alpha}' \boldsymbol{H}^{-1}\sqrt{N}(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u)\right)$$
$$= \frac{1}{\sqrt{N}}\left(\frac{1}{T}\sum_{u=1}^{T} w_u \hat{\boldsymbol{f}}_u \hat{\boldsymbol{f}}_u'\right)^{-1} \left(\frac{1}{\sqrt{T}}\sum_{u=1}^{T} w_u \boldsymbol{H}\boldsymbol{f}_u \boldsymbol{\alpha}' \boldsymbol{H}^{-1}\sqrt{N}(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u)\right) + \frac{1}{\sqrt{N}}o_p(1)$$
$$= O_p(\frac{1}{\sqrt{N}})$$

The last equality holds using Slutsky because: $\left(\frac{1}{T}\sum_{u=1}^{T} w_u \hat{\boldsymbol{f}}_u \hat{\boldsymbol{f}}_u'\right)^{-1}$ is a constant in the limit; $\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u = o_p(1)$ and therefore $w_u \hat{\boldsymbol{f}}_u \boldsymbol{\alpha}' \boldsymbol{H}^{-1}\sqrt{N}(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u) = \boldsymbol{H}\boldsymbol{f}_u \boldsymbol{\alpha}' \boldsymbol{H}^{-1}\sqrt{N}(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u)w_u + o_p(1)$; in the limit $\{w_u\}$ is a deterministic sequence; and $\sqrt{N}(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u)$ is independent of the true $\boldsymbol{f}_u$. All these statements then imply that $\frac{1}{\sqrt{T}}\sum_{u=1}^{T} w_u \boldsymbol{H}\boldsymbol{f}_u \boldsymbol{\alpha}' \boldsymbol{H}^{-1}\sqrt{N}(\hat{\boldsymbol{f}}_u - \boldsymbol{H}\boldsymbol{f}_u)$ is a bounded mean zero random variable for N, T large – but there is an extra $\frac{1}{\sqrt{N}}$ out front, and so the whole term is $O_p(\frac{1}{\sqrt{N}})$. The second term of (A2) is identical to the third term we just handled, except for the $(\dot{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}\boldsymbol{H}^{-1})$ term which is itself $O_p(\frac{1}{\sqrt{T}})$ – therefore the second term of (A2) is $O_p(\frac{1}{\sqrt{NT}})$. This means that $\sqrt{T}(\hat{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1}) = \sqrt{T}(\dot{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1}) + o_p(1)$.

Now $\sqrt{T}\hat{\boldsymbol{\Sigma}}_{PCQR,2}^{-1/2}(\dot{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1}) \xrightarrow[T\to\infty]{d} N(\boldsymbol{0}, \boldsymbol{I})$ by White (1994) and Engle and Manganelli (2004). Furthermore, since $\hat{\boldsymbol{f}} = \boldsymbol{H}\boldsymbol{f}_t + o_p(1)$, we can consistently estimate the asymptotic variance of $\left[\sqrt{T}(\dot{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1})\right]\frac{1}{\sqrt{T}}\boldsymbol{H}\boldsymbol{f}_t$ by $\frac{1}{T}\hat{\boldsymbol{f}}_t'\hat{\boldsymbol{\Sigma}}_{PCQR,2}\hat{\boldsymbol{f}}_t$ given in the theorem statement.

Recalling the asymptotic independence of the two terms in (A1), we therefore have the result that

$$\frac{\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{f}}_t - \boldsymbol{\alpha}'\boldsymbol{f}_t}{\frac{1}{N}\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{\Sigma}}_{PCQR,1}\hat{\boldsymbol{\alpha}} + \frac{1}{T}\hat{\boldsymbol{f}}_t'\hat{\boldsymbol{\Sigma}}_{PCQR,2}\hat{\boldsymbol{f}}_t} \xrightarrow[N,T\to\infty]{d} N(0,1).$$

□

A.2.3 Sketch of proof of Theorem 2

The first-stage orthogonal quantile regression estimate $\hat{\phi}_i$ is given by

$$(\hat{\phi}_{0i}, \hat{\phi}_i) = arg\min_{\phi_0,\phi_i}\frac{1}{T}\sum_{t=1}^{T}\rho_\tau\left(\frac{y_{t+1} - \phi_{0i} - \phi_i x_{it}}{\sqrt{1+\phi^2 c_i}}\right).$$

For a consistent estimate of c_i, Liang and Li's (2009) theorem 3 and assumption 1 imply that $\hat{\phi}_i \xrightarrow[T\to\infty]{p} \frac{\alpha}{\phi_i}$ with a rate of convergence $O(\frac{1}{\sqrt{T}})$.

The second-stage cross-sectional OLS regression of x_{it} on a constant and $\hat{\phi}_i$ yields a slope coefficient that is consistent for an affine function $a + bf_t$ for each t, for N large. Call this estimate $\hat{f}_t$. Using this estimate we can construct an estimate $\hat{c}_i$, plug this into the first-stage, and repeat the first and second stages until convergence.

