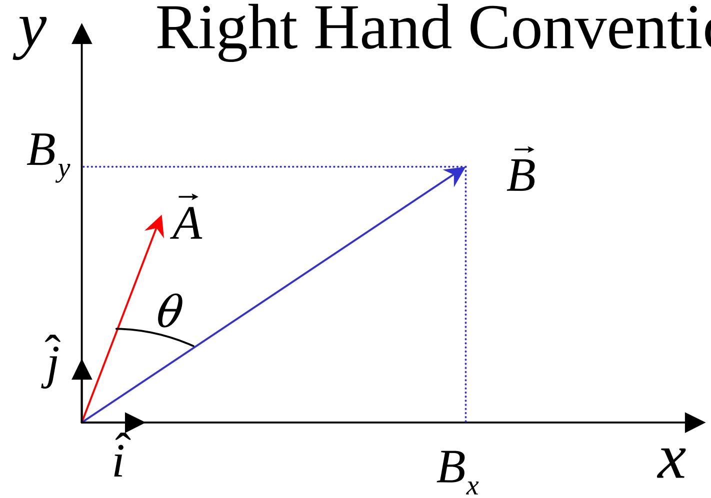
# ME 3700 – Fluid Mechanics

## Mathematics Review

# Vector Review

Right Hand Convention




$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

1. Dot Product

$$A \bullet B = A_x B_x + A_y B_y + A_z B_z = AB\cos\theta$$

2. Cross Product

$$A \times B = \left(A_y B_z - A_z B_y\right)\hat{i} + \left(A_z B_x - A_x B_z\right)\hat{j} + \left(A_x B_y - A_y B_x\right)\hat{k}$$

$$\left|A \times B\right| = AB\sin\theta$$

# Integral Calculus Review

1. Gauss Theorem – Divergence theorem

$$\int_{\forall} \nabla \bullet \vec{A} d\forall = \int_{S} \left(\hat{n} \bullet \vec{A}\right) dS$$

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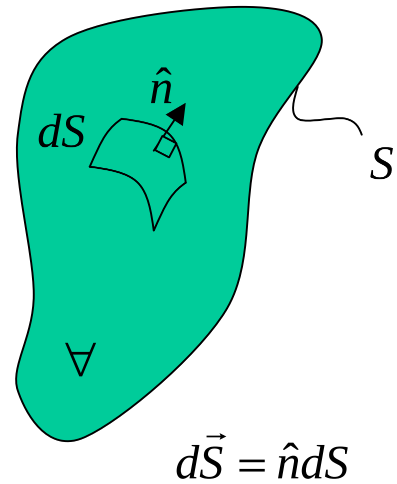
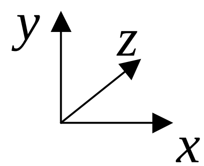
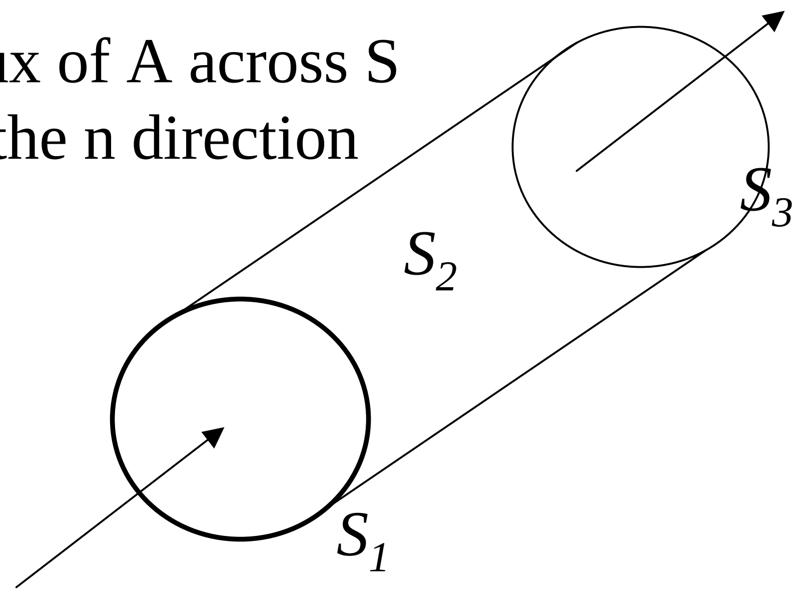



