The Geometry of Spacetime

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Zen Master's eyes
twinkled
as he gave me
the one-word koan
"Hyperspace"

Abstract

The fundamental fabric of spacetime is revealed by deep Dimensional Analysis of the Planck Units of mass, energy, and electromagnetism. Using a little-known expression derived by James Clerk Maxwell for the dimensional reduction of mass and charge into units of length and inverse-time (frequency), all of the physical quantities can be expressed in terms of metres and inverse-seconds (Hz).

On arranging these quantities into a 2D log-log space/time matrix, simple (but compelling) patterns emerge in the mathematical relationship between fundamental units. The space/time matrix requires five spatial dimensions to accommodate the physical units, two of which are shown to be imaginary spatially-gauged wavelengths, i.e. unobservable dimensions of complex 5+1D spacetime, measured in metres, which exist (mathematically), but are not real.

Deep Dimensional Analysis

Introduction

During the late 1850's, James Clerk Maxwell's ideas about electromagnetism gradually became more mathematically complex. His spacetime geometry contained two imaginary dimensions to accommodate the electromagnetic potentials **E** and **A**, plus another imaginary dimension for the gravitational potential **V**, all three being mathematically orthogonal to real Euclidean space. In his discussion of the findings of the great electromagnetic experimentalist Michael Faraday, these comprised six spatial dimensions (three real plus three imaginary)…

"*I am getting converted to Quaternions, and have put some in my book, in a heretical form…*"[1]

In his scientific description of electromagnetism, Maxwell used what he called a "heretical form" of quaternionic algebra, which explicitly separated the three imaginary dimensions (*i,j,k*) from the real part representing a radial length or coordinate triplet (*x,y,z*). He stated emphatically that tensors and vectors were inadequate mathematical tools to correctly encapsulate the electromagnetic fields and forces. He also quietly discussed with colleagues how one might detect and measure "non-observable" or "hidden" spatial dimensions, which he conceived of as "storing energy", both kinetic and potential, in the elastic fabric of space itself.[2]

"The peculiarity of our space is that of its three dimensions, none is before or after another. As is 'x', so is 'y', and so is 'z'. If you have 4 dimensions, this becomes a puzzle. For first, if three of them are in our space, then which three? Also, if we lived in space of 'm' dimensions, but were only capable of thinking 'n' of them, then first, which 'n'? Second, if so, things would happen requiring the rest to explain them, and so we should either be stultified or made wiser. I am quite sure that the kind of continuity which has four dimensions all co-equal, is not to be discovered by merely generalising Cartesian space equations." — James Clerk Maxwell, in correspondence with C.J. Monro, Esq., 15 Mar 1871[1]

His preferred quaternionic notation was eliminated from *A Treatise on Electricity and Magnetism*[1] at the insistence of his publisher (over his strenuous objections), because very few physicists at the time understood the quaternion calculus. Maxwell regrettably passed away in 1879 at age 48, when he was only partway through his revision for the second edition.

Maxwell played a major role in establishing the modern use of dimensional analysis by distinguishing mass, length, and time as fundamental units, while referring to other units as derived.[4] Although he declared length, time and mass to be *"the three fundamental units"*, he also noted that gravitational mass can be derived from length and time by assuming a form of Newton's law of universal gravitation in which the gravitational constant *G* is taken as unity, giving $M = L^3 \cdot T^{-2}$.[3]

> If, as in the astronomical system, the unit of mass is defined with respect to its attractive power, the dimensions of $[M]$ are $[L^3\, T^{-2}]$.

By assuming Coulomb's constant k_e to also be unity, and dimensionless, Maxwell then determined that the dimensions of an electrostatic unit of charge were $Q = L^{3/2}·M^{1/2}·T^{-1}$,[5] which, after substituting his equation for mass, results in charge having the same fundamental dimensions as mass, viz. $Q = L^3·T^{-2}$.

This equality of dimensions for mass and charge must have intrigued and puzzled him, but he apparently never mentioned it in his lectures, correspondence or scientific writings. Perhaps because he'd intuitively understood that the dimensions of volume defining mass and charge must have (at least) one orthogonal dimension of unit length. But "what's the particular go o'that?", he must have wondered.

The fundamental physical constants

The primary physical constant is the speed of light in vacuum c_0, which has unitary 2D space-time dimensions of L/T, i.e. metres per second.

Another fundamental is Newton's gravitational constant G, having SI dimensions of $L^3·M^{-1}·T^{-2}$. When Maxwell's dimensions for mass ($M = L^3·T^{-2}$) are substituted into the SI dimensions, the gravitational constant is shown to be dimensionless in 2D space-time, viz. $L^0·T^0$.

Planck's constant h, the fundamental ratio of a quantum of energy to its wavefunction's frequency (T^{-1}), has SI dimensions of $L^2·M·T^{-1}$. Substituting Maxwell's dimensions for mass shows that the Planck elementary quantum of Action has fundamental dimensions of $L^5·T^{-3}$.

