174 (multiplication table) Given n: *nat* and variable M: [*[*nat*]] , write a program to assign to M a multiplication table of size n without using multiplication. For example, if $n=4$, then

$M' = [$ [0];
[0; 1];
[0; 2; 4];
[0; 3; 6; 9]]

§ Define function $mul = \langle i, j: nat \rightarrow i \times j \rangle$. Define $row = \langle i: nat \rightarrow mul\ i\ [0;..i+1] \rangle$. For example,

$row\ 3 = mul\ 3\ [0; 1; 2; 3] = [mul\ 3\ 0; mul\ 3\ 1; mul\ 3\ 2; mul\ 3\ 3] = [0; 3; 6; 9]$

So the problem is $M' = row\ [0;..n]$. Introduce new variables i, j: *nat* and R: [*nat*] .

$M' = row\ [0;..n] \Longleftarrow M:= [nil].\ i:= 0.\ M' = M + row\ [i;..n]$

$M' = M + row\ [i;..n] \Longleftarrow$
if $i=n$ **then** *ok*
else $R' = row\ i \wedge M'=M \wedge i'=i.\ M:= M + [row\ i].\ i:= i+1.\ M' = M + row\ [i;..n]$ **fi**

$R' = row\ i \wedge M'=M \wedge i'=i \Longleftarrow$
$R:= [nil].\ j:= 0.\ R' = R + mul\ i\ [j;..i+1] \wedge M'=M \wedge i'=i$

$R' = R + mul\ i\ [j;..i+1] \wedge M'=M \wedge i'=i \Longleftarrow$
if $j = i+1$ **then** *ok*
else $R:= R + [i \times j].\ j:= j+1.\ R' = R + mul\ i\ [j;..i+1] \wedge M'=M \wedge i'=i$ **fi**

Recursive time requires $t:= t+1$ in the inner loop.

$t' \le t + n \times (n-1)/2 \Longleftarrow M:= [nil].\ i:= 0.\ t' \le t + n \times (n-1)/2 - i \times (i-1)/2$

$t' \le t + n \times (n-1)/2 - i \times (i-1)/2 \Longleftarrow$
if $i=n$ **then** *ok*
else $t' \le t+i+1 \wedge i'=i.\ M:= M + [row\ i].\ i:= i+1.\ t' \le t + n \times (n-1)/2 - i \times (i-1)/2$ **fi**

$t' \le t+i+1 \wedge i'=i \Longleftarrow$
$R:= [nil].\ j:= 0.\ t' \le t+i+1-j \wedge i'=i$

$t' \le t+i+1-j \wedge i'=i \Longleftarrow$
if $j = i+1$ **then** *ok*
else $R:= R + [i \times j].\ j:= j+1.\ t:= t+1.\ t' \le t+i+1-j \wedge i'=i$ **fi**