International Journal of Emerging Technology and Advanced Engineering
Website: www.ijetae.com (ISSN 2250-2459, ISO 9001:2008 Certified Journal, Volume 6, Issue 10, October 2016)

Determination of Field Intensities Belonging to the Wedge Regions Adjacent to A Convex Triangular Obstacle Subject to Asymmetric Conditions

Sanjay Kumar[1], Dr. Amita Sharma[2]

[1]P. G. Centre Department of Mathematics, [2]P.G. Department of Physics, Ram Dayalu Singh College, Muzaffarpur-842002, Bihar, B. R. A. Bihar University, Muzaffarpur, Bihar, India

Abstract : **Electromagnetic (EM) field intensities happen to exist as solutions of Maxwell's equations in a three dimensional space. In the present paper, an attempt has been made to determine the components of EM field intensities belonging to a pair of groove regions adjacent to a convex triangular prism. Field intensities are supposed to be asymmetric in the space R^3, and the triangular prism forms a part of an echellete grating of fixed period. The governing Maxwell's equation is solved subject to the Dirichlet conditions of the filed intensity $\boldsymbol{F} = (\boldsymbol{H} \vee \boldsymbol{E})$ on the boundaries of the said groove regions. The concerning mathematical ideas happen to be associated with the properties of associated Legendre function. Twelve spherical wave functions have been determined for finding the components of the said field intensities. Two existence theorems, concerning an asymmetric spherical wave, have been established. Finally, the expressions of the field intensities $\boldsymbol{H}$ and $\boldsymbol{E}$ have been utilized for determining the current density.**

Keywords: **Electromagnetic field intensities, convex triangular prism, Maxwell's equations.**

I. INTRODUCTION

A convex triangular obstacle forms a vital part of a periodic echellete antenna. In recent years [1-8] quite a good number of results have been reported pertaining to the groove field estimates and the efficiency of the said grating. The present paper deals with a general convex triangular prismatic obstacle K(Figure 1) having an open rectangular base, a flare angle β, the groove depth 'h' and the grating period 'd' (Figure 2). In the present paper a model M, has been allowed to interact with an asymmetric EM field $\boldsymbol{F} = (\boldsymbol{H} \vee \boldsymbol{E})$ satisfying the Maxwell's equations

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} = \sigma \boldsymbol{E} + \in \frac{\partial \boldsymbol{E}}{\partial t}, \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\mu \frac{\partial \boldsymbol{H}}{\partial t}$$

and $$\nabla^2 \boldsymbol{F} = \mu\left(\sigma \frac{\partial \boldsymbol{F}}{\partial t} + \in \frac{\partial^2 \boldsymbol{F}}{\partial t^2}\right)$$

The equations have been transformed by using spherical polar coordinates $x_1 = r\sin\theta\cos\phi$, $x_2 = r\sin\theta\sin\phi$ and $x_3 = z = r\cos\theta$ resulting to an asymmetric spherical wave. The solutions of the Maxwell's equation have been determined from the following forms of EM problems :

$$\nabla^2 \boldsymbol{F} = \mu\left(\sigma \frac{\partial \boldsymbol{F}}{\partial t} + \in \frac{\partial^2 \boldsymbol{F}}{\partial t^2}\right)$$

$$\boldsymbol{F}\big|_{\partial K} = \boldsymbol{f} \qquad \text{Dirichlet's problem)}$$

Where ∂K stands for the plane bounding faces of the model M. Two existence theorems, concerning an asymmetric spherical wave have been established. Twelve spherical wave functions have been determined in terms of the components of $\boldsymbol{E}$ and $\boldsymbol{H}$. The concerning mathematical ideas happen to be associated with the properties of associated Legendre function $P_n^m(x)$ $(m, n \in J^+)$ and oblique coordinate transformations.