Maxwell determined[5] that the unit of elementary charge $e^{\pm}$ has dimensions of $(L^3·M·T^{-2})^{1/2}$. Substituting his mass dimensions ($M = L^3·T^{-2}$) reveals that charge has the fundamental dimensions of $(L^6·T^{-4})^{1/2}$, i.e. $Q = L^3·T^{-2}$. The electric constant (vacuum permittivity) ε_0 and the Coulomb constant k_e are therefore dimensionless.

The Boltzmann constant (k_B) is defined as the energy in Joules per degree of temperature (Θ), having SI dimensions of $L^2·M·T^{-2}·\Theta$. Substituting $M = L^3·T^{-2}$ reveals the Boltzmann constant to have space-time dimensions of $L^5·T^{-4}$ (energy) per degree K.

Universal physical constants normalised in two dimensions

Constant	Symbol	SI Dimensions	L/T Dimensions
Speed of light in vacuum	c_0	LT^{-1}	LT^{-1}
Gravitational constant	G	$L^3M^{-1}T^{-2}$	L^0T^0
Planck constant	h	L^2MT^{-1}	L^5T^{-3}
Coulomb constant	k_e	$L^3MT^{-2}Q^{-2}$	L^0T^0
Boltzmann constant	k_B	$L^2MT^{-2}\Theta^{-1}$	$L^5T^{-4}\Theta^{-1}$

The Planck Units

The Planck Units are "natural units" of measurement defined exclusively in terms of five universal physical constants, viz. c, G, $\hbar$, k_e and k_B, such that these constants have the numerical value of 1 when expressed in terms of the Planck units.

The base spatial unit is the Planck length (ℓ_P), defined as the distance traveled by light in vacuum during one Planck time (t_P). The numerical value of ℓ_P is calculated from $(\hbar G c^{-3})^{1/2}$, the fundamental space-time dimensions of which resolve as $(L^5T^{-3}·L^{-3}T^3)^{1/2} = L$.

The five base Planck units, viz. length, time, mass, charge and temperature, have traditionally been dimensioned in terms of the base SI units L, T, M, Q and Θ. However, Maxwell's factoring[1] of mass and charge into the more fundamental space-time dimensions of L^3T^{-2} permits a deep two-dimensional analysis of the base and derived Planck units.

Since the gravitational constant G and the vacuum permittivity (electric constant) ε_0 are dimensionless in L/T space-time units, they can be factored out of the Planck units, thereby simplifying the dimensional analysis. For example, Planck area is defined as $\hbar G/c^3$, which simplifies to L^2. Similarly, Planck current is defined as $(4\pi\varepsilon_0 c^6/G)^{1/2}$, which resolves to L^3T^{-3}.

Thus, the fundamental 2D space-time dimensions for each of the Planck units can be derived from their defining expressions. However, it is considerably easier to simply substitute L^3T^{-2} for M and Q in the conventional SI dimensions of the Planck quantities, as follows:

The Planck Units normalised in two dimensions (L,T)

Quantity	SI Dimensions	L/T Dimensions
Planck length	L	L
Planck time	T	T
Planck area	L^2	L^2
Planck volume	L^3	L^3
Planck mass	M	L^3T^{-2}
Planck charge	Q	L^3T^{-2}
Planck momentum	LMT^{-1}	L^4T^{-3}
Planck force	LMT^{-2}	L^4T^{-4}
Planck energy	L^2MT^{-2}	L^5T^{-4}
Planck power	L^2MT^{-3}	L^5T^{-5}
Planck density	ML^{-3}	T^{-2}
Planck intensity	MT^{-3}	L^3T^{-5}
Planck frequency	T^{-1}	T^{-1}
Planck pressure	$L^{-1}ML^{-2}$	L^2T^{-4}
Planck current	QT^{-1}	L^3T^{-3}
Planck voltage	$L^2MT^{-2}Q^{-1}$	L^2T^{-2}
Planck resistance	$L^2MT^{-1}Q^{-2}$	$L^{-1}T$

To facilitate further analysis, these quantities can be arranged into a log-log space/time matrix, whose columns represent incrementing powers of Planck length (L^n) and whose rows represent increasing powers of inverse-time (T^{-m}):

L/T	L^0	L^1	L^2	L^3	L^4	L^5
T^0	Constants G, ε_0	Length	Surface area	Volume	4D-volume	5D-volume
T^{-1}	Frequency	Velocity	Flux	Volumetric flow rate		
T^{-2}	Mass density ρ_m	Acceleration	Voltage $\Delta\varphi$	Mass, Charge		
T^{-3}				Current **I**	Momentum **p**	Planck Action h
T^{-4}			Pressure p		Force **F**	Energy **E**
T^{-5}				Intensity		Power **P**

The fundamental mathematical pattern underlying this 6D matrix can be discerned from the "G-normalised" expressions for the various Planck units. The most striking feature of the spacetime fabric's geometry is its central diagonal, comprising increasing powers of c_0 (the speed of light in vacuum).

