

33 variables are often categorical factors and there are very many possible habi-
34 tat factors that could determine each species. Integration of continuous and
35 categorical models into a single model can be a challenge.

36 On the basis of these and other studies we hypothesized in [18] that seg-
37 mented models could be used to fit arbitrary responses, would be at least as
38 accurate as continuous models, and be of greater accuracy when mining large
39 environmental data sets that contain mixed habitat and climatic variables. The
40 second hypothesis was confirmed using the segmented modelling method called
41 **WhyWhere** [18, 17] (WW1) and both are again addressed in this new version
42 (WW2). Moreover, we present theoretical and practical advances of the algo-
43 rithm enabling the goal of incorporating species distribution modelling into an
44 artificial intelligence environment [2].

45 2 Methods

46 A package for species modelling in R called **dismo** includes a number of popular
47 methods including MaxEnt, Bioclim, Domain, GLM, GAM, and RandomForest.
48 This package implements elements of the SDM work flow originally developed
49 in the GARP machine learning system [14], such as pseudo-absences, whereby
50 a random sample of the environment is in lieu of true absence values. The
51 **dismo** package contains bioclimatic variables from the WorldClim database [8]
52 and terrestrial biome data on terrestrial ecoregions [11].

53 Other R packages that have made **WhyWhere** more accessible are the **raster**
54 package for handling gridded spatial data [19] and a new package called **data.table**
55 for fast aggregation of large data sets [3].

56 Any species modelling method must address three main stages, and problems
57 arising at any of these stages can lead to poor results: (1) getting the environ-
58 mental data into a uniform form for analysis (2) determining the best type of
59 model to use to represent the response of the species (3) the interpretation of
60 the results.

61 2.0.1 Environmental variables

62 As geographic variables come in varying extents, projections and resolutions,
63 they must generally be unified by coregistration before modelling. But this entails
64 additional processing and inefficiencies. Smaller resolution sets must be enlarged
65 redundantly and information lost when contracted. Memory needs burgeon
66 when coregistering many variables to the finest scale. An ideal method would
67 utilize each data set at its native projection and resolution.

68 After coregistration of geographic layers, most statistical models input a
69 'wide' file format, i.e. with variables in columns and locations in rows. This
70 adds an intermediate processing step where all environmental data must be then
71 be held in memory, which can hit memory limitations at the prediction stage
72 when the models are applied across the geographic space.

73 The approach in WW1 was to transform geographic variables into a compact
74 image format and then to combine at most three variables into the image in the
75 red, green and blue channels. While fast image processing packages could then
76 be used for segmentation, the range of the variables were scaled between 0-255,
77 and also did not obviate the need for coregistration of geographic data. One
78 advance in WW2 is the manner of building multidimensional models. The key
79 insight is that the evaluation of a combination of two or more variables can
80 be performed on the predictions of the models, instead of being performed in
81 the high-dimensional model model space; a higher dimensional model is not
82 required.

83 For example, the response on a single variable is called a membership function
84 (for reasons explained later). Prediction assigns a membership value to each
85 data point in the training set, producing a vector of values in the range [0,1]
86 which can be evaluated (using the ROC or AUC). The membership vectors for
87 two single variables can be combined using a fuzzy AND operator to produce a
88 new membership vector. This membership vector can then be evaluated (using
89 the ROC or AUC), thus evaluating the performance of the conjunction with-
90 out calculating the two-dimensional membership function. The combination of
91 membership functions follows the AND, OR Zadeh operators [20]:

$$AND : x \wedge y = \min(f(x), f(y)) \quad (1)$$

$$OR : x \vee y = \max(f(x), f(y)) \quad (2)$$

92 This eliminates the need to develop and express a higher dimensional model,
93 enabling a data mining approach where we can explore large databases for a
94 parsimonious model of the species in analysis. Because of this we here focus on
95 single variable models.

