Fluid Mechanics

Attendance

Fluid Mechanics

A fluid is either a liquid or a gas. Air and Water are both fluids.

This experiment has four parts:

1. **Archimedes' Principle – Why things float (or not)**
2. **Venturi Tube – Fluid velocity**
3. **Blood Pressure – Bernoulli's equation**
4. **Diffusion and Osmosis – Fick's law**

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \qquad P = \frac{F}{A} \qquad \text{Units:} \frac{\text{N}}{\text{m}^2} = \text{Pascal (Pa)}$$

$$F = PA$$

Pressure as a function of depth in a fluid: $P = P_0 + \rho g h$

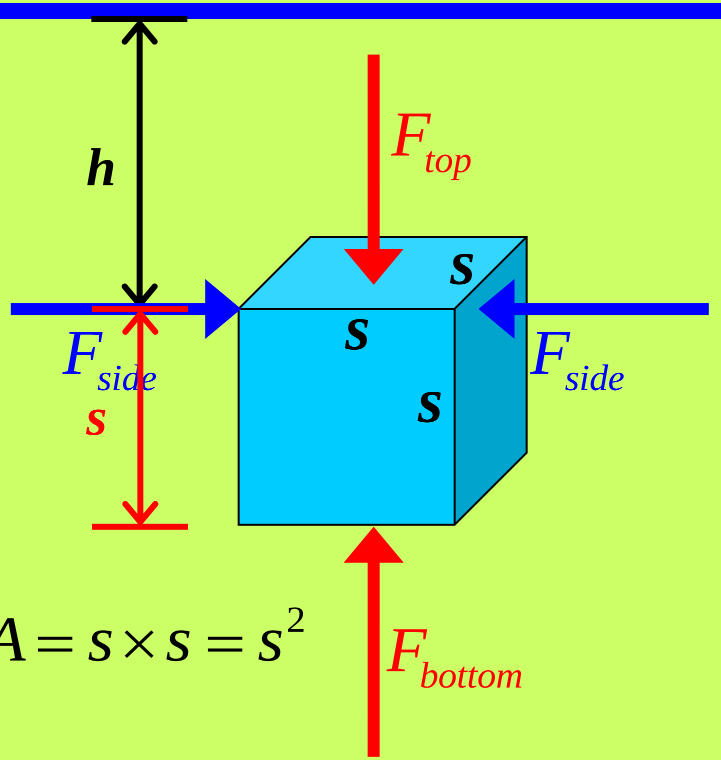
$P_0 = \text{Pressure at the top of the fluid}$

$\rho = \text{Density of the fluid}$

$h = \text{Depth}$

Archimedes' Principle:

Picture a cube with sides *s* submerged in a fluid.



$$A = s \times s = s^2$$

Pressure at the top of the cube:

$$P_{top} = P_0 + \rho g h$$

Force:

$$F_{top} = P_{top} A = \left(P_0 + \rho g h\right) s^2$$

Pressure at the bottom of the cube:

$$P_{bottom} = P_0 + \rho g \left(h + s\right)$$

Force:

$$F_{bottom} = P_{bottom} A = \left(P_0 + \rho g \left(h + s\right)\right) s^2$$

The force on a vertical side varies with the depth, but the forces on opposite sides cancel each other out.

$$\sum F_x = 0$$

Buoyant Force: $F_B = F_{bottom} - F_{top}$

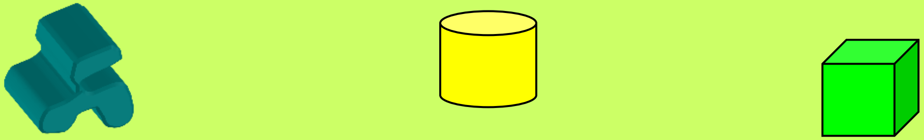
$$F_B = F_{bottom} - F_{top}$$

$$F_B = \left(P_0 + \rho g\left(h+s\right)\right)s^2 - \left(P_0 + \rho g h\right)s^2$$