L/T	L^0	L^1	L^2	L^3	L^4	L^5
T^0	1	$\sqrt{\frac{\hbar}{c^3}}$	$\frac{\hbar}{c^3}$	$\sqrt{\frac{\hbar^3}{c^9}}$	$\frac{\hbar^2}{c^6}$	$\sqrt{\frac{\hbar^5}{c^{15}}}$
T^{-1}	$\sqrt{\frac{c^5}{\hbar}}$	c	$\sqrt{\frac{\hbar}{c}}$	$\frac{\hbar}{c^2}$	$\sqrt{\frac{\hbar^3}{c^7}}$	$\frac{\hbar^2}{c^5}$
T^{-2}	$\frac{c^5}{\hbar}$	$\sqrt{\frac{c^7}{\hbar}}$	c^2	$\sqrt{\hbar c}$	$\frac{\hbar}{c}$	$\sqrt{\frac{\hbar^3}{c^5}}$
T^{-3}	$\sqrt{\frac{c^{15}}{\hbar^3}}$	$\frac{c^6}{\hbar}$	$\sqrt{\frac{c^9}{\hbar}}$	c^3	$\sqrt{\hbar c^3}$	$\hbar$
T^{-4}	$\frac{c^{10}}{\hbar^2}$	$\sqrt{\frac{c^{17}}{\hbar^3}}$	$\frac{c^7}{\hbar}$	$\sqrt{\frac{c^{11}}{\hbar}}$	c^4	$\sqrt{\hbar c^5}$
T^{-5}	$\sqrt{\frac{c^{25}}{\hbar^5}}$	$\frac{c^{11}}{\hbar^2}$	$\sqrt{\frac{c^{19}}{\hbar^3}}$	$\frac{c^8}{\hbar}$	$\sqrt{\frac{c^{13}}{\hbar}}$	c^5

The relationship between the two universal constants *c* and *ħ* define the fundamental units of space and time, viz. the Planck length and the Planck frequency (inverse-time). Similarly to the central diagonal, all parallel diagonals within the matrix follow the same principle, i.e. increasing powers of *c* as the diagonal is traversed (in the down-right direction).

Electromagnetism

Substitution of Maxwell's L^3T^{-2} for mass (M) and charge (Q) in the SI dimensions of the electromagnetic quantities[8] effects a "flattening" of their dimensionality to just spatial length and time, as follows:

Quantity	Symbol	Unit	SI Dimensions	L/T Dimensions
Electric charge	Q	coulomb	Q	L^3T^{-2}
Electric current	I	ampere	QT^{-1}	L^3T^{-3}
Electric potential	$\Delta\phi$	volt	$L^2MT^{-2}Q^{-1}$	L^2T^{-2}
Magnetic field strength	**H**	ampere per metre	$QL^{-1}T^{-1}$	L^2T^{-3}
Electric displacement	**D**	coulomb per metre²	QL^{-2}	LT^{-2}
Electric field strength	**E**	volt per metre	$LMT^{-2}Q^{-1}$	LT^{-2}
Current density	**J**	ampere per metre²	$QT^{-1}L^{-2}$	LT^{-3}
Charge density	ρ_q	coulomb per metre³	QL^{-3}	T^{-2}
Electric resistance	R	ohm	$L^2MT^{-1}Q^{-2}$	$L^{-1}T$
Capacitance	C	farad	$T^2Q^2L^{-2}M^{-1}$	L
Permittivity	ε	farad per metre	$T^2Q^2L^{-3}M^{-1}$	L^0T^0
Magnetic flux density	**B**	tesla	$MT^{-1}Q^{-1}$	T^{-1}
Magnetic potential	**A**	weber per metre	$LMT^{-1}Q^{-1}$	LT^{-1}
Magnetic flux	Φ	weber	$L^2MT^{-1}Q^{-1}$	L^2T^{-1}
Energy density	ρ_e	joule per meter³	$ML^{-1}T^{-2}$	L^2T^{-4}
Inductance	L	henry	ML^2Q^{-2}	$L^{-1}T^2$
Permeability	μ	henry per metre	$ML^{-1}Q^{-2}$	$L^{-2}T^2$
Magnetic momentum	**p**	newton-second	LMT^{-1}	L^4T^{-3}
Electromagnetic force	**F**	newton	LMT^{-2}	L^4T^{-4}
Electromagnetic Action	S	joule-second	L^2MT^{-1}	L^5T^{-3}
Electric potential energy	U	joule	L^2MT^{-2}	L^5T^{-4}
Radiant EM energy	E	joule	L^2MT^{-2}	L^5T^{-4}
Poynting vector	**S**	watt per metre²	MT^{-3}	L^3T^{-5}
Power	P	watt	L^2MT^{-3}	L^5T^{-5}

As with the Planck Units, these quantities can be arranged in a 6-dimensional space/time matrix, with columns representing powers of Planck length (L^n), and rows which represent powers of inverse-time (T^{-m}).