96 2.0.2 Choice of model structure

97 Ecological theory maintains that the response of species to the environment is
98 generally 'humped' around an ideal, or restricted to a range of a variable (e.g.
99 the range of temperature tolerances). Such a model must be at least quadratic
100 to represent a unimodal response, and order three to incorporate skewness. An
101 ideal method would be robust to any non-linear response type. Many environ-
102 mental variables are categorical particularly soil and vegetation type. Ideally a
103 modeling system integrates both types: continuous (e.g. temperature, rainfall)
104 or categorical (e.g. biome or soil type) but very few do.

105 MaxEnt solves this problem by providing a range of potential response types
106 [12]. The approach in **WhyWhere** is cutting up the range of the variable into
107 discrete categorical factors. WW1 [18] used a color quantization algorithm in the
108 GIF image format. WW2 segments a single continuous or categorical variables
109 into open-closed intervals. For example, the output of the R cut function on the
110 numbers 1..4 into 2 levels as open-closed intervals:

```
> cut(1:4,2)
```

-Submitted-

```
[1] (0.997,2.5] (0.997,2.5] (2.5,4]      (2.5,4]
Levels: (0.997,2.5] (2.5,4]
```

111 The unique 4 categorical values could also be represented in the same open-
112 closed syntax.

```
> cut(1:4,4)
```

```
[1] (0.997,1.75] (1.75,2.5] (2.5,3.25] (3.25,4]
Levels: (0.997,1.75] (1.75,2.5] (2.5,3.25] (3.25,4]
```

113 The cuts do not need to be uniformly spaced. The default method of de-
114 termining the cut locations in WW1 followed the median cut algorithm which
115 assigns equal numbers of points to a limited number of categories [7] in the
116 red, green, blue 3D space. This has been shown to retain good visual appear-
117 ance, but also has a statistical justification of minimizing the variance across the
118 range, by minimizing the variance in each category. There are other methods
119 of multi-interval discretization of continuous-valued attributes for classification
120 learning [4] implemented in the `discretization` R package.

121 The calculation of the intensity of species' response is straightforward on a
122 segmented variable. The locations where the species occurs can be thought of
123 as a sampling of the environmental space with a count of values in the each
124 environmental category (S), and expressed as a density by normalizing the sum
125 over the values in the categories to one. The prior density of values in each
126 environmental category is G . The change in the density from G to S indicates
127 strength of the the response of the species to its environment in that category –
128 the membership function M . A membership function is a fuzzy truth value as
129 a function $f : \mathbb{R} \rightarrow [0, 1]$ from a numeric domain to the real unit interval. The
130 membership function we used in WW1 and in WW2 for each category i in f is:

$$M_i = S_i / (S_i + G_i) \quad (3)$$

131 What we would like is the conditional probability of species being present
132 given the environmental $P(S|G)$ for each category of G . Frequentest calculations
133 give us $P(G|S)$. While $P(S|G)$ could be obtained using Bayes Theorem, it
134 requires an estimate of probability of the occurrence of a species $P(S)$, which in
135 opportunistic data is not well defined, and dependent on season, search effort and
136 other uncontrolled variables. The best we can do is an approximate equivalence
137 that holds under certain conditions (principally independence of variables) and
138 has been shown to be sufficient and useful in modelling such relationships [15,
139 16]. Where β is a normalization factor:

$$P(S|E) = \beta P(E|S) \quad (4)$$

140 One may ask why not calculate S/G and not $S/(S + G)$? Because we are
141 developing a model on a background set, developed from a random sample of
142 points in G , this can result in points in S_i that are not in G_i . That is, the
143 species occurs in environments that are not represented in the background set.

144 Use of $S/(S + G)$ avoids a divide by zero error. The membership function is a
145 proportional relationship which is sufficient to compare the efficiency of single
146 variables.