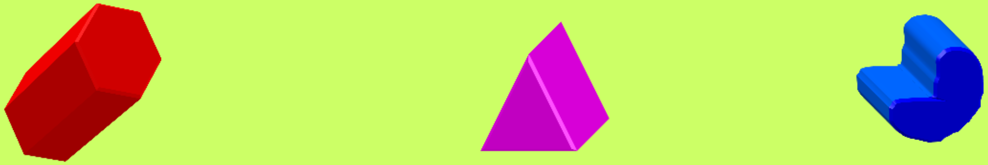
$$F_B = \left(\cancel{P_0} + \cancel{\rho g h} + \rho g s - \cancel{P_0} - \cancel{\rho g h}\right)s^2$$

$$F_B = \rho g s^3 = \rho g V$$

$$F_B = \rho g V$$

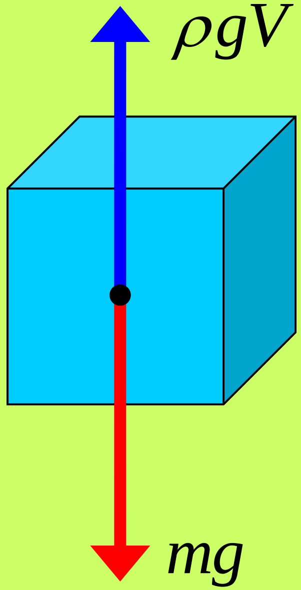


Though this was derived for the case of a cube but it is true for any shape of the volume.



$$\sum F_y = ma_y = ?$$

$$\sum F_y = ma_y = \rho g V - mg$$



$\rho V =$ mass of water displaced by the object

$\rho V > m$ Float

$\rho V < m$ Sink

Note that the buoyancy force does not depend on the depth of object in the fluid.

Is it possible for a balloon filled with air to sink in water?

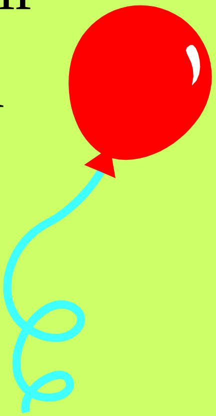
1) Yes **2) No**

$m_b =$ mass of balloon and air

$V_b =$ volume of the balloon

$\rho =$ density of water

weight > buoyant force



$$m_b g > F_B = \rho g V_b$$

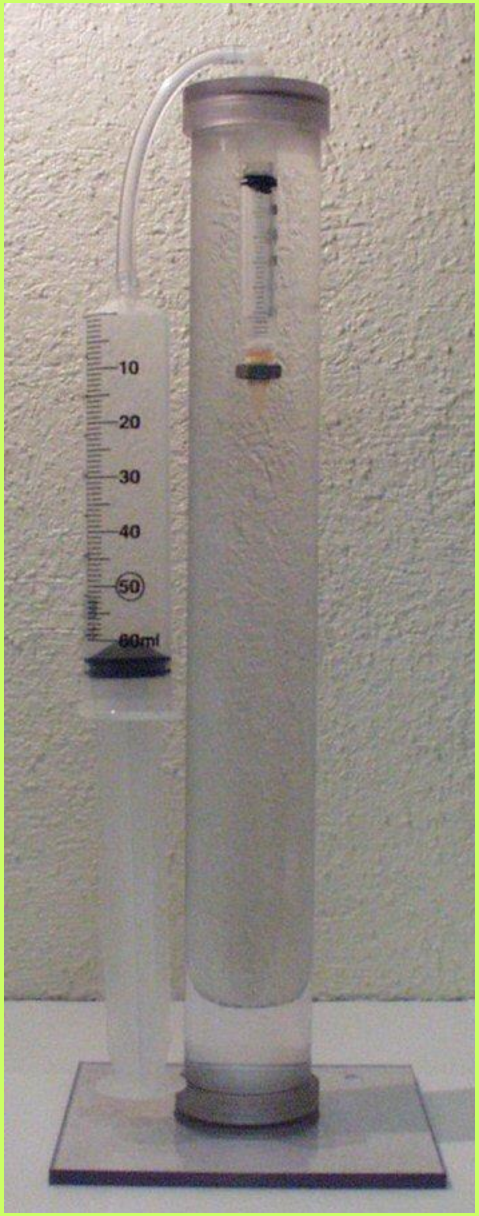
$$m_b \cancel{g} > \rho \cancel{g} V_b$$