L/T	L^0	L^1	L^2	L^3	L^4	L^5
T^0	Electric constant ϵ_0	Wavelength λ Capacitance C	Surface area	Volume	4D-volume	5D-volume
T^{-1}	Magnetic flux density **B**	Magnetic vector potential **A**	Magnetic flux Φ	$\Phi \cdot \lambda$		
T^{-2}	Charge density $\rho_q = \nabla \cdot \mathbf{E}$	Electric field strength **E** (**D**)	Electrical potential $\Delta\varphi$	Charge Q	$Q \cdot \lambda$	
T^{-3}		Current density $\mathbf{J} = \nabla \times \mathbf{H}$	Magnetic field strength **H**	Current **I**	Momentum **p**	Planck Action h
T^{-4}			Energy density $\rho_e = \mathbf{B} \cdot \mathbf{H}$	$\mathbf{v} \times \mathbf{H}$	Force $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	Energy $U_\varepsilon = q \cdot \Delta\varphi$
T^{-5}				Poynting vector **S**	$\nabla\mathbf{P} = d\mathbf{F}/dt$	Power $\mathbf{P} = \Delta\varphi \cdot \mathbf{I}$

Discussion

Five mutually-orthogonal spatial dimensions are required to accommodate all the Planck units, notably the "higher dimensional" (L^4, L^5) quantities of momentum, force, action, energy and power. Three of the spatial dimensions are the real linear dimensions of Euclidean *x,y,z* space, viz. length, breadth and height.

Like the "time dimension" of special relativity, defined by Einstein as $\sqrt{-1} \cdot c \cdot t$,[6] the two extra spatial dimensions must be mathematically imaginary by virtue of their orthogonality, i.e. being Wick-rotated relative to all the other dimensions.

Extra spatial dimensions

In 2006, Paul Wesson determined[7] that an extra spatial coordinate x_4 could be identified as $\ell = Gm/c^2$, which he termed the "Einstein gauge". Formulated in terms of momentum, i.e. $\ell = Gp/c^3$, this gauge corresponds to the L^4/T^4 spatial dimension of the space/time matrix, from which emerges momentum, force, and pressure.

Wesson also identified another spatial coordinate as $\ell = \hbar/mc$ (dimensionally identical to $\ell = \hbar/qc$), which he termed the "Planck gauge". This $\hbar/qc$ gauge corresponds to the L^5/T^5 spatial coordinate in the space/time matrix. The physical quantities of action, energy, power and intensity emerge from this imaginary dimension.

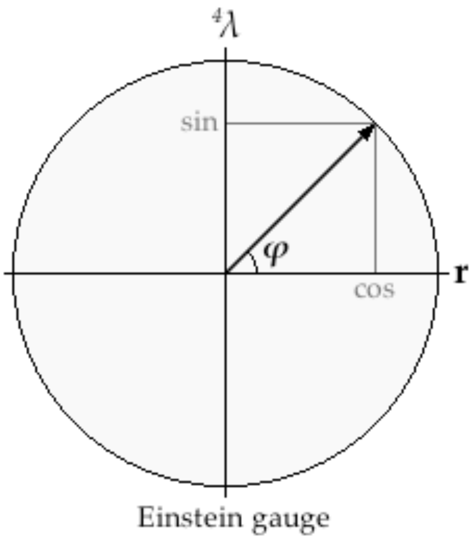
Imaginary spatial dimensions in complex 6D space/time may be formulated using various equivalent expressions, all of which resolve to the dimension of spatial distance (L). The Einstein gauge is canonically formulated[7] as Gm/c^2 (one-half of the Schwarzschild radius), but is more usefully expressed in terms of momentum, viz. $\ell = p/c^3$, particularly in the context of 6-dimensional Special Relativity. It can also be expressed in terms of kinetic energy as $\ell = E_K/c^4$. In the context of electromagnetism, this metric is best formulated in terms of magnetic momentum divided by current (flow of charge): $\ell = p_m/I$. In quantum mechanics, the Einstein gauge can be formulated in terms of a wavefunction's frequency of oscillation: $\ell = i\hbar f/c^4$ (where frequency $f = c/\lambda$ and energy $E = \hbar f$).

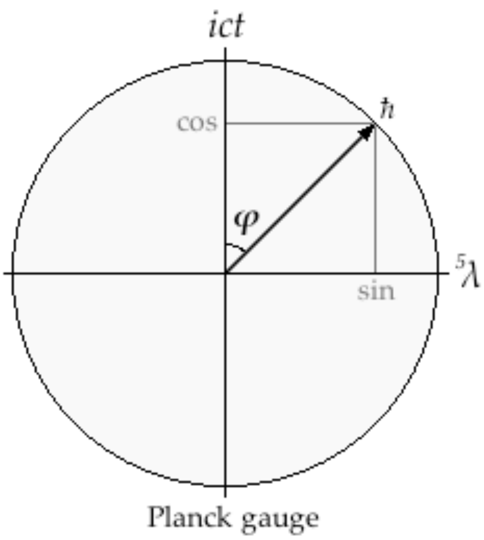
The Planck gauge was originally formulated[7] as $i\hbar/mc$ in the context of higher-dimensional (Kaluza–Klein) gravitation. In electromagnetism, this imaginary dimension may be equivalently expressed as $\ell = i\hbar/qc = iq/c^2$. In regards to electrical potential (voltage), it is formulated by $\ell = q/V = i\hbar c/qV$, and in quantum mechanics it is most usefully expressed in terms of potential energy, viz. $\ell = i\hbar c/U_0 = iq^2/U_0$ (where $U_0 = mc^2 = qV$). The fifth spatial dimension is associated with potential energy and time, analogous to the fourth dimension's association with spatial position and momentum (kinetic energy).