Divergence of A in
The volume V

Flux of A across S
in the n direction

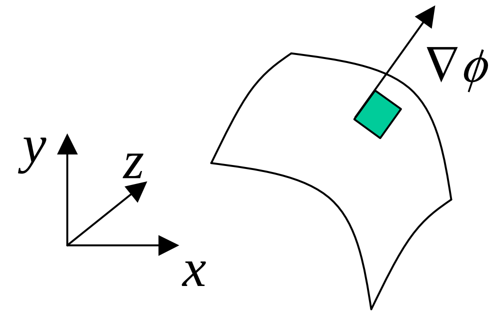


# Integral Review

2. Leibnitz Rule

$$\int_{\forall} \frac{\partial}{\partial t} F d\forall = \frac{\partial}{\partial t} \int_{\forall} F d\forall$$

# Differential Calculus Review




1. Gradient Operator

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Gradient of a Scalar (i.e., density, temperature,etc)

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

2. Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

3. Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}$$

# Differential Calculus Review

4. Laplacian -

$$\nabla^2 \phi = \nabla \bullet \nabla \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

5. Total Differential

$$d\vec{V} = \frac{\partial \vec{V}}{\partial x} dx + \frac{\partial \vec{V}}{\partial y} dy + \frac{\partial \vec{V}}{\partial z} dz$$

# <u>Error Propagation</u>- Functions of more than one variable

Apply Taylor Series to functions of multiple variables, I.e., *f(x,y,z)*

$$f(x_{i+1}, y_{i+1}, z_{i+1}) = f(x_i, y_i, z_i) + \left.\frac{\partial f}{\partial x}\right|_i (x_{i+1} - x_i) + \left.\frac{\partial f}{\partial y}\right|_i (y_{i+1} - y_i) + \left.\frac{\partial f}{\partial z}\right|_i (z_{i+1} - z_i) + H.O.T.$$

Neglecting 2$^{nd}$ order and higher terms, the error in *f* is:

$$\Delta f(\tilde{x}, \tilde{y}, \tilde{z}) = \left|\frac{\partial f}{\partial x}\right| \Delta\tilde{x} + \left|\frac{\partial f}{\partial y}\right| \Delta\tilde{y} + \left|\frac{\partial f}{\partial z}\right| \Delta\tilde{z}$$

Where $\Delta\tilde{x}\ \Delta\tilde{y}\ \Delta\tilde{z}$ are estimates of the error in *x*, *y* and *z*

In general, 1$^{st}$ order approximation of the error in *f* is:

$$\Delta f(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_n) = \left|\frac{\partial f}{\partial x_1}\right| \Delta\tilde{x}_1 + \left|\frac{\partial f}{\partial x_2}\right| \Delta\tilde{x}_2 + \left|\frac{\partial f}{\partial x_3}\right| \Delta\tilde{x}_3 + \ldots + \left|\frac{\partial f}{\partial x_n}\right| \Delta\tilde{x}_n$$

# Error Propagation- Reynolds Number Example

$$\mathrm{Re} = \frac{UD}{\nu}$$

Where: V = average fluid velocity (m/s)
D = pipe diameter (m)
ν = kinematic viscosity ($m^2$/s) water

Estimate the error in Re for the Given data:

$\tilde{U} = 0.5$ $\quad \Delta\tilde{U} = 0.01$ m/s

$\tilde{D} = 0.1$ $\quad \Delta\tilde{D} = 0.001$ m

$\tilde{\nu} = 1.0\times10^{-6}$ $\quad \Delta\tilde{\nu} = 0.005\times10^{-6}$ $m^2$/s

$$\mathrm{Re} = \frac{UD}{\nu} = \frac{(.5)(.1)}{1\times10^{-6}} = 50{,}000$$

$$\Delta\mathrm{Re}(\tilde{U},\tilde{D},\tilde{\nu}) = \left|\frac{\partial \mathrm{Re}}{\partial U}\right|\Delta\tilde{U} + \left|\frac{\partial \mathrm{Re}}{\partial D}\right|\Delta\tilde{D} + \left|\frac{\partial \mathrm{Re}}{\partial \nu}\right|\Delta\tilde{\nu}$$

$$\Delta\mathrm{Re}(\tilde{U},\tilde{D},\tilde{\nu}) = \left|\frac{D}{\nu}\right|\Delta\tilde{U} + \left|\frac{U}{\nu}\right|\Delta\tilde{D} + \left|\frac{-UD}{\nu^2}\right|\Delta\tilde{\nu}$$

$$\Delta\mathrm{Re}(\tilde{U},\tilde{D},\tilde{\nu}) = \left|\frac{.1}{1\times10^{-6}}\right|.01 + \left|\frac{.5}{1\times10^{-6}}\right|.001 + \left|\frac{(.5)(.1)}{(1\times10^{-6})^2}\right|(0.005\times10^{-6})$$

$$\Delta\mathrm{Re}(\tilde{U},\tilde{D},\tilde{\nu}) = 1000 + 500 + 250$$

$$\mathrm{Re} = 50{,}000 \pm 1750$$

$$\mathrm{Re} = 50{,}000 \pm 3.5\%$$