Fitted values form the third-stage quantile regression of y_{t+1} on a constant and the second-stage $\{\hat{f}_t\}$ should be asymptotically normal for the conditional quantile αf_t.

A.3 Tables and Figures

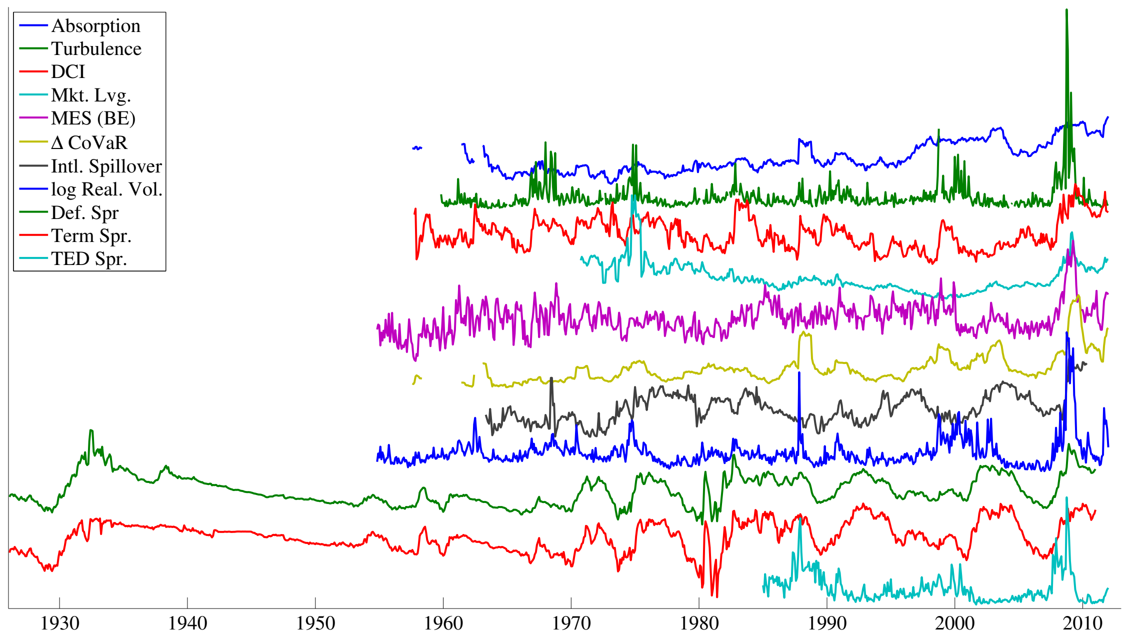

Figure 1: Systemic Risk Measures

Notes: The figure plots a subset of our panel of systemic risk measures. All measures have been standardized to have equal variance.

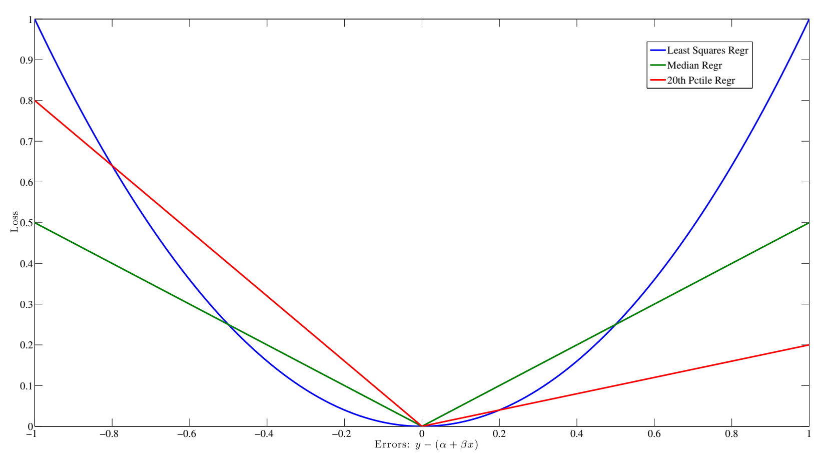

Figure 2: Loss Functions

Notes: The figure plots three regression loss functions including that of least squares, quantile regression with $\tau = 0.5$ (median regression), and quantile regression with $\tau = 0.2$