The spatial hyper-dimensions associated with potential and kinetic energy are obviously orthogonal, since total energy squared is given by $E^2 = E_P^2 + E_K^2$ (i.e. the Pythagorean solution of a right triangle). Heisenburg's Uncertainty Principle can also be expressed in terms of position and momentum, as $\sigma_x\sigma_p \geq \hbar/2$, or in terms of energy and time, viz. $\sigma_E\sigma_t \geq \hbar/2$, revealing the fundamental "principle of complementarity" as originally defined by Bohr.[9]

The simplest model for such extra spatial dimensions is provided by the complex plane ("Argand diagram"), a geometric representation of complex numbers bounded by a real axis orthogonal to an imaginary axis. Within this complex plane, the unit vector sweeps out a circle in accordance with an exponential function of the form $e^{i\varphi} = \cos\varphi + i\sin\varphi$ isomorphic to the circle group **U(1)**, having a radial modulus (absolute value) of $\hbar$.

The circumferential length of this circle is $2\pi\hbar$, i.e. Planck's constant h, representing the wavelength of a harmonic oscillator having the minimal quantum of energy hf. The surface area of a complex 3-sphere having radius $\hbar$ is given by h^2/π, that of a 4-sphere by $h^3/4\pi$, and that of the complex 5-sphere by $h^4/6\pi^2$. These various expressions relating hyper-spherical surface areas to Planck's constant have significance for Maldacena's anti-de Sitter/conformal field theory (AdS/CFT) correspondence, referred to as the "holographic principle",[10] and for the Bekenstein-Hawking entropy of black holes. Quantum resonance phenomena such as spin, Zitterbewegung,[11] and electron orbitals can be understood as hyper-spherical harmonics[12] within such N-dimensional surfaces.





The Einstein gauge has one real axis representing spatial position, canonically unitary (the Planck length). Its imaginary axis, designated ${}^4\lambda$, is determined canonically by unit Planck momentum divided by c^3, or equivalently by unit Planck energy divided by c^4.

The Planck gauge has no real axes, so is entirely imaginary. Its "time" axis is canonically the Planck time multiplied by ic, orthogonal to which is an imaginary axis designated ${}^5\lambda$, canonically determined as $\hbar/m_Pc$, the Compton wavelength of the Planck mass (or the wavelength of the Planck frequency). It can equivalently be expressed as $\hbar c/U_0$, where unit rest-mass energy U_0 is defined as that of the Planck mass, m_Pc^2. The energy of a photon having angular frequency ω is given by $\hbar\omega$, which also equates Planck energy with the Planck angular frequency ω_P multiplied by the reduced Planck constant.

Each Planck quantity within the 6D matrix holds both a "mass-gravity unit" and an "electromagnetic unit", which are orthogonal to each other, but have the same dimensions of space and time.

The Phase Angle

The dimensionless phase angle (φ) of these "imaginary circular dimensions" is well-known in electromagnetics, wherein a magnetic field causes a change in magnetic phase given by $\Delta\varphi_m = -(q/h)\cdot\int\mathbf{A}\cdot ds,$[13] where **A** is the vector potential in the Einstein gauge. Likewise, an electrostatic field changes the electrical phase of a sinusoidal waveform by $\Delta\varphi_e = -(q/h)\cdot\int\varphi\cdot dt,$[13] where φ is the scalar electric potential (voltage) in the Planck gauge.

Analogously, a gravitational potential rotates the phase angle of an electrically-neutral particle such as a neutron. This was convincingly demonstrated by Colella, Overhauser and Werner in 1975,[14] and strongly suggests the existence of a gravitational phase angle, φ_g.

General Relativity relates gravitational curvature to mass-energy volumetric density, so the gravitational phase angle is related to energy density by a function of $\rho_E = mc^2/ds^3$, i.e. rest-mass energy divided by volume. It can be shown that the phase angle delta in a gravitational potential is expressed by $\Delta\varphi_g = -(m/h)\cdot\int\rho_E\cdot dt^3$ in the Einstein gauge. Similarly, in the Planck gauge, the "temporal" phase angle delta is given by $\Delta\varphi_t = -(m/h)\cdot\int(dL_G/dt^3)\cdot ds^3$ where L_G is the quantity of gravitational inductance (dimensioned as T^2/L), i.e. the resistance of vacuum to establishing a field.

The gravitational phase angle can also be shown to be a function of mass and radial distance. The gravitational phase angle reaches $\pi/2$ when the mass density in a volume of space reaches one-quarter Planck density ($\rho_P/4$), at which point an event horizon forms, where time is infinitely dilated and space is infinitely Lorentz-contracted. Since the Schwarzschild radius is given by $2GM/c^2$ where $\varphi_g = \pi/2,$ it can be shown that $\sin(\varphi_g) = GM/\mathbf{r}c^2$.