147 There are many ways to evaluate skill of a model. The approach to evaluating
148 the strength of the response in WW1 was significance with the Chi-squared test
149 or a K-S test, however we find it more convenient to use the area under the
150 curve of the receiver operating characteristic, or AUC. The receiver operating
151 curve (ROC) is widely used to compare classification models, while the AUC
152 provides a robust measure of skill. It is the probability that a model correctly
153 classifies a random draw of a positive and negative example.

154 **2.0.3 Ecological interpretation**

155 How does the structure of a multi-dimensional model embody the ecology of the
156 species? Many methods used in species modelling are based in other domains
157 (e.g. linear regression) without clear ecological interpretation. For example,
158 when inconsistent units such temperature and rainfall are combined in a gener-
159 alized linear model, how are they to be interpreted?

160 Ecological principles such as competitive interactions [13] tend to be in the
161 form of logical expressions. The principle of Liebig's Law of the Minimum states
162 that growth is controlled by the scarcest necessary resource. This is logically a
163 fuzzy conjunction of limiting factors – a Zadeh AND. Another law of ecology
164 is Gauss's law of competitive exclusion. This is a proposition that two species
165 competing for the same resource cannot coexist at constant population, due to
166 effect of slight advantages magnified over generations. This is a fuzzy disjunction
167 – a Zadeh OR. Thus fuzzy AND and OR can represent elements of established
168 ecological theory. The approach of modelling with a logical expression of a small
169 number of variables has utility in interpreting as ecological theory.

170 **2.1 WhyWhere Algorithm**

171 The first step is to 'presample' which when given a set of presence data locations,
172 and a geographic file that serves as a mask, produces a combined list of the
173 presence data and a randomly sampled list of sites of both presence and absence.
174 The mask file defines the geographic extent for sampling the background data.
175 If absences are available then they need not be generated.

176 The inputs to WW2 are: a **data.table** from presample with the longitude
177 and latitude of known locations, and a list of environmental data files that
178 may be read into the **raster** package. This file is then input to the main
179 routine with a list of the geographic files. Parameters include **multi** for searching
180 conjunctions of variables and **segment** to select the form of segmentation.

181 The algorithm proceeds by looping through the environmental variables and
182 creating and evaluating a membership function on each one. A table of the
183 variables so-far is retained. In the current implementation of a multi-dimensional
184 model, only the best variable is combined with each new variable using the fuzzy
185 minimum and the AUC recalculated. Alternative approaches to searching the

186 space of conjunctions may be implemented in future. It is possible to monitor
187 the progress with the plot option. This plots out the best model so far and
188 prints out a list of the best models considered in a streaming work flow.

Listing 1: Listing of the main algorithm

```

189 input locations
190 input a mask file
191 prepare background points and combine with presence points
192 for all environmental files do
193   develop membership function for file
194   insert AUC and variable name into ordered result
195   test conjunct of this variable with best so far
196   if result is better then insert into ordered result
197 output table of results
198

```

200 We show the results for the brown-throated three-toed sloth (*Bradypus variegatus*)
201 that is documented and modeled in `dismo`. The environmental data consist of 9
202 environmental files from the WorldClim data set covering the South American
203 continent. Figure 1 shows (A) the best variable, (B) the AUC of the best model,
204 (C) the membership function, and (D) the predicted distribution with the pres-
205 ence points. The highest rated variable in the *Bradypus* data set is *bio7* = the
206 temperature annual range (*bio5* – *bio6*) where *bio5* = maximum temperature
207 of the warmest month and *bio6* = minimum temperature of the coldest month.
208 The result is very similar to the results from GARP and MaxEnt in [12]. Table 2
209 lists the results for all variables.