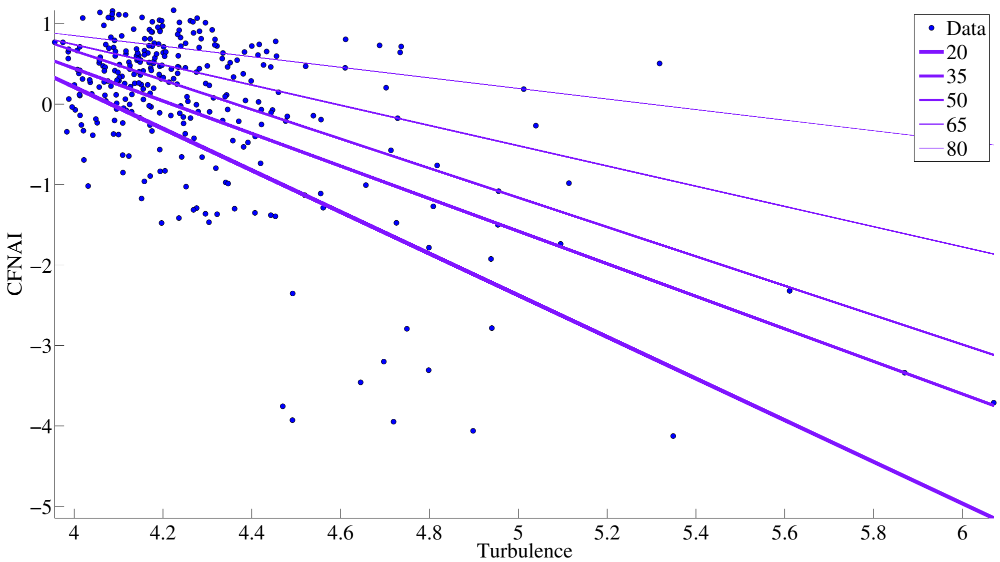

Figure 3: Example of Quantile Fit: Turbulence

Notes: The figure show a scatter plot of financial sector turbulence at time t against CFNAI shocks aggregated from $t+1$ to $t+3$. It also shows fitted quantile regression forecast lines for quantiles between 0.2 and 0.8.

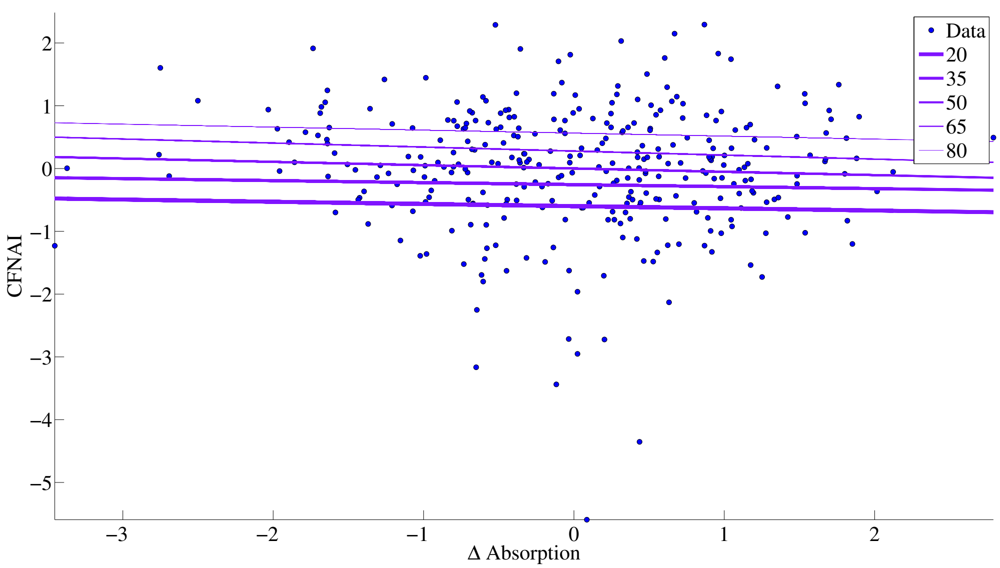

Figure 4: Example of Quantile Fit: Δ Absorption Ratio

Notes: The figure show a scatter plot of Δ absorption ratio at time t against CFNAI shocks aggregated from $t+1$ to $t+3$. It also shows fitted quantile regression forecast lines for quantiles between 0.2 and 0.8.

Table 2: Sample Start Dates

	US	UK	EU
Absorption	1957	1973	1973
Δ Absorp.	1957	1973	1973
Turbulence	1959	1978	1978
dci	1957	1975	1975
avgmessrisk	1954	1973	1973
avgmesappr	1971	1973	1973
avgcov	1957	1974	1974
avgdcov	1957	1974	1974
mktherf	1926	1973	1973
realvol	1954	1973	1973
diebold	1963	-	-
def	1926	-	-
ts	1926	-	-
ted	1984	-	-
avgaim	1954	-	-
avgsysriskappr	1971	-	-
booklev	1970	-	-
mktlev	1970	-	-

Notes: Measures begin in the stated year and are available through 2012 unless otherwise specified (in parenthesis).