IJETAE EXPLORING RESEARCH AND INNOVATIONS

International Journal of Emerging Technology and Advanced Engineering
Website: www.ijetae.com (ISSN 2250-2459, ISO 9001:2008 Certified Journal, Volume 6, Issue 10, October 2016)

1. Formulation of the problem

Consider the Maxwell's equation [9]

$$\nabla^2 \boldsymbol{F} = \frac{\partial^2 \boldsymbol{F}}{\partial x_1^2} + \frac{\partial^2 \boldsymbol{F}}{\partial x_2^2} + \frac{\partial^2 \boldsymbol{F}}{\partial x_3^2} = \mu\left(\sigma \frac{\partial \boldsymbol{F}}{\partial t} + \in \frac{\partial^2 \boldsymbol{F}}{\partial t^2}\right) \qquad (1)$$

Satisfied by the asymmetric field intensity vector

$$\boldsymbol{F} = \boldsymbol{F}(x_1, x_2, x_3, t)$$

Using the spherical polar coordinate transformation

$$x_1 = r\sin\theta\cos\phi,\ x_2 = r\sin\theta\sin\phi,\quad x_3 = r\cos\theta = z$$

one can transform the equation (1) in the form

$$\nabla^2 \boldsymbol{F} = \frac{1}{r^2\sin\theta}\left[r^2\sin\theta\frac{\partial^2 \boldsymbol{F}}{\partial r^2} + 2r\sin\theta\frac{\partial \boldsymbol{F}}{\partial r} + \sin\theta\frac{\partial^2 \boldsymbol{F}}{\partial \theta^2} + \cos\theta\frac{\partial \boldsymbol{F}}{\partial \theta} + \frac{1}{\sin\theta}\frac{\partial^2 \boldsymbol{F}}{\partial \phi^2}\right] = \mu\left(\sigma\frac{\partial \boldsymbol{F}}{\partial t} + \in\frac{\partial^2 \boldsymbol{F}}{\partial t^2}\right) \qquad (2)$$

Now, applying variable separable method for the equation (2), one can arrive at the solution

$$F(r,\theta,\phi,t) = F_1(r)F_2(\theta,\phi)G(t) \qquad (3)$$

Where the functions $F_1(r)$, $F_2(\theta,\phi)$ and $G(t)$ satisfy the equations

$$\left[\left(r^2F_1'' + 2rF_1'\right)\Big/ F_1 + \frac{\partial^2 F_2}{\partial\theta^2} + \cot\theta\frac{\partial F_2}{\partial\theta} + \cos ec^2\theta\frac{\partial^2 F_2}{\partial\phi^2}\Big/ F_2\right]\frac{1}{r^2} = \mu\left(\sigma G'(t) + \in G''(t)\right)/G(t) = -k^2 \qquad (4)$$

and

$$\left(r^2F_1'' + 2rF_1'\right)\Big/ F_1 + k^2r^2 = -\left(\frac{\partial^2 F_2}{\partial\theta^2} + \cot\theta\frac{\partial F_2}{\partial\theta} + \cos ec^2\theta\frac{\partial^2 F_2}{\partial\phi^2}\right)\Big/ F_2 = \xi \quad (5)$$

Where k and ξ are independent of $r,\ \theta,\ \phi$ and t.

In particular, assuming $\xi = n(n+1) \forall n \in J^+$, the equation (5) gives rise to the surface harmonic function $F_2(\theta,\phi)$ satisfying the PDE

$$\frac{\partial^2 F_2}{\partial\theta^2} + \cot\theta\frac{\partial F_2}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2 F_2}{\partial\phi^2} + n(n+1)F_2 = 0 \qquad (6)$$

Now, separating 'F_2'further in the form of the product

$$F_2(\theta,\phi) = F_3(\theta)F_4(\phi) \qquad (7)$$

One can arrive at the following ordinary differential equations (ODE)

$$F_4''(\phi) + l^2F_4(\phi) = 0 \qquad (8)$$

And

$$\left(1-x^2\right)F_3''(\theta)-2xF_3'(\theta)+\left[n(n+1)-l^2/1-x^2\right]F_3(\theta)=0 \qquad (9)$$

Where $x=\cos\theta$ and 'l' is zero or positive integer. The ODE (9) may be identified as associated Legendre equation which furnishes the associated Legendre's function $P_n^l(\cos\theta)$ as one of its solutions.

Thus, combining (7), (8) and (9), the surface harmonic function $F_2(\theta,\phi)$ may be expressed in the form

$$F_2(\theta,\phi)=C_3 e^{jl\phi}P_n^l(x) \qquad \left(j=\sqrt{-1}\right) \qquad (10)$$

where $$P_n^l(x)=\left(1-x^2\right)^{l/2}D^l P_n(x) \qquad \left(D\equiv\frac{d}{dx}\right) \qquad (11)$$

for $|x|<1$, and 'C_3' is an arbitrary constant.

Hence, considering the asymmetric EM field intensity $F(r,\theta,\phi,t)$ it follows that the surface harmonic function $F_2(\theta,\phi)$ forms a part of the EM field.