Table 1: The AUC of environmental variables on the *Bradypus* data: WW is WhyWhere and GLM is a third order generalised linear model.

	name	WW	GLM
1	bio7	0.76	0.74
2	bio12	0.73	0.70
3	bio6	0.72	0.67
4	bio17	0.70	0.67
5	bio16	0.68	0.67
6	biome	0.67	0.65
7	bio5	0.63	0.61
8	bio1	0.63	0.59
9	bio8	0.59	0.57
10	limit	0.00	0.00

210 The membership function shown graphically in Figure 1.C is represented
211 internally as a lookup table, shown in Table2. The prior distribution (*G1*) is the
212 blue line, and the distribution of presences (*G2*) is the red line. The membership
213 function is shown by the grey bars (*G1/(G1 + G2)*). Note the almost uniform
214 distribution of background classes in the variable width quantile cuts.

-Submitted-

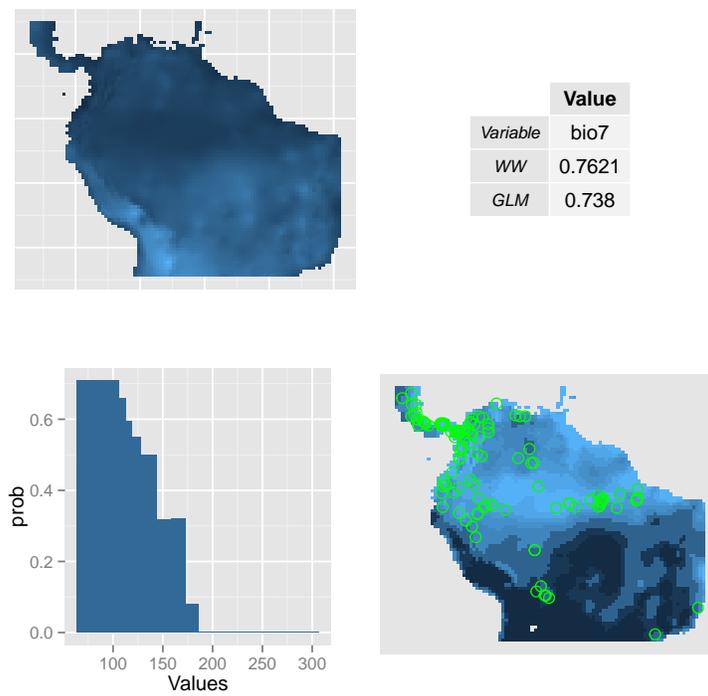


Figure 1: Output of WW2 on the Bradypus data set. (A) the best variable, (B) the AUC of the best model, (C) the membership function, and (D) the predicted distribution with the presence points in green.

Table 2: The lookup table for the membership function on the *Bradypus* data. Each row is a segment of the range of the variable (factors). The density of points in each segment in the background (g1) and the points where species occur (g2) is used to calculate the odds ration, and then membership in each segment. The width is the size of the segment interval

	factors	levels	g1	g2	odds	prob	width
1	(64,106]	1	0.13	0.31	2.44	0.71	42.00
2	(106,113]	2	0.11	0.22	1.94	0.66	7.00
3	(113,119]	3	0.11	0.16	1.47	0.59	6.00
4	(119,128]	4	0.10	0.12	1.21	0.55	9.00
5	(128,144]	5	0.10	0.10	1.00	0.50	16.00
6	(144,159]	6	0.09	0.04	0.47	0.32	15.00
7	(159,173]	7	0.09	0.04	0.47	0.32	14.00
8	(173,186]	8	0.10	0.01	0.09	0.08	13.00
9	(186,200]	9	0.08	0.00	0.00	0.00	14.00
10	(200,307]	10	0.09	0.00	0.00	0.00	107.00

215 3 Experiments

216 3.1 K-fold Validation

217 K-fold validation provides a robust evaluation of the accuracy of a method
 218 on independent data. The data was prepared by generating a data set using
 219 presample, and then labeling each row from one to 5. K-fold validation proceeds
 220 by sequentially holding back one fifth of the data each time for evaluation, and
 221 developing the model using the remaining four-fifths. Table 3 shows the results
 222 of the five-fold validation.