Table 3: Correlations, US

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
Absorption	(1)	1.00	−0.46	0.23	0.07	0.27	0.71	0.71	0.77	0.41	0.38	0.38	0.44	0.36	0.07	−0.40	0.70	0.10	−0.20
Δ Absorp.	(2)	−0.46	1.00	0.00	−0.04	−0.18	−0.39	−0.33	−0.35	−0.14	0.08	−0.15	−0.23	−0.22	0.05	0.24	−0.37	−0.10	0.11
Turbulence	(3)	0.23	0.00	1.00	0.21	0.37	0.36	0.34	0.28	0.10	0.70	0.12	0.15	0.00	0.52	−0.03	0.37	0.14	0.21
DCI	(4)	0.07	−0.04	0.21	1.00	0.29	0.44	0.30	0.31	−0.13	0.29	0.15	0.22	0.15	0.17	0.00	0.45	0.23	0.52
MES (BE)	(5)	0.27	−0.18	0.37	0.29	1.00	0.50	0.29	0.36	0.10	0.44	0.17	0.41	0.34	0.34	−0.31	0.49	−0.05	0.05
MES (APPR)	(6)	0.71	−0.39	0.36	0.44	0.50	1.00	0.93	0.95	0.22	0.60	0.43	0.47	0.31	0.18	−0.15	0.99	0.19	0.22
CoVaR	(7)	0.71	−0.33	0.34	0.30	0.29	0.93	1.00	0.96	0.35	0.58	0.40	0.49	0.29	0.20	−0.23	0.93	0.24	0.24
Δ CoVaR	(8)	0.77	−0.35	0.28	0.31	0.36	0.95	0.96	1.00	0.26	0.51	0.43	0.53	0.37	0.21	−0.33	0.95	0.18	0.18
Size Conc.	(9)	0.41	−0.14	0.10	−0.13	0.10	0.22	0.35	0.26	1.00	0.20	0.13	0.00	0.08	−0.23	−0.29	0.21	0.21	−0.29
log Real. Vol	(10)	0.38	0.08	0.70	0.29	0.44	0.60	0.58	0.51	0.20	1.00	0.16	0.27	0.08	0.49	0.02	0.61	0.13	0.25
Intl. Spill.	(11)	0.38	−0.15	0.12	0.15	0.17	0.43	0.40	0.43	0.13	0.16	1.00	0.39	0.35	−0.16	−0.26	0.46	−0.03	0.16
Def Spr.	(12)	0.44	−0.23	0.15	0.22	0.41	0.47	0.49	0.53	0.00	0.27	0.39	1.00	0.86	0.03	−0.41	0.46	0.03	0.06
Term Spr.	(13)	0.36	−0.22	0.00	0.15	0.34	0.31	0.29	0.37	0.08	0.08	0.35	0.86	1.00	−0.11	−0.36	0.30	−0.02	−0.07
TED Spr.	(14)	0.07	0.05	0.52	0.17	0.34	0.18	0.20	0.21	−0.23	0.49	−0.16	0.03	−0.11	1.00	0.04	0.19	−0.01	0.27
AIM	(15)	−0.40	0.24	−0.03	0.00	−0.31	−0.15	−0.23	−0.33	−0.29	0.02	−0.26	−0.41	−0.36	0.04	1.00	−0.14	0.27	0.42
SysRisk	(16)	0.70	−0.37	0.37	0.45	0.49	0.99	0.93	0.95	0.21	0.61	0.46	0.46	0.30	0.19	−0.14	1.00	0.16	0.25
Book Lvg.	(17)	0.10	−0.10	0.14	0.23	−0.05	0.19	0.24	0.18	0.21	0.13	−0.03	0.03	−0.02	−0.01	0.27	0.16	1.00	0.51
Mkt Lvg.	(18)	−0.20	0.11	0.21	0.52	0.05	0.22	0.24	0.18	−0.29	0.25	0.16	0.06	−0.07	0.27	0.42	0.25	0.51	1.00

Notes: Each pairwise correlation is calculated using the longest available coinciding sample for that pair.

Table 4: Correlations, UK and EU

UK	Absorption	Δ Absorp.	turb	dci	avgmessrisk	avgmesappr	avgcov	avgdcov	mktherf	realvol
Absorption	1.00	−0.50	0.10	0.40	0.45	0.62	0.57	0.69	0.17	0.34
Δ Absorp.	−0.50	1.00	0.06	−0.23	−0.15	−0.35	−0.31	−0.37	−0.04	0.12
Turbulence	0.10	0.06	1.00	0.03	0.46	0.36	0.40	0.35	0.16	0.69
DCI	0.40	−0.23	0.03	1.00	0.40	0.45	0.34	0.37	0.38	0.21
MES (BE)	0.45	−0.15	0.46	0.40	1.00	0.66	0.49	0.54	0.59	0.66
MES (APPR)	0.62	−0.35	0.36	0.45	0.66	1.00	0.92	0.93	0.50	0.66
CoVaR	0.57	−0.31	0.40	0.34	0.49	0.92	1.00	0.97	0.32	0.69
Δ CoVaR	0.69	−0.37	0.35	0.37	0.54	0.93	0.97	1.00	0.32	0.65
Size Conc.	0.17	−0.04	0.16	0.38	0.59	0.50	0.32	0.32	1.00	0.40
log Real. Vol.	0.34	0.12	0.69	0.21	0.66	0.66	0.69	0.65	0.40	1.00
EU										
Absorption	1.00	−0.51	0.02	0.39	0.53	0.78	0.68	0.77	0.30	0.34
Δ Absorp.	−0.51	1.00	0.09	−0.20	−0.25	−0.41	−0.34	−0.38	−0.21	0.18
Turbulence	0.02	0.09	1.00	0.14	0.16	0.08	0.11	0.09	−0.02	0.42
DCI	0.39	−0.20	0.14	1.00	0.39	0.54	0.51	0.53	0.42	0.33
MES (BE)	0.53	−0.25	0.16	0.39	1.00	0.63	0.51	0.64	0.23	0.35
MES (APPR)	0.78	−0.41	0.08	0.54	0.63	1.00	0.94	0.96	0.37	0.52
CoVaR	0.68	−0.34	0.11	0.51	0.51	0.94	1.00	0.96	0.42	0.57
Δ CoVaR	0.77	−0.38	0.09	0.53	0.64	0.96	0.96	1.00	0.46	0.51
Size Conc.	0.30	−0.21	−0.02	0.42	0.23	0.37	0.42	0.46	1.00	0.10
log Real. Vol.	0.34	0.18	0.42	0.33	0.35	0.52	0.57	0.51	0.10	1.00