However, looking to the wide utility of Ferrar's function $T_n^l(x)$ for physical applications one is led to convert $P_n^l(x)$ in terms of $T_n^l(x)$ by means of the relation

$$T_n^l(x)=(-1)^{l/2}P_n^l(x) \qquad (12)$$

for $|x|<1$, and as such the surface harmonic function $F_2(\theta,\phi)$ given by (10) may be further expressed in the form

$$F_2(\theta,\phi)=C_3\exp\left(Jl(\phi-\pi/2)\right)T_n^l(\cos\theta) \qquad (13)$$

Now, recalling the equation (6), one can arrive at the linear ODE

$$r^2F_1''+2rF_1'+\left\{k^2r^2-n(n+1)\right\}F_1=0 \qquad (14)$$

Possessing the only regular singular point (RSP) at origin r = 0, consequently one can arrive at Frobenius solution in series

$$F_1(r)=\sum_{m=0}^{\infty}a_{2m}r^{2m+p} \qquad (15)$$

Around the origin, the series being convergent within a sphere $|r|=A$ of arbitrarily finite radius A. The values of the identical roots happen to be

$p = n$ and $-(n+1)$ (16)

And the coefficients of the series (15) may be determined by means of the recurrence relation

$$a_{2m}=\frac{-K^2a_{2(m-1)}}{\left[(p+2m)(p+2m+1)-n(n+1)\right]}\ \forall m\in J^+ \qquad (17)$$

Now, considering the right hand side of the equation (4) one can arrive at the solution

$$G(t)=A\exp\left(j\omega-\left(\frac{\sigma}{2\in}\right)\right)t \qquad (18)$$

Where 'A' is an arbitrary constant, 'ω' stands for the frequency of the EM wave satisfying the relation $\omega = \dfrac{\rho}{2\mu \in}$ and $\rho = \sqrt{-\mu^2\sigma^2 + 4\mu \in k^2}$ subject to the restriction $2k\sqrt{\in} > \sigma\sqrt{\mu}$ and $(\sigma/2\in)$ stand for time attenuation of the EM wave.

Therefore, combining (3), (13), (15) and (18), one can finally arrive at the asymmetric EM field

$$F(r,\theta,\phi,t) = \sum_{m=0}^{\infty} a_{2m} r^{2m+p} T_n^l(\cos\theta)\exp\left(jl(\pi h - \phi)\right)G(t) \qquad (19)$$

For an arbitrary choice of '*l*'

Dirichelet Conditions

In order to match the initial value of the field intensity (19) with the prescribed initial values of EM fields on the bounding faces ∂K of the model M one can arrive at the Dirichlet's condition

$$F(r,\theta,\phi,0)\big|_{OA} = F_1(x',0,x_3), F(r,\theta,\phi,0)\big|_{AC} = F_2(a,y',x_3)$$

$$F(r,\theta,\phi,0)\big|_{OB} = F_3(0,y',x_3) \text{ and } F(r,\theta,\phi,0)\big|_{BC'} = F_4(x',-b,x_3) \qquad (20)$$

Where the field intensity $F(r,\theta,\phi,t)$ is essentially expressed in the form

$$F(r,\theta,\phi,t) = \sum_{n=1}^{\infty} B_n^l T_n^l(\cos\theta) F_1(r)\exp\left(jl(\phi - \pi/2)\right)G(t) \qquad (21)$$

Now, making use of the transformation [3]

$$\begin{aligned} x'\sin\beta &= \rho\sin(\theta_0 + \beta + \phi) \\ y'\sin\beta &= \rho\sin(\theta_0 + \phi) \qquad (22) \\ \rho &= r\sin\theta \end{aligned}$$

Over the bounding faces *AC* and BC' of the model 'M', one can arrive at the values

$$\begin{aligned} F_1(r)\big|_{AC} &= F_1(a\sin\beta\cos ec\theta\cos ec(\theta_0 + \phi + \beta)) \\ &= \sum_{m=0}^{\infty} a_{2m}(a\sin\beta)^{2m+p}(\cos ec\theta\cos ec(\theta_0 + \phi + \beta))^{2m+p} \end{aligned} \qquad (23)$$

And

$$F_1(r)\big|_{BC'} = \sum_{m=0}^{\infty} a_{2m}(-b\sin\beta)^{2m+p}(\cos ec\theta\cos ec(\theta_0 + \phi))^{2m+p} \qquad (24)$$

Spherical wave functions and the components of electric and magnetic intensity vectors:

The expression (21) represents a spherical wave function

$$\psi(r,\theta,\phi,t) = \psi^F(r,\theta,\phi)e^{-t(\sigma/2\in - j\omega)} \qquad (25)$$

Where $\psi^F(r,\theta,\phi) = \sum_{n=1}^{\infty} B_n^l F_1(r) T_n^l(\cos\theta)e^{jl(\phi - \pi/2)}$ stands for the free space spherical wave formed by the superimposition of spherical waves of amplitude $B_n^l(F)$. The nature of these waves are similar to that given by (19).