$$m_b > \rho V_b$$

$$h = \frac{P_0}{\rho g}\left(\frac{\rho V_o}{m_b} - 1\right) = \frac{1.013\times10^5 \text{ Pa}}{\left(1000 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)}\left(\frac{\left(1000 \text{ kg/m}^3\right)\frac{4\pi}{3}\left(0.1 \text{ m}\right)^3}{0.01 \text{ kg}} - 1\right)$$

$$h = 4320 \text{ m} = 2.68 \text{ mi}$$

Cartesian Diver

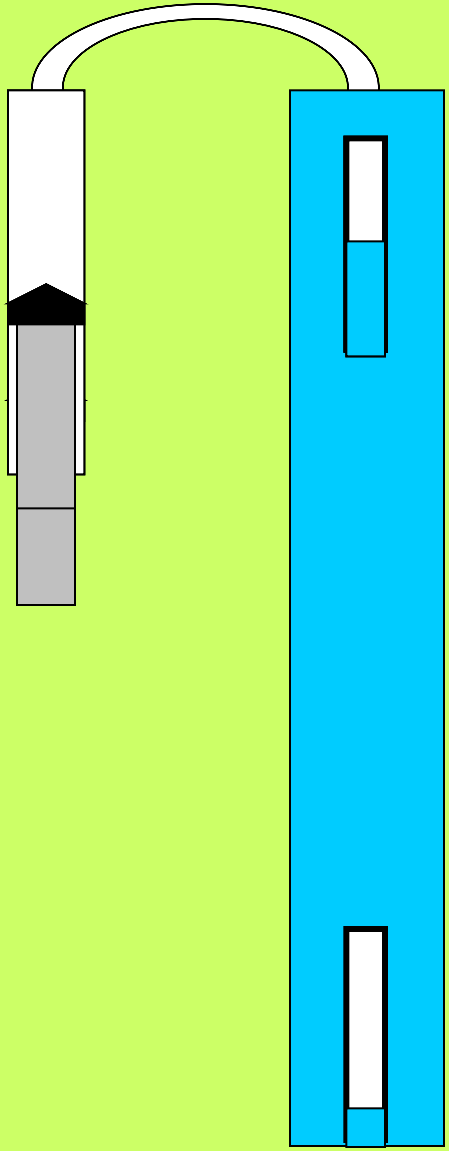


Increase the pressure at the top of the column of water and the volume of air inside the diver will decrease.

Sink!

Decrease the pressure at the top of the column of water and the volume of air inside the diver will increase.

Float!



How much do you have to change the volume of air in the diver to make it sink?