Notes: Each pairwise correlation is calculated using the longest available coinciding sample for that pair.

Table 5: Variance Decomposition, 1984-2011

	Principal Component				
	1	2	3	4	5
US					
Absorption	0.58	0.20	0.08	0.00	0.00
Δ Absorp.	0.13	0.19	0.00	0.14	0.18
Turbulence	0.29	0.16	0.19	0.04	0.02
DCI	0.47	0.05	0.00	0.17	0.04
MES (BE)	0.31	0.33	0.00	0.00	0.02
MES (APPR)	0.89	0.01	0.00	0.01	0.02
CoVaR	0.84	0.04	0.01	0.00	0.03
Δ CoVaR	0.85	0.02	0.00	0.00	0.03
Size Conc.	0.02	0.52	0.14	0.11	0.00
log Real. Vol.	0.51	0.10	0.13	0.14	0.00
Intl. Spillover	0.36	0.06	0.12	0.00	0.12
Def Spr.	0.40	0.00	0.40	0.09	0.00
Term Spr.	0.14	0.00	0.66	0.08	0.01
TED Spri.	0.10	0.37	0.17	0.00	0.01
AIM	0.00	0.00	0.00	0.44	0.00
SysRisk	0.91	0.00	0.00	0.01	0.02
Book Lvg.	0.23	0.22	0.03	0.00	0.36
Mkt Lvg.	0.56	0.12	0.03	0.05	0.14
UK					
Absorption	0.45	0.21	0.04	0.02	0.10
Δ Absorp.	0.11	0.52	0.08	0.14	0.10
Turbulence	0.24	0.42	0.08	0.18	0.00
DCI	0.28	0.08	0.39	0.02	0.14
MES (BE)	0.65	0.02	0.05	0.11	0.01
MES (APPR)	0.90	0.01	0.00	0.04	0.00
CoVaR	0.80	0.00	0.05	0.10	0.00
Δ CoVaR	0.85	0.01	0.06	0.06	0.00
Size Conc.	0.46	0.00	0.30	0.01	0.16
log Real. Vol	0.61	0.28	0.00	0.00	0.00
EU					
Absorption	0.75	0.03	0.04	0.00	0.00
Δ Absorp.	0.25	0.35	0.06	0.20	0.04
Turbulence	0.03	0.55	0.00	0.34	0.06
DCI	0.39	0.00	0.21	0.04	0.21
MES (BE)	0.49	0.00	0.08	0.03	0.18
MES (APPR)	0.92	0.00	0.01	0.02	0.00
CoVaR	0.86	0.00	0.00	0.04	0.04
Δ CoVaR	0.93	0.00	0.00	0.02	0.01
Size Conc.	0.23	0.05	0.55	0.00	0.04
log Real. Vol.	0.31	0.49	0.00	0.07	0.00

Notes: We standardize all systemic risk measures so that they have equal variance, and calculate the principal components of the standardize measures. We then report the fraction of each measure's variance attributable to each principal component.

Table 6: Granger Causality Tests

	US		UK		EU	
	Causes	Caused by	Causes	Caused by	Causes	Caused by
Absorption	4	1	1	1	1	6
Δ Absorp.	7	1	5	0	4	0
Turbulence	9	5	6	1	5	1
DCI	1	7	0	5	3	1
MES (BE)	1	12	1	8	1	6
MES (APPR)	5	6	3	6	2	5
CoVaR	5	4	4	3	3	3
Δ CoVaR	7	4	3	3	4	3
Size Conc.	0	0	2	0	1	0
log Real. Vol.	10	4	6	4	6	5
Intl. Spillover	0	7	–	–	–	–
Def. Spr.	3	4	–	–	–	–
Term Spr.	2	10	–	–	–	–
TED Spr.	4	1	–	–	–	–
AIM	1	0	–	–	–	–
SysRisk	7	4	–	–	–	–
Book Lvg.	1	0	–	–	–	–
Mkt Lvg.	3	0	–	–	–	–

Notes: For each pair of variables, we conduct two-way Granger causality tests. The table reports the number of variables that each measure significantly causes (left column) or is caused by (right column) in a Granger sense at the 2.5% one-sided significance level (tests are for *positive* causation only).