This simple mathematical relationship is equivalent to Newton's laws of gravitation in flat 3D Euclidean space, and to Einstein's General Theory of Relativity in 4D Minkowski space. Spacetime curvature (the "gravitational field") is thereby seen to be a 4D artefact of the *rotated* gravitational phase angle φ_g at every point of 6D spacetime proximal to a gravitating mass. Each such infinitesimal point *induces* (forces) a gravitational phase shift $\Delta\varphi_g$ in neighbouring spacetime points, this induction effect diminishing as the inverse-square of radial distance ($1/r^2$) from the mass *m*.

Analogously, electromagnetic "fields" can be conceptualised as spacetime curvature due to the magnetic and electric phase angles being rotated by EM potentials (Φ and **A**). Such EM phase-curvature explains the Aharonov-Bohm effect,[15] when a charged particle's wavefunction is phase-shifted while passing through a region shielded from electromagnetic fields, i.e. **B** and **E** are zero.

In the absolute limit of mass density (Planck density ρ_P), where Planck mass is compressed into a Planck volume, the gravitational phase angle reaches 2π. Presumably, this represents the extreme state of spacetime and compressed matter-energy at the core of a black hole, where space and time are entirely imaginary: mirror-inverted, complex-conjugated spacetime.

Mathematically, such a spacetime *exists*, but it can in no way be considered *real.* It is the (un-)physical state referred to by Hawking[16] as "imaginary time".

Geometrical Calculus

Within the log-log Space/Time matrix, obtaining the quantity to the lower-right (on the diagonal) is performed by multiplying it by *c* (speed of light). Conversely, a unit to the upper-left is obtained by dividing by *c*.

Multiplication of quantities is effected by simply adding the two units' dimensional indices, i.e. $L^aT^b \times L^cT^d = L^{(a+c)}T^{(b+d)}$. Division of one unit by another is performed by subtracting the denominator's space/time indices from the numerator's.

Noting that the differential (gradient) of any quantity with respect to time (d/dt) is the unit immediately below it, and that the differential of a quantity with respect to space (d/ds or ∇) is the unit to its left, the Maxwell-Heaviside equations can be discerned in the relationships between these electromagnetic quantities.

Conversely, integrals of quantities within the 6D matrix can easily be obtained. Integration with respect to time gives the unit above, while integrating over space gives the unit to the right (increasing the logarithmic index of L).

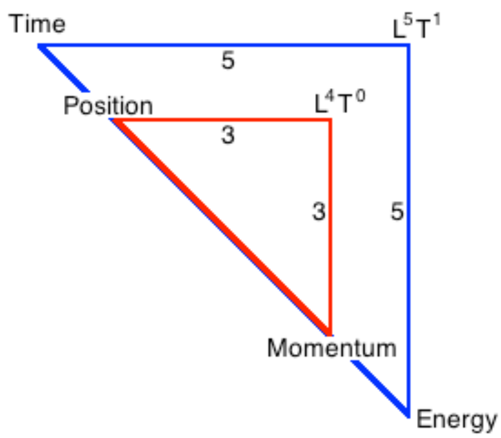
L/T	L^0	L^1	L^2	L^3	L^4	L^5
T^0	Electric constant ϵ_0	Wavelength λ Capacitance C	Surface area	Volume	4D-volume	5D-volume
T^{-1}	Magnetic flux density $\nabla \times \mathbf{A}$	Magnetic vector potential $\nabla \phi$	Magnetic flux $\Phi = \int \mathbf{B} \cdot d\mathbf{A}$			
T^{-2}	Charge density $d\mathbf{B}/dt = \nabla \cdot \mathbf{E}$	Electric field strength $d\mathbf{A}/dt$	Electrical potential $d\phi/dt = \nabla Q$	Charge Q		
T^{-3}		Current density $d\mathbf{E}/dt = \nabla \times \mathbf{H}$	Magnetic field strength $\nabla \mathbf{I}$	Current $dQ/dt = \nabla \mathbf{p}$	Momentum $\mathbf{p}$	Planck Action h
T^{-4}			Energy density $d\mathbf{H}/dt$	$d\mathbf{I}/dt = \nabla \mathbf{F}$	Force $d\mathbf{p}/dt = \nabla U$	Energy U_ε
T^{-5}				Poynting vector $\mathbf{S}$	$d\mathbf{F}/dt = \nabla \mathbf{P}$	Power $\mathbf{P}$

This simple geometrical calculus is highly effective in helping understand the core mathematical relationships which correlate and connect all the fundamental physical units and quantities.

Because 6D spacetime geometry contains three imaginary dimensions, three orthogonal Argand planes exist. As a consequence of this, the Space/Time matrix displays internal symmetries corresponding to the **SU(3)** × **SU(2)** × **U(1)** unitary group, consistent with the Standard Model.

An intriguing digression

The 4D Einstein gauge is bounded by position and momentum (kinetic energy) axes, whereas the 5D Planck gauge is bounded by time and inverse-energy. Those quantities are separated (in the 6D matrix) by three orders of magnitude of length and three of time, and by five orders of magnitude of length and five of time, respectively.