Table 3: Results of a five-fold validation of *Bradypus* model. WW is WW2 and GLM is generalised linear regression. the suffix tr is the AUC on the training set and te is the AUC on the test set.

	name	WWtr	GLMtr	WWte	GLMte
1	bio7	0.77	0.73	0.77	0.78
2	bio7	0.77	0.74	0.74	0.73
3	bio7	0.76	0.73	0.74	0.78
4	bio7	0.76	0.73	0.78	0.77
5	bio7	0.77	0.74	0.74	0.75
6	mean	0.77	0.73	0.75	0.76
7	sd	0.00	0.00	0.02	0.02

223 The accuracies were similar on test and training sets and between the WW2
 224 and the GLM models, and the best variable *bio7* was chosen consistently. The
 225 performance of WW2 not dissimilar to the GLM.

226 **3.2 Multi-dimensional option**

227 Table 4 lists the results for multidimensional models developed by combining
228 the prediction of two or more variables with a fuzzy AND operator and then
229 evaluating the resulting AUC.

Table 4: The AUC of environmental variables on multi-dimensional prediction of the Bradypus data: WW is WhyWhere and GLM is a third order generalised linear model.

	name	WW	GLM
1	bio7.biome	0.85	0.90
2	bio7	0.78	0.75
3	bio12	0.73	0.70
4	bio17	0.73	0.68
5	bio6	0.72	0.67
6	bio12.bio6	0.72	0.71
7	bio12.bio17	0.72	0.68
8	bio16	0.68	0.66
9	bio12.bio16	0.68	0.66
10	biome	0.67	0.65
11	bio5	0.64	0.61
12	bio1	0.62	0.59
13	bio8	0.59	0.57
14	bio12.bio5	0.52	0.72
15	bio7.bio8	0.49	0.71
16	limit	0.00	0.00

230 The best result is a combination of *bio7* and *biome* variables (the AUC of
231 WW=0.7969 and GLM=0.906). The variable *biome* is a categorical variable
232 expressing ecosystem type, and so more like a habitat variable than a numeric
233 climatic variable.

234 **3.3 Alternate segmentation**

235 We also evaluated some alternative methods of segmenting the response function,
236 shown on Table 5: an even distribution of cuts over the range of the
237 variable, distribution by quantile frequency, and an entropy optimizing method.
238 The even method segments the variable evenly over the range. The quantile
239 method segments the variable so that as far as possible each segment contains
240 an equal number of data.

241 The number of segments is determined using the Freedman-Diaconis rule
242 [5] for optimal binning of histograms. The entropy method uses the R package
243 `discretization` and the routine `cutPoints()` that perform cuts for the Minimum
244 Description Length Principle. This analysis is not a comprehensive evaluation
245 of discretization method, but serves to validate the performance of the quantile
246 approach in comparison of some readily available alternatives.

Table 5: Results of a five-fold validation of these data with other segmentation approaches: even, quantile and entropy.

	name	WWtr	GLMtr	WWte	GLMte	segment
1	mean	0.77	0.73	0.75	0.76	even
2	mean	0.77	0.73	0.75	0.76	quantile
3	mean	0.75	0.72	0.71	0.72	entropy

247 The segmentation methods that performs best on the test set are the even
 248 and quantile methods, and their performance is similar on the training set.
 249 The entropy method performed less well. This provides support for the quan-
 250 tile approach that was used in WW1, although further testing of segmentation
 251 approaches may lead to improvements.

252 3.4 Prediction on World Data

253 To demonstrate the system on a larger data set we use the Global Ecosystems
 254 Database, a set of 940 global data sets of environmental variables, previously
 255 prepared and used in the WW1. The multi-agency distribution of the original
 256 CD includes many groups of variables listed in [17] including satellite green-
 257 ing, monthly temperature and rainfall and many others in a range of different
 258 resolutions in raster and vector formats.