Table 7: In-Sample Quantile Forecasts, CFNAI and its Components, 20th Percentile, Quarter Shocks

	Mean Loss Relative to Historical Quantile				
	Total	EUH	PH	PI	SOI
Hist. Quantile	-	-	-	-	-
Absorption	0.9789	0.9611	0.9859	0.9530	0.9899
AIM	1.0000	0.9977	0.9986	0.9992	0.9965
CoVaR	0.9849	0.9704	0.9839	0.9644	0.9957
Δ CoVaR	0.9975	0.9963	0.9796	0.9865	0.9996
MES (APPR)	0.9950	0.9900	0.9780	0.9862	0.9987
MES (BE)	0.9869	0.9904	0.9733	0.9875	0.9832
Book Lvg.	0.9741	0.9634	0.9582	0.9624	0.9956
DCI	0.9795	0.9662	0.9042	0.9864	0.9994
Def. Spr.	0.9988	0.9952*	0.9946	0.9997	0.9993
Δ Absorption	0.9990	0.9956	0.9956	0.9998	1.0000
Intl. Spillover	0.9935	0.9947	0.9954	0.9978	0.9925**
Size Conc.	0.9994	0.9837	0.9890	0.9885	0.9982
Mkt. Lvg.	0.9371	0.9338	0.8555	0.9457	0.9797
log Real. Vol.	0.9155	0.8922	0.9576	0.9062	0.9404
TED Spr.	0.9226	0.9475	0.9425	0.9269	0.9473
Term Spr.	0.9770**	0.9563***	0.9981	0.9747***	0.9840**
Turbulence	0.8781	0.8694	0.9185	0.8819	0.9052
Multiple QR	0.7156	0.7093	0.7568	0.7267*	0.7802
PCQR1	0.9630	0.9464	0.9472	0.9455	0.9819
PCQR2	0.9430***	0.9384***	0.9194***	0.9445***	0.9683**
PQR	0.8613***	0.8632***	0.8871***	0.8722***	0.8922***
cPQR	0.8893***	0.8632***	0.8866***	0.8806***	0.8911***

Notes: The table reports in-sample average quantile forecast losses relative to the historical quantile model. Lower average losses represent more accurate quantile forecasts.

Table 8: Out-of-Sample Quantile Forecasts, CFNAI and its Components, 20th Percentile, Quarter Shocks

	Mean Loss Relative to Historical Quantile				
	Total	EUH	PH	PI	SOI
Hist. Quantile	-	-	-	-	-
Absorption	1.0176	0.9880	1.0020	0.9654	1.0195
AIM	1.0080	1.0119	1.0037	1.0123	0.9990
CoVaR	1.0146	0.9997	1.0049	0.9810	1.0227
Δ CoVaR	1.0294	1.0341	1.0063	1.0103	1.0258
MES (APPR)	1.0420	1.0309	1.0215	1.0326	1.0329
MES (BE)	1.0171	1.0548	1.0265	1.0260	1.0348
Book Lvg.	1.0151	0.9936	0.9720*	0.9810	1.0145
DCI	1.0060	1.0085	0.9014**	1.0146	1.0443
Def. Spr.	1.0284	1.0234	1.0124	1.0245	1.0250
Δ Absorption	1.0070	1.0022	0.9976	1.0105	1.0136
Intl. Spillover	1.0256	1.0414	1.0234	1.0301	1.0237
Size Conc.	1.0519	1.0294	1.0542	1.0439	1.0503
Mkt. Lvg.	0.9928	0.9648	0.8429***	1.0010	1.0585
log Real. Vol.	0.9158**	0.9219**	1.0013	0.9139**	0.9479*
TED Spr.	0.9717	0.9979	0.9444*	0.9508**	0.9725*
Term Spr.	1.0044	0.9735	1.0259	0.9882	0.9962
Turbulence	0.8956*	0.8643**	0.9559	0.8905**	0.9144**
Multiple QR	1.0773	1.0742	1.0352	1.0638	1.0838
PCQR1	0.9977	0.9773	0.9641**	0.9517	1.0275
PCQR2	0.9093*	0.9185*	0.9138**	0.9184*	0.9310
PQR	0.9116**	0.8980***	0.8667***	0.9273**	0.9535
cPQR	0.9309*	0.8980***	0.8668***	0.9141**	0.9517

Notes: The table reports out-of-sample average quantile forecast losses relative to the historical quantile model. Lower average losses represent more accurate quantile forecasts. Out-of-sample begins 1990.

