Double, Double, Double

by Doug French

Knowing multiplication tables by heart is an important requirement for success in mathematics and effective ways of learning them should be a key component of any policy for improving standards of numeracy. It is widely assumed that the reason students do not know their tables is because they have not been made to learn them, and that the problem can be solved by making them do so. As every teacher of mathematics knows only too well, it is not so simple, and it would not be so even if every child were well motivated and worked hard at their mathematics.

In *Recent Research in Mathematics Education 5–16*, Askew and Wiliam (1995) discuss evidence which suggests that 'knowing by heart' and 'figuring it out' are two complementary aspects of developing students' 'progression in number'. Students should gradually come to know more number facts by heart, but it is also essential that they develop the ability to use their existing knowledge to work out new results or ones they do not remember. I would suggest that students are more likely to remember number facts if they have the confidence and security that comes from knowing that they can always figure out a result that they have inadvertently forgotten.

The ways in which teachers help students 'learn their tables' are very important, but the form taken by all subsequent attempts at reviewing and reinforcing, or indeed recreating, that early learning, in both primary and secondary school, is also of crucial importance. It should involve learning useful calculating strategies which are of more general application and it should help reinforce links and connections between results, rather than convey the idea that multiplication tables are a set of isolated facts. Simple exhortations to 'learn your tables' together with frequent testing, may be counter-productive, because they often reinforce failure rather than bring about improvements, and they certainly do little to improve attitudes and motivation. More imaginative approaches are needed and the purpose of this article is to offer a few suggestions, which are not particularly original, but which might help to provide new approaches for some readers.

I vividly remember getting 5×7 wrong in a test when I was at primary school and then being kept in to write out $5 \times 7 = 35$ fifty times. I have not forgotten this result since, but then I was a reasonably confident individual who did not make such errors very often. Others grew to hate mathematics as a result of such treatment and as a consequence did not learn very much either. Fortunately schools are more humane places today, but students still have trouble with their tables! When I reflect on this incident now, it occurs to me how much more useful it would have been if my teacher had pointed out to me that I could work out 5×7 by halving the answer to 10×7 or, better still, encouraged me to figure that out for myself. Not only would I have been able to work out that particular answer quickly for subsequent tests, but I would have had a strategy which enabled me to multiply any number by 5 easily.

Any of the number facts in the tables can be arrived at in a wide variety of ways and all students, even those who are secure with their tables, can benefit from encountering and discussing that richness of possibilities from time to time. By way of an example let us look at 8×9. The most elementary strategy is to count up in nines:

9, 18, 27, 36, 45, 54, 63, **72**,

or in eights:

8, 16, 24, 32, 40, 48, 56, 64, **72**.

At a slightly more sophisticated level a wide range of possibilities opens up:

- Counting back from 10×8: 80, **72**, or, from 10×9: 90, 81, **72**. or, more simply, just subtracting 8 from 10×8 or 18 from 10×9.
- Starting from another known fact: $8 \times 8 = 64$, so 8×9 is 8 more, or, $9 \times 9 = 81$, so 8×9 is 9 less.
- Multiplying by 8 is 'double, double, double', so $2 \times 9 = 18$, $2 \times 18 = 36$ and $2 \times 36 = 72$.
- 9 is 3 squared ('treble, treble'), and so $3 \times 8 = 24$ and $3 \times 24 = 72$.
- Using addition of known results: $5 \times 9 = 45$ and $3 \times 9 = 27$ to give $45 + 27 = 72$.
- Doubling and halving: 8×9 is the same as 4×18, which is the same as 2×36, which is 72.

Asking students of all ages to devise and discuss their own methods of doing calculations – ways of 'figuring it out' for themselves – helps to develop their understanding of number, builds up their fund of number facts and encourages a problem-solving approach to all mathematical work. Ideas like this were developed at some length in the Mathematical Association's publication *Mental Methods in Mathematics: A First Resort* (1992).

I should like to look a little more systematically at the multiplication tables and show how a few simple strategies give most of the multiplication table facts. Most of these strategies have been used in considering the example of 8×9, but their application extends to far more than just multiplying single-digit numbers.

- The commutative law. The name is not important, but the fact that it tells you that $8 \times 9 = 9 \times 8$ is, for this immediately nearly halves the number of facts that have to be learnt!
- The distributive law. Again the name does not matter, but it is important to understand that any number can be split up in a variety of ways and that this both simplifies multiplication and underlies standard methods of multiplication, beside being of key importance in understanding algebra. The law is illustrated here with three different ways of calculating 2×37:

  $2 \times 37 = 2 \times 30 + 2 \times 7 = 60 + 14 = 74$
  $2 \times 37 = 2 \times 40 - 2 \times 3 = 80 - 6 = 74$
  $2 \times 37 = 2 \times 35 + 2 \times 2 = 70 + 4 = 74$.

