1. Create a(n)

   (a) Addition table mod 5

   Answer:

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	1	1
3	3	4	0	1	2
4	4	0	1	2	3

   (b) Multiplication table mod 5

   Answer:

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

2. Decide which of the following are groups. Justify for answer by showing that either all group axioms hold, or by giving a specific example showing an axiom fails

   (a) The set of rational numbers under addition

   Answer:

   Checking that addition is a binary operation on the rational numbers: $a, b \in \mathbb{Q}$, and $a + b \in \mathbb{Q}$

   - **Associativity:**
     $(a + b) + c = a + (b + c)$
   - **Identity:**
     0 is the additive identity, and $0 \in \mathbb{Q}$.
   - **Inverses**
     If $a \in \mathbb{Q}$, then $a + (-a) = -a + a = 0$, and $-a \in \mathbb{Q}$

   Thus the rational numbers under addition are a group.

(b) The set of complex numbers S=$\{1, -1, i, -i\ \}$

Answer:

- **Associativity:**
  $(ab)c = a(bc)$ is true for all elements in the set
- **Identity:**
  1 is the additive identity, and $1 \in \mathbb{S}$.
- **Inverses**
  $1(1) = (1)1 = 1,\ 1 \in S$
  $-1(-1) = (-1) - 1 = 1,\ -1 \in S$
  $i(-i) = (-i)i = 1,\ -i \in S$
  $(-i)i = i(-i) = 1,\ i \in S$

Thus the set S under multiplication is a group.

(c) The set of even numbers under multiplication

Answer:

- **Inverses:**

1 is the multiplicative identity, but $1 \notin S$ (where S is the even numbers)

3. An element a in a group G under multiplication is called an *idempotent* if $a^2 = a$. Prove that the only idempotent in a group is the identity element. (Hint: We're assuming a is in the group, so it must have an inverse.)

Answer:

$$a^2 = a$$
$$a^-1(a^2) = a^-1(a)$$
$$a = 1$$

Proof: We know that 1 is the identity of a group G under multiplication. Since we showed $a = 1$ above, bo other element in group G can be idempotent. We disregard 0 because even though $0^2 = 0$, 0 has no multiplication, there for there is no possibility that $0 \in G$. Therefore, the only idempotent element in a multiplicative group G is the identity element.

4. A *permutation* of a finite set of numbers {1,2, ... ,n} is an arrangement of the numbers in the set. We can express this arrangement using an ordered list. For instance, all possible permutations of the set {1,2,3} would be (1,2,3),(1,3,2),(2,1,3),(2,3,1),(3,1,2), and (3,2,1).

   (a) Find all possible permutations of the set {1,2,3,4}

   Answer:

(1,2,3,4)	(2,1,3,4)	(3,1,2,4)	(4,1,2,3)
(1,2,4,3)	(2,1,4,3)	(3,1,4,2)	(4,2,1,3)
(1,3,2,4)	(2,3,1,4)	(3,2,1,4)	(4,3,1,2)
(1,3,4,2)	(2,3,4,1)	(3,2,4,1)	(4,1,3,2)
(1,4,3,2)	(2,4,3,1)	(3,4,2,1)	(4,2,3,1)
(1,4,2,3)	(2,4,1,3)	(3,4,2,1)	(4,3,2,1)

   (b) Give a conjecture (ie, a guess) of the number of possible permutations on a set with n numbers. You do not need to prove your conjecture, you have n choices for the first number, which leaves *n-1* choices for the second, *n-3* for the third, etc.)

   Answer: For a given number n, you could figure out the number of permutations. If you have n numbers in your set, your permutations would be equal to $n(n-1)(n-2)(n-3)...1$ For example in part a, a set with 4 elements gives you 24 permutations, which $4*3*2*1 = 24$ In other words, you could have *n!*, where n is the number of permutations.