Table 9: In-Sample Quantile Forecasts, CFNAI and its Components, Median, Quarter Shocks

	Mean Loss Relative to Historical Quantile				
	Total	EUH	PH	PI	SOI
Hist. Quantile	-	-	-	-	-
Absorption	0.9919	0.9941	0.9961	0.9962	0.9999
AIM	0.9991	0.9992	0.9990	0.9999	0.9995
CoVaR	0.9985	1.0000	0.9994	0.9986	0.9986
Δ CoVaR	0.9997	0.9980	0.9994	0.9996	0.9943
MES (APPR)	0.9998	0.9974	0.9999	0.9984	0.9951
MES (BE)	0.9988	0.9989	0.9994	1.0000	0.9995
Book Lvg.	0.9917	0.9997	0.9885	0.9968	0.9963
DCI	0.9970	0.9992	0.9932	0.9989	0.9995
Def. Spr.	0.9907**	0.9920	0.9957	0.9893**	0.9925*
Δ Absorption	1.0002	0.9981	0.9994	0.9938	0.9969
Intl. Spillover	0.9922**	0.9945	0.9996	0.9822***	0.9925
Size Conc.	0.9949	0.9968	0.9997	0.9966	0.9945
Mkt. Lvg.	0.9930	1.0002	0.9749	0.9957	0.9986
log Real. Vol.	0.9729	0.9766	0.9926	0.9684	0.9907
TED Spr.	0.9888	0.9912	0.9904	0.9721	0.9929
Term Spr.	0.9765***	0.9796***	0.9922	0.9741***	0.9738***
Turbulence	0.9563	0.9527	0.9660	0.9642	0.9675
Multiple QR	0.8333	0.8414	0.8967	0.8243*	0.8601
PCQR1	0.9982	1.0002	0.9961	0.9990	0.9994
PCQR2	0.9966	0.9997	0.9927**	0.9921**	0.9973
PQR	0.9366***	0.9325***	0.9473***	0.9305***	0.9486***
cPQR	0.9573***	0.9313***	0.9450***	0.9311***	0.9480***

Notes: The table reports in-sample average quantile forecast losses relative to the historical quantile model. Lower average losses represent more accurate quantile forecasts.

Table 10: Out-of-Sample Quantile Forecasts, CFNAI and its Components, Median, Quarter Shocks

	Mean Loss Relative to Historical Quantile				
	Total	EUH	PH	PI	SOI
Hist. Quantile	-	-	-	-	-
Absorption	1.0071	1.0156	1.0157	1.0123	1.0201
AIM	1.0217	1.0180	1.0088	1.0150	1.0052
CoVaR	1.0082	1.0250	1.0075	1.0136	1.0163
Δ CoVaR	1.0047	1.0169	1.0078	1.0053	1.0024
MES (APPR)	1.0081	1.0248	1.0173	1.0109	1.0152
MES (BE)	1.0170	1.0135	1.0157	1.0142	1.0091
Book Lvg.	1.0090	1.0180	1.0032	1.0103	1.0151
DCI	1.0170	1.0202	1.0082	1.0399	1.0145
Def. Spr.	1.0111	1.0103	1.0073	1.0045	1.0068
Δ Absorption	1.0031	1.0060	1.0043	0.9958	1.0023
Intl. Spillover	1.0115	1.0197	1.0144	0.9903	0.9995
Size Conc.	1.0152	1.0193	1.0196	1.0230	1.0197
Mkt. Lvg.	1.0443	1.0539	0.9993	1.0493	1.0311
log Real. Vol.	0.9956	1.0129	1.0160	0.9840	1.0088
TED Spr.	1.0055	1.0094	1.0081	0.9859*	1.0047
Term Spr.	0.9878	0.9880	1.0029	0.9834*	0.9855
Turbulence	0.9715	0.9657	0.9895	0.9722	0.9729
Multiple QR	1.1500	1.2042	1.2162	1.1321	1.1044
PCQR1	1.0222	1.0441	1.0264	1.0290	1.0285
PCQR2	1.0045	1.0323	1.0238	1.0161	1.0102
PQR	0.9674	0.9858	1.0225	0.9542*	0.9793
cPQR	0.9513*	0.9840	1.0215	0.9520*	0.9793

Notes: The table reports out-of-sample average quantile forecast losses relative to the historical quantile model. Lower average losses represent more accurate quantile forecasts. Out-of-sample begins 1990.

Table 11: In-Sample Quantile Forecasts, Industrial Production Growth, 20th Percentile, Quarter Shocks

	Mean Loss Relative to Historical Quantile		
	US	UK	EU
Hist. Quantile	-	-	-
Absorption	0.9484	0.9922	0.9587
AIM	0.9959	1.0001	0.9952
CoVaR	0.9110	0.9270	0.9715
Δ CoVaR	0.9455	0.9427	0.9711
MES (APPR)	0.9390	0.9404	0.9711
MES (BE)	0.9516	0.9314	0.9772
Book Lvg.	0.9536	0.9371	0.9830
DCI	0.9557	1.0016	0.9750
Def. Spr.	0.9847	0.9650	0.9122
Δ Absorption	0.9998	0.9949	0.9898*
Intl. Spillover	0.9890	0.9617	0.9811
Size Conc.	0.9918	0.8807	0.9447
Mkt. Lvg.	0.8932	0.9455	0.9960
log Real. Vol.	0.8451	0.9372	0.9355
TED Spr.	0.9229	0.9531	0.9849
Term Spr.	0.9955	0.9987	0.9534
Turbulence	0.8429	0.9498	0.9549
Multiple QR	0.6716*	0.7798*	0.7321**
PCQR1	0.8805	0.9120	0.9232
PCQR2	0.8712***	0.8862***	0.9047***
PQR	0.8840***	0.9265***	0.9358***
cPQR	0.8840***	0.9265***	0.9358***

Notes: The table reports in-sample average quantile forecast losses relative to the historical quantile model. Lower average losses represent more accurate quantile forecasts.