Now, recalling the Maxwell's equations.

$\nabla\times\boldsymbol{H}=\sigma\boldsymbol{E}+\in\frac{\partial\boldsymbol{E}}{\partial t}$ and $\nabla\times\boldsymbol{E}=-\mu\frac{\partial\boldsymbol{H}}{\partial t}$, one can arrive at following relations :

$$\begin{aligned}\frac{\partial H_3}{\partial x_2}-\frac{\partial H_2}{\partial x_3}&=\sigma E_1+\in\frac{\partial E_1}{\partial t}\\ \frac{\partial H_1}{\partial x_3}-\frac{\partial H_3}{\partial x_1}&=\sigma E_2+\in\frac{\partial E_2}{\partial t}\\ \frac{\partial H_2}{\partial x1}-\frac{\partial H_1}{\partial x_2}&=\sigma E_3+\in\frac{\partial E_3}{\partial t}\end{aligned}\qquad(26)$$

$$\begin{aligned}\frac{\partial E_3}{\partial x_2}-\frac{\partial E_2}{\partial x_3}&=-\mu\frac{\partial H_1}{\partial t}\\ \frac{\partial E_1}{\partial x_2}-\frac{\partial E_3}{\partial x_1}&=-\mu\frac{\partial H_2}{\partial t}\\ \frac{\partial E_2}{\partial x_1}-\frac{\partial E_1}{\partial x_2}&=-\mu\frac{\partial H_3}{\partial t}\end{aligned}\qquad(27)$$

Replacing $\boldsymbol{F}$ by $\boldsymbol{H}$ and $\boldsymbol{E}$ successively in (25) one can recast (26) and (27) in the form

$$\left(\frac{1}{2}\sigma+j\omega\in\right)E_1=\left[\psi_2^{H_3}(r,\theta,\phi,1)-\psi_3^{H_2}(r,\theta,\phi,1)\right]G(t)\qquad(28)$$

$$\left(\frac{1}{2}\sigma+j\omega\in\right)E_2=\left[\psi_3^{H_1}(r,\theta,\phi,1)-\psi_1^{H_3}(r,\theta,\phi,1)\right]G(t)\qquad(29)$$

$$\left(\frac{1}{2}\sigma+j\omega\in\right)E_3=\left[\psi_1^{H_2}(r,\theta,\phi,1)-\psi_2^{H_1}(r,\theta,\phi,1)\right]G(t)\qquad(30)$$

$$\mu\left(\frac{1}{2}\sigma-j\omega\right)H_1=\left[\psi_2^{E_3}(r,\theta,\phi,1)-\psi_3^{E_2}(r,\theta,\phi,1)\right]G(t)\qquad(31)$$

$$\mu\left(\frac{1}{2}\sigma-j\omega\right)H_2=\left[\psi_3^{E_1}(r,\theta,\phi,1)-\psi_1^{E_3}(r,\theta,\phi,1)\right]G(t)\qquad(32)$$

$$\mu\left(\frac{1}{2}\sigma-j\omega\right)H_3=\left[\psi_1^{E_2}(r,\theta,\phi,1)-\psi_2^{E_1}(r,\theta,\phi,1)\right]G(t)\qquad(33)$$

where

$$\begin{aligned}\psi_2^{H_3}(r,\theta,\phi,1)=\sum_{n=1}^{\infty}B_n^{l,3}(H)e^{jl(\phi-\pi/2)}\Big\{&F_1'(r)T_n^l(\cos\theta)\sin\theta\sin\phi\\ &-F_1(r)\left(T_n^l(\cos\theta)\right)'\frac{\sin\theta\cos\theta\sin\phi}{r}\\ &+F_1(r)T_n^l(\cos\theta)(jl)\frac{\cos\phi}{r\sin\theta}\Big\}\end{aligned}\qquad(34)$$