- The inverse nature of multiplication and division. Understanding this reduces what has to be learnt. Knowing one of the three facts $3 \times 4 = 12$, $12 \div 3 = 4$ and $12 \div 4 = 3$ immediately tells you the others, so only one fact needs to be learnt. In spite of this we still find science books (not mathematics books I hope), which ask students to learn the three formulae $V = IR$, $I = \frac{V}{R}$ and $R = \frac{V}{I}$, and others like them, as though they were separate and unrelated!
- Doubling and halving. This is the simplest example of the inverse relationship between multiplication and division. The two times table is not problematic for most students, which means that it is not difficult for them to learn how to double two-digit numbers mentally. Alongside this halving should be considered – again most students can learn to halve two-digit numbers successfully. Successively doubling or halving a number can provide challenges at many different levels:

  $1 \to 2 \to 4 \to 8 \to 16 \to 32 \to 64 \to 128$
  $100 \to 50 \to 25 \to 12.5 \to 6.25 \to 3.125 \to 1.5625$.
- Multiplying and dividing by 10 lies at the heart of the way our base 10 number system is structured and it is therefore important to make sense of it from an early stage. It is easy to remember the ten times table, and to extend it, and the corresponding divisions, to much larger whole numbers and then at a later stage to decimal fractions. Again successively multiplying or dividing serves to reinforce the ideas at different stages:

  $1 \to 10 \to 100 \to 1000 \to 10\,000 \to 100\,000$
  $300 \to 30 \to 3 \to 0.3 \to 0.03 \to 0.003$.
- Multiplying by 5 follows from our knowledge of multiplying by 10 and dividing by 2. Whilst the five times table is not usually regarded as a difficult one, it is worth noting that multiplying by 10 and halving is equivalent to multiplying by 5 because, besides providing a way of retrieving forgotten table facts, it applies more generally. For example, it is convenient to calculate 3.5×5 as $35 \div 2$ or 2.48×5 as $24.8 \div 2$. Doubling and dividing by 10 as a neat way of dividing by 5 provides a useful reinforcement of the idea of inverse operations.
- Multiplying and dividing by 4, and by 8, follow on nicely from doubling and halving. Multiplying by 4 is *double, double* and dividing by 4 is *halve and halve again*:

  7×4: $7 \to 14 \to 28$
  $74 \div 4$: $74 \to 37 \to 18.5$.

  In the same way multiplying by 8 is *double, double, double* and dividing by 8 is *halve, halve and halve again.*

  17×8: $17 \to 34 \to 68 \to 136$
  $2.5 \div 8$: $2.5 \to 1.25 \to 0.625 \to 0.3125$.

  This idea extends readily to multiplying and dividing by 16, 32 and other higher powers of 2.
- Multiplying by 9 is easily done by subtraction from the corresponding multiple of 10:

  $7 \times 9 = 7 \times 10 - 7 = 70 - 7 = 63$.

  Again this extends to larger numbers, but the same strategy also enables you to multiply by numbers like 99 or 19:

  $7 \times 99 = 7 \times 100 - 7 = 700 - 7 = 693$
  $7 \times 19 = 7 \times 20 - 7 = 140 - 7 = 133$.
- The square numbers are an important and special set of numbers with many interesting properties which students commonly investigate and discuss at various stages in mathematics lessons.

  1 4 9 16 25 36 49 64 81 100.

  One consequence of looking at them in a variety of ways is that the square numbers become familiar and fill in some gaps in the multiplication tables that are not filled by the strategies I have outlined here so far.

If we now look at a multiplication table square with the squares that have been accounted for so far shaded in, we see that only three facts remain unshaded:

3×6 3×7 6×7

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

The doubling principle is useful for the 6 times table provided that the corresponding multiple of 3 is familiar. Thus 3×6 is obtained by doubling 3×3 and 6×7 is double 3×7. So, perhaps surprisingly, the only thing that there is left to learn is 3×7! Everything else can be worked out quickly and easily! Contrary to popular opinion perhaps 7×8, 7×9 and 9×8 should not be viewed as the hardest products in the multiplication tables: they have only become so because students attempt to learn them as isolated facts. In the sense that it has fewer links to other products, 3×7 is actually much harder, although in practice multiples of 3 are perhaps easier to make sense of than products involving only numbers near to 10.

I have emphasized strongly the 'figuring it out' aspect of learning about multiplication tables here. Many people would suggest that more emphasis is needed on 'knowing' or 'remembering', so that the facts are available for instant recall. I wholeheartedly agree that the facts should be known in this sense, because they provide an essential basis from which to generate further numerical results. However, more students are likely to achieve this desirable state if they come to know these number facts through learning general strategies which provide them with secure ways of retrieving any fact that they may have forgotten. Such strategies have the important added bonus of enabling them to do a much wider range of calculations effectively. When students have failed to 'learn their tables' at an earlier stage of their education, I would suggest that emphasizing these strategies and providing plenty of opportunities to use the table facts in meaningful ways is more likely to be successful than direct attempts to encourage memorization.

References

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