This geometry causes the ratios 3/5 and 5/3 to emerge as numerical factors in the Planck values of some electromagnetic quantities, notably voltage and current, as if these ratios are "baked into" the structure of spacetime itself. It also appears in the equation for gravitational binding energy, viz. $U = 3GM^2/5R$. The unit of Planck length is defined as a function of c^3, whereas the unit of Planck time is a function of c^5. Thus, the same 3/5 ratio appears in these fundamental space and time indices.

Conclusions

The "mass-energy stuff" comprising our universe is, in reality, nothing more substantial than space and time, or more fundamentally, quantum wavelengths and frequencies. All physically observable, measurable quantities consist of relationships between "mathematical entities"[17] having various configurations of spatial dimensions and temporal dimensions. Thus, mass, charge and energy are emergent phenomena, not fundamental building blocks.

The ontology of mass, charge, quantum spin and energy are thereby revealed to be purely geometrical: the fundamental ontological entities include lines (radius and circumference), angles (phase), simple shapes (e.g. triangles, squares, circles), surface areas and volumes.

The primary ontological entities are n-spherical surfaces embedded in higher dimensions, e.g. the 4-sphere and 5-sphere. Such hyper-spheres are weakly coupled to real 3D Euclidean space, so n-dimensional quantum wavefunctions appear to behave probabilistically. Were we able to view such wavefunctions in all their higher dimensions, they would appear to be unfolding entirely deterministically.

We inhabit a 3D hologram, embedded in an infinitely-dimensioned imaginary hyperspace. Matter, forces and energy emerge from the geometry and curvature of this complex spacetime. The bedrock foundation of the entire universe *is* multi-dimensional mathematics.

c^{-5}	$\sqrt{\frac{\hbar}{c^{13}}}$	$\frac{\hbar}{c^{8}}$	$\sqrt{\frac{\hbar^3}{c^{19}}}$	$\frac{\hbar^2}{c^{11}}$	$\sqrt{\frac{\hbar^5}{c^{25}}}$	$\frac{\hbar^3}{c^{14}}$	$\sqrt{\frac{\hbar^7}{c^{31}}}$	$\frac{\hbar^4}{c^{17}}$	$\sqrt{\frac{\hbar^9}{c^{37}}}$	$\frac{\hbar^5}{c^{20}}$
$\sqrt{\frac{1}{\hbar c^5}}$	c^{-4}	$\sqrt{\frac{\hbar}{c^{11}}}$	$\frac{\hbar}{c^{7}}$	$\sqrt{\frac{\hbar^3}{c^{17}}}$	$\frac{\hbar^2}{c^{10}}$	$\sqrt{\frac{\hbar^5}{c^{23}}}$	$\frac{\hbar^3}{c^{13}}$	$\sqrt{\frac{\hbar^7}{c^{29}}}$	$\frac{\hbar^4}{c^{16}}$	$\sqrt{\frac{\hbar^9}{c^{35}}}$
$\frac{1}{\hbar}$	$\sqrt{\frac{1}{\hbar c^3}}$	c^{-3}	$\sqrt{\frac{\hbar}{c^{9}}}$	$\frac{\hbar}{c^{6}}$	$\sqrt{\frac{\hbar^3}{c^{15}}}$	$\frac{\hbar^2}{c^{9}}$	$\sqrt{\frac{\hbar^5}{c^{21}}}$	$\frac{\hbar^3}{c^{12}}$	$\sqrt{\frac{\hbar^7}{c^{27}}}$	$\frac{\hbar^4}{c^{15}}$
$\sqrt{\frac{c^5}{\hbar^3}}$	$\frac{c}{\hbar}$	$\sqrt{\frac{1}{\hbar c}}$	c^{-2}	$\sqrt{\frac{\hbar}{c^{7}}}$	$\frac{\hbar}{c^{5}}$	$\sqrt{\frac{\hbar^3}{c^{13}}}$	$\frac{\hbar^2}{c^{8}}$	$\sqrt{\frac{\hbar^5}{c^{19}}}$	$\frac{\hbar^3}{c^{11}}$	$\sqrt{\frac{\hbar^7}{c^{25}}}$
$\frac{c^5}{\hbar^2}$	$\sqrt{\frac{c^7}{\hbar^3}}$	$\frac{c^2}{\hbar}$	$\sqrt{\frac{c}{\hbar}}$	c^{-1}	$\sqrt{\frac{\hbar}{c^{5}}}$	$\frac{\hbar}{c^{4}}$	$\sqrt{\frac{\hbar^3}{c^{11}}}$	$\frac{\hbar^2}{c^{7}}$	$\sqrt{\frac{\hbar^5}{c^{17}}}$	$\frac{\hbar^3}{c^{10}}$
$\sqrt{\frac{c^{15}}{\hbar^5}}$	$\frac{c^6}{\hbar^2}$	$\sqrt{\frac{c^9}{\hbar^3}}$	$\frac{c^3}{\hbar}$	$\sqrt{\frac{c^3}{\hbar}}$	1	$\sqrt{\frac{\hbar}{c^{3}}}$	$\frac{\hbar}{c^{3}}$	$\sqrt{\frac{\hbar^3}{c^{9}}}$	$\frac{\hbar^2}{c^{6}}$	$\sqrt{\frac{\hbar^5}{c^{15}}}$
$\frac{c^{10}}{\hbar^3}$	$\sqrt{\frac{c^{17}}{\hbar^5}}$	$\frac{c^7}{\hbar^2}$	$\sqrt{\frac{c^{11}}{\hbar^3}}$	$\frac{c^4}{\hbar}$	$\sqrt{\frac{c^5}{\hbar}}$	c	$\sqrt{\frac{\hbar}{c}}$	$\frac{\hbar}{c^{2}}$	$\sqrt{\frac{\hbar^3}{c^{7}}}$	$\frac{\hbar^2}{c^{5}}$
$\sqrt{\frac{c^{25}}{\hbar^7}}$	$\frac{c^{11}}{\hbar^3}$	$\sqrt{\frac{c^{19}}{\hbar^5}}$	$\frac{c^8}{\hbar^2}$	$\sqrt{\frac{c^{13}}{\hbar^3}}$	$\frac{c^5}{\hbar}$	$\sqrt{\frac{c^7}{\hbar}}$	c^2	$\sqrt{\hbar c}$	$\frac{\hbar}{c}$	$\sqrt{\frac{\hbar^3}{c^{5}}}$
$\frac{c^{15}}{\hbar^4}$	$\sqrt{\frac{c^{27}}{\hbar^7}}$	$\frac{c^{12}}{\hbar^3}$	$\sqrt{\frac{c^{21}}{\hbar^5}}$	$\frac{c^9}{\hbar^2}$	$\sqrt{\frac{c^{15}}{\hbar^3}}$	$\frac{c^6}{\hbar}$	$\sqrt{\frac{c^9}{\hbar}}$	c^3	$\sqrt{\hbar c^3}$	$\hbar$
$\sqrt{\frac{c^{35}}{\hbar^9}}$	$\frac{c^{16}}{\hbar^4}$	$\sqrt{\frac{c^{29}}{\hbar^7}}$	$\frac{c^{13}}{\hbar^3}$	$\sqrt{\frac{c^{23}}{\hbar^5}}$	$\frac{c^{10}}{\hbar^2}$	$\sqrt{\frac{c^{17}}{\hbar^3}}$	$\frac{c^7}{\hbar}$	$\sqrt{\frac{c^{11}}{\hbar}}$	c^4	$\sqrt{\hbar c^5}$
$\frac{c^{20}}{\hbar^5}$	$\sqrt{\frac{c^{37}}{\hbar^9}}$	$\frac{c^{17}}{\hbar^4}$	$\sqrt{\frac{c^{31}}{\hbar^7}}$	$\frac{c^{14}}{\hbar^3}$	$\sqrt{\frac{c^{25}}{\hbar^5}}$	$\frac{c^{11}}{\hbar^2}$	$\sqrt{\frac{c^{19}}{\hbar^3}}$	$\frac{c^8}{\hbar}$	$\sqrt{\frac{c^{13}}{\hbar}}$	c^5