Table 6: The top ten variables in a single variable WhyWhere model of *Bradypus* using the 940 variable World dataset. AUC is the WW AUC and BAUC is the AUC from GLM.

	name	AUC	BAUC
1	<i>fnocwat</i>	0.85	0.80
2	<i>srzsoil</i>	0.85	0.68
3	<i>wrzsoil</i>	0.85	0.58
4	<i>wrroot</i>	0.84	0.71
5	<i>lcprc08</i>	0.84	0.74
6	<i>lwerr05</i>	0.84	0.76
7	<i>wrcla01</i>	0.84	0.57
8	<i>wrcla03</i>	0.84	0.54
9	SALINITY_ANN_AVG	0.84	0.68
10	<i>wrsil02</i>	0.83	0.65

259 Table 6 lists the top 10 variables identified by the algorithm in predicting
 260 *Bradypus* on the World dataset. Out of 940 variables, the top variables were
 261 *fnocwat*: Navy Terrain Data - Percent Water Cover. The next three variables
 262 are soil classifications – *srzsoil*: Staub and Rosenzweig Zobler Soil Units, and
 263 Webb et al Soil Particle Size Properties Zobler Soil Types. The fifth variable is
 264 climatic: Leemans and Cramer August Precipitation (mm/month) which corre-
 265 sponds to the dry season in Amazonia. Note that the accuracy of the GLM is

266 less than the WW2 in this case.

267 It might be inferred from this limited study that habitat features have greater
268 predictive power than climatic variables over the region of distribution for this
269 species. By way of interpretation, *Bradypus variegatus* does leave the trees
270 in search of food and while it crawls along the forest floor poorly, it does swim
271 well [6]. Its distribution may be closely linked to the flooded forest (defined as a
272 seasonal inundation of the forest floor) facilitating access to other trees for food.
273 The identification of soil classification may be indicative of the soils supporting
274 a the flooded forest ecosystem.

275 Habitat variables are proximal causes of species which necessarily produce
276 higher accuracy than the more distal climatic variables. It is well known that
277 habitat features are crucial in identifying suitable areas of land for the conser-
278 vation of threatened species, and due to the proximal relationship should be a
279 more important determinant of species decline than distal factors such as cli-
280 mate change. This is not a definitive examination of determinants of Bradypus,
281 but serves to illustrate the potential expositions available from this approach.

282 4 Discussion

283 This study evaluated the accuracy of a segmented model and algorithm in an
284 updated version of **WhyWhere** against a generalized linear model, and also mod-
285 elling species response to climate variables and accuracy on a large data set
286 containing mixtures of continuous and categorical environmental data.

287 The accuracies were similar on the Bradypus data set and **WhyWhere** was
288 superior on the large World dataset, attributed to selection of the best WW2
289 variables and handling of categorical as well as continuous variables. The ben-
290 efit of WW2 are more accurate species prediction, and potential insight into
291 proximal cause of the species occurrences. The results verify the findings of
292 the previous version of the **WhyWhere**, showing progress that could be made
293 in modelling species response to the environment by using segmented models of
294 few variables mined from large databases of environmental variables.

295 The recoding of **WhyWhere** into R has greatly improved the programs' utility.
296 Refinements to the algorithm reduce the steps in the species modelling work-
297 flow and support more efficient higher dimensional models using novel fuzzy set
298 operators.

299 It is interesting to note that that the two variables identified in the multi-
300 variable mode are habitat variables (*biome*) and a climate variable (*bio7*). We
301 hypothesize that climate and habitat factors are independent causal factors
302 that together determine species distribution, and that the multi-dimensional
303 WW2 can correctly identify such independent determinants of species response,
304 yielding a parsimonious explanation of the species' response to its environment.

305 More complex expressions of species distribution models in the R package
306 **WhyWhere** are planned, as are improvements in computational efficiency and
307 testing over a greater range of higher resolution environmental data.

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