International Journal of Emerging Technology and Advanced Engineering
Website: www.ijetae.com (ISSN 2250-2459, ISO 9001:2008 Certified Journal, Volume 6, Issue 10, October 2016)

$$\psi_3^{H_2}(r,\theta,\phi,1)=\sum_{n=1}^{\infty}B_n^{l.2}(H)e^{jl(\phi-\pi/2)}\Big\{F_1'(r)T_n^l(\cos\theta)\cos\theta + F_1(r)\left(T_n^l(\cos\theta)\right)'\frac{\sin^2\theta}{r}\Big\} \quad (35)$$

$$\psi_3^{H_1}(r,\theta,\phi,1)=\sum_{n=1}^{\infty}B_n^{l.1}(H)e^{jl(\phi-\pi/2)}\Big\{F_1'(r)T_n^l(\cos\theta)\cos\theta + F_1(r)\left(T_n^l(\cos\theta)\right)'\frac{\sin^2\theta}{r}\Big\} \quad (36)$$

$$\psi_1^{H_3}(r,\theta,\phi,1)=\sum_{n=1}^{\infty}B_n^{l.3}(H)e^{jl(\phi-\pi/2)}\Big\{F_1'(r)T_n^l(\cos\theta)\sin\theta\cos\phi - F_1(r)\left(T_n^l(\cos\theta)\right)'\frac{\sin\theta\cos\theta\cos\phi}{r} - F_1(r)T_n^l(\cos\theta)(jl)\frac{\sin\phi}{r\sin\theta}\Big\} \quad (37)$$

$$\psi_1^{H_2}(r,\theta,\phi,1)=\sum_{n=1}^{\infty}B_n^{l.2}(H)e^{jl(\phi-\pi/2)}\Big\{F_1'(r)T_n^l(\cos\theta)\sin\theta\cos\phi - F_1(r)\left(T_n^l(\cos\theta)\right)'\frac{\sin\theta\cos\theta}{r\cos\phi} - F_1(r)T_n^l(\cos\theta)(jl)\frac{\sin\phi}{r\sin\theta}\Big\} \quad (38)$$

$$\psi_2^{H_1}(r,\theta,\phi,1)=\sum_{n=1}^{\infty}B_n^{l.1}(H)e^{jl(\phi-\pi/2)}\Big\{F_1'(r)T_n^l(\cos\theta)\sin\theta\sin\phi - F_1(r)T_n^l(\cos\theta)'\frac{\sin\theta\cos\theta\sin\phi}{r} + F_1(r)T_n^l(\cos\theta)(jl)\frac{\cos\phi}{r\sin\theta}\Big\} \quad (39)$$

Replacing H_1, H_2, H_3 and H by E_1, E_2, E_3 and E respectively one can easily find the value of $\psi_2^{E_3}(r,\theta,\phi,1)$, $\psi_3^{E_2}(r,\theta,\phi,1)$, $\psi_3^{E_1}(r,\theta,\phi,1)$, $\psi_1^{E_3}(r,\theta,\phi,1)$, $\psi_1^{E_2}(r,\theta,\phi,1)$ $\psi_2^{E_1}(r,\theta,\phi,1)$.

Hence, one can arrive at the following theorems :

Theorem 1: An asymmetric electric intensity vector E is said to be associated with time dependent damped spherical wave $\psi^E(r,\theta,\phi,t)$ of frequency ω and the damping factor $(\sigma/2\in)$ iff the bounding surfaces of ∂K are conducting ($\sigma=0$) and the components of magnetic intensity vector ***H*** are given by (31) to (33) and the frequency ω and the wave number k are mutually related by the non-linear relation $4\in k^2=\mu(4\in\omega^2+\sigma^2)$ subject to the restriction $2k\sqrt{\in}>\sqrt{\mu}\ \sigma$.

International Journal of Emerging Technology and Advanced Engineering
Website: www.ijetae.com (ISSN 2250-2459, ISO 9001:2008 Certified Journal, Volume 6, Issue 10, October 2016)

Theorem 2: An asymmetric magnetic intensity vector H is said to be associated with a time dependent damped spherical wave $\psi^H(r,\theta,\phi,t)$ of frequency ω with damped factor $(\sigma/2\in)$ iff the bounding surfaces of ∂K are conducting ($\sigma = 0$) and the components of electric intensity vector ***E*** are given by (28) to (30) and the frequency ω and k are mutually related by the non-linear relation $4\in k^2 = \mu(4\in\omega^2+\sigma^2)$ subject to the restriction that $2k\sqrt{\in} > \sqrt{\mu}\ \sigma$.