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Appendix A

Numeric values for the "G-normalised" Planck units may be calculated using a simple spreadsheet, as illustrated below. The Planck length ℓ_P is obtained from the "hard-coded" values for light-speed c [L/T] and the Planck constant $\hbar$ [L^5/T^3] using the expression $\ell_P = \sqrt{(\hbar/c^3)}$. The value of Planck time t_P (calculated from $t_P = \ell_P/c$) is used to obtain the value for Planck frequency [1/T]. These units are then squared, cubed, etc, to populate the length and frequency (inverse-time) axes. Each of the diagonals can then be populated via successive multiplications by c.

L/T	L^0	L^1	L^2	L^3	Λ^4	Λ^5	Λ^6
F^0	$G = \varepsilon_0 = 1$	1.978E-30	3.914E-60	7.743E-90	1.532E-119	3.031E-149	0
F^1	1.515E+38	**299792458**	5.931E-22	1.173E-51	2.321E-81	4.592E-111	1.797E-170
F^2	2.296E+76	4.543E+46	8.988E+16	1.778E-13	3.518E-43	6.959E-73	4.128E-94
F^3	3.480E+114	6.884E+84	1.362E+55	2.694E+25	5.331E-05	**1.055E-34**	6.255E-56
F^4	5.273E+152	1.043E+123	2.064E+93	4.083E+63	8.078E+33	1.598E+04	2.841E-09
F^5	7.990E+190	1.581E+161	3.127E+131	6.187E+101	1.224E+72	2.422E+42	1.436E+21
F^6	∞	3.630E+237	1.421E+178	2.811E+148	5.561E+118	1.100E+89	2.176E+59

More spatial dimensions can easily be added to such a spreadsheet, although some of the numeric values therein become quite surreal (either extremely large, or infinitesimally small). It is thereby possible to calculate the values of the Planck units out to any number of spacetime dimensions.

Exercise for the gentle reader: determine the value of the Planck unit of gravitational inductance (L_G). Hint: its dimensions are T^2/L (seconds^2/metre). Also calculate the Planck unit of gravitational permeability (μ_G), i.e. inductance per metre. Hint: $\mu_G = 1/c^2$ (seconds^2/metre^2).