Table 12: Out-of-Sample Quantile Forecasts, Industrial Production Growth, 20th Percentile, Quarter Shocks

	Mean Loss Relative to Historical Quantile		
	US	UK	EU
Hist. Quantile	-	-	-
Absorption	0.9808	1.0154	0.9804
AIM	1.0043	1.0291	1.0033
CoVaR	0.9388	0.9446**	0.9870
Δ CoVaR	0.9762	0.9603**	0.9976
MES (APPR)	0.9671	0.9537**	0.9910
MES (BE)	1.0025	0.9567*	0.9965
Book Lvg.	0.9734*	0.9337**	1.0108
DCI	0.9929	1.0302	0.9922
Def. Spr.	1.0119	0.9925	0.9282**
Δ Absorption	1.0037	0.9968	0.9966
Intl. Spillover	1.0409	1.0024	1.0235
Size Conc.	1.0358	0.8940***	0.9628**
Mkt. Lvg.	0.9190	1.0019	1.0262
log Real. Vol.	0.8504***	0.9872	0.9642*
TED Spr.	0.9376***	0.9807	1.0043
Term Spr.	1.0023	1.0318	0.9657*
Turbulence	0.8460***	1.0297	0.9826
Multiple QR	0.9337	1.2230	0.9951
PCQR1	0.8979*	0.9480**	0.9492*
PCQR2	0.8834**	0.9066***	0.9898
PQR	0.9154**	0.9484*	0.9774
cPQR	0.9154**	0.9484*	0.9774

Notes: The table reports out-of-sample average quantile forecast losses relative to the historical quantile model. Lower average losses represent more accurate quantile forecasts. Out-of-sample begins 1990.

Table 13: In-Sample Quantile Forecasts, Industrial Production Growth, Median, Quarter Shocks

	Mean Loss Relative to Historical Quantile		
	US	UK	EU
Hist. Quantile	-	-	-
Absorption	0.9871	0.9981	0.9817
AIM	0.9962	1.0006	0.9993
CoVaR	0.9871	0.9859	0.9959
Δ CoVaR	0.9950	0.9897	0.9956
MES (APPR)	0.9980	0.9901	0.9983
MES (BE)	0.9993	0.9491	0.9987
Book Lvg.	0.9696	0.9636	0.9963
DCI	0.9846	0.9999	0.9909
Def. Spr.	0.9997	0.9971	0.9693
Δ Absorption	0.9989	0.9956	0.9943
Intl. Spillover	0.9968	0.9963	0.9969
Size Conc.	0.9961	0.9492	0.9914
Mkt. Lvg.	0.9561	0.9730	0.9951
log Real. Vol.	0.9597	0.9714	0.9964
TED Spr.	0.9790	0.9953	0.9998
Term Spr.	0.9862**	0.9759***	0.9833
Turbulence	0.9522	0.9805	1.0000
Multiple QR	0.7713*	0.8247	0.8668
PCQR1	0.9807	0.9738	0.9769
PCQR2	0.9732***	0.9629**	0.9765**
PQR	0.9291***	0.9347***	0.9724***
cPQR	0.9291***	0.9347***	0.9724***

Notes: The table reports in-sample average quantile forecast losses relative to the historical quantile model. Lower average losses represent more accurate quantile forecasts.

Table 14: Out-of-Sample Quantile Forecasts, Industrial Production Growth, Median, Quarter Shocks

	Mean Loss Relative to Historical Quantile		
	US	UK	EU
Hist. Quantile	-	-	-
Absorption	0.9951	1.0102	0.9842
AIM	0.9989	1.0434	1.0013
CoVaR	0.9992	1.0255	1.0057
Δ CoVaR	1.0057	1.0208	1.0036
MES (APPR)	1.0247	1.0177	1.0079
MES (BE)	1.0171	0.9589**	1.0105
Book Lvg.	0.9753*	0.9733	1.0041
DCI	1.0080	1.0123	0.9952
Def. Spr.	1.0196	1.0418	0.9763*
Δ Absorption	0.9990	1.0116	0.9955
Intl. Spillover	1.0125	1.0298	1.0034
Size Conc.	1.0173	0.9709*	0.9991
Mkt. Lvg.	0.9603	1.0439	1.0158
log Real. Vol.	0.9794	0.9816	1.0161
TED Spr.	0.9826***	1.0138	1.0060
Term Spr.	0.9970	1.0081	0.9901
Turbulence	0.9553**	1.0119	1.0101
Multiple QR	1.0400	1.2380	1.0910
PCQR1	0.9997	1.0007	0.9919
PCQR2	0.9902	0.9790	1.0097
PQR	0.9546**	0.9724	1.0355
cPQR	0.9546**	0.9723	1.0355

Notes: The table reports out-of-sample average quantile forecast losses relative to the historical quantile model. Lower average losses represent more accurate quantile forecasts. Out-of-sample begins 1990.