Determination of Current Density I

A current density ***I*** consists of displacement current and the conduction current according to Maxwell's theory in electromagnetics. Hence one can express ***I*** in the form

$$\boldsymbol{I} = \boldsymbol{I}_c + \boldsymbol{I}_d = \sigma\boldsymbol{E}(r,\theta,\phi) + \in\frac{\partial \boldsymbol{E}}{\partial t}(r,\theta,\phi,t) \qquad (40)$$

Now, combining the relations (25) and (40), ***I*** may be finally expressed in the form

$$\boldsymbol{I} = \psi^E(r,\theta,\phi,t)e^{-t(\sigma/2\in - j\omega)}(\sigma/2 + j\omega\in) \qquad (41)$$

Which represents a spherical wave with its amplitudes and phase given by the following expressions :

$|\boldsymbol{I}| = \frac{1}{2}\psi^E(r,\theta,\phi)e^{-\sigma t/2\in}\sqrt{(\sigma^2+4\omega^2\in^2)}$ and phase $(\boldsymbol{I}) = \delta + \omega t$ where $\tan\delta = \frac{2\omega\in}{\sigma}$

II. Conclusions

The present paper furnishes the existence of asymmetric EM waves associated with an echellete model. The concerning wave functions happen to be derived from the governing Maxwell's equation in spherical coordinates (r,θ,ϕ). The waves associated with such wave functions may be identified as spherical waves. The present field of study happens to be equivalent to EM boundary value problems. The foregoing results have been applied for finding the components of electric and magnetic intensity vectors ***E*** and ***H***. Two existence theorems regarding the spherical mode of polarisation of a EM wave have been established. Finally, the expression of the field intensities ***H*** and ***E*** have been utilized for computing the current density.

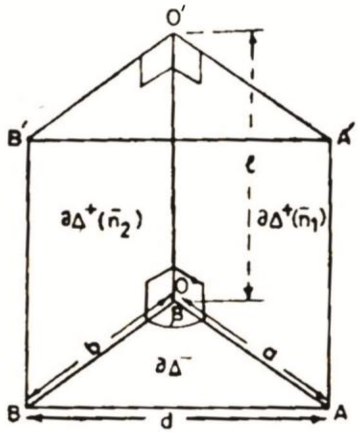

Figure 1

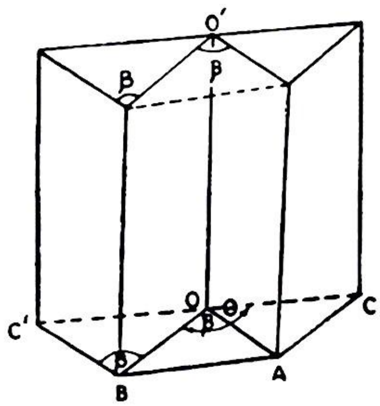

Figure 2

REFERENCES

[1] K N Bhowmick and K K Dey J. Electronics and Telecom. Engineers 30 112 (1984)

[2] K N Bhowmick and K K Dey J. Electronics and Telecom. Engineers 31 19 (1985)

[3] K N Bhowmick J. Electronics and Telecom. Engineers 34 (4) 319 (1988)

[4] K N Bhowmick and Sanjay Kumar Indian J. Phys. 65B (4) 329 (1991)

[5] Sanjay Kumar Proc. of Math. Soc., B.H.U 26 35 (2010)

[6] Sanjay Kumar Proc. of Math. Soc., B.H.U 25 71 (2009)

[7] S I Ghobrial and M S Sami IEEE Trans. Antenna Propagation Ap35 418 (1987)

[8] J S Mandeep and M T Islam Indian J Phys 88(6) 541 (2014)

[9] C R Wylic Advanced Engineering Mathematics, MaCGraw Hill Book Co., 2nd ed 418 (1960).

Captions of the Figures

Figure 1.

A convex triangular prism of dimensions a, b, d and with it's flare angle 'β', OO' is perpendicular to the planes Δs OAB and $O'A'B'$.

Figure 2.

A model 'M' consists of a triangular prism formed by Δs OAB and $O'A'B'$ and its adjacent groove regions formed by the sides BC' and AC and the sides parallel to OO', OA and OB.