$$m = \text{mass of diver}$$

$$\rho = \text{density of water} = 1.0 \ \text{g/mL}$$

$$V_0 = \text{initial volume of air in the diver}$$

Float: $\rho \cancel{g} V_0 > m \cancel{g}$

$$\rho V_0 > m$$

In order for the diver to sink: $\rho\left(V_0 - \Delta V\right) < m$

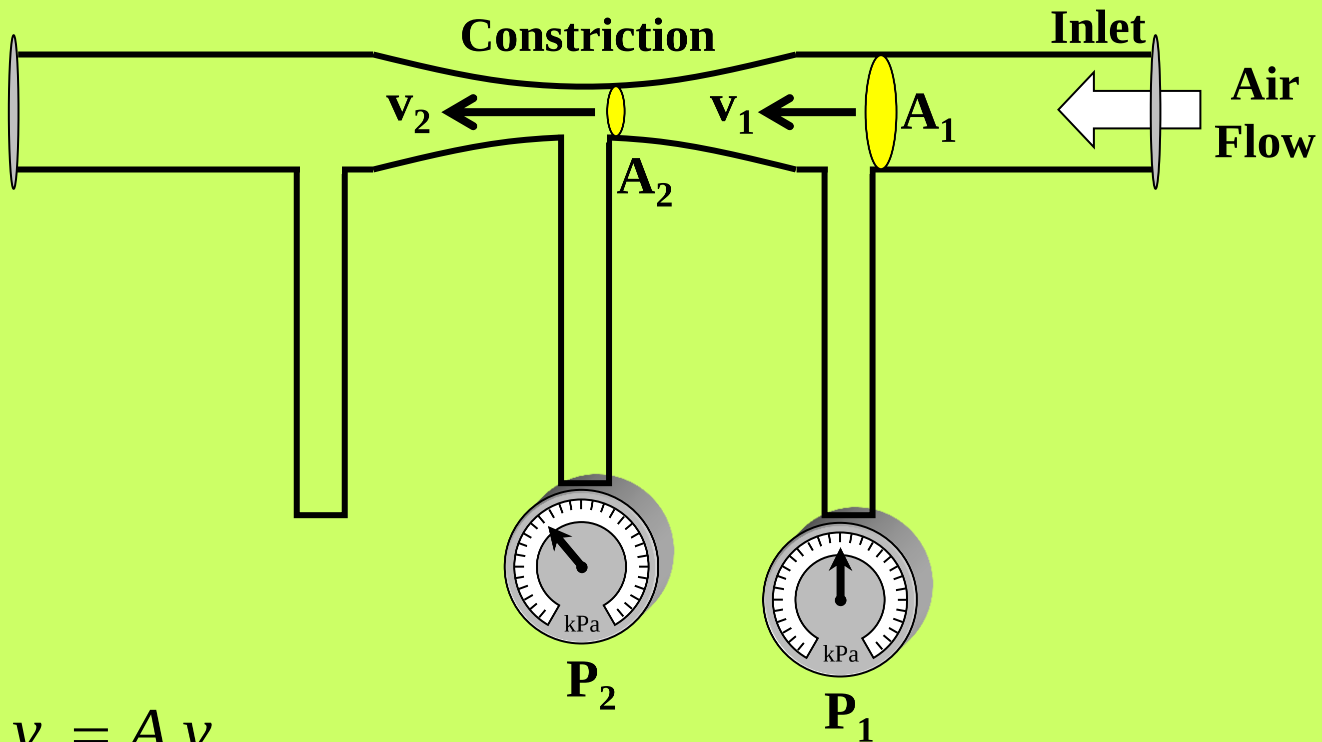
$$\Delta V = \text{change in the volume of air in the diver}$$

$$\Delta V > V_0 - \frac{m}{\rho}$$

Say that: $m = 6.0 \ \text{g}$, $V_0 = 6.5 \ \text{mL}$

Then: $\Delta V > 6.5 \ \text{mL} - \dfrac{6.0 \ \text{g}}{1.0 \ \text{g/mL}} > 0.5 \ \text{mL}$

The Venturi Tube



Continuity Equation: $A_1 v_1 = A_2 v_2$

Bernoulli's Equation: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

Assume: $y_1 = y_2$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Solve the continuity equation for v_2: $v_2 = \frac{A_1}{A_2} v_1$

Substitute into Bernoulli's equation: $P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho \left(\frac{A_1}{A_2} v_1 \right)^2$

Solve for v_1: $v_1 = \sqrt{\dfrac{2(P_1 - P_2)}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}}$

The cross sectional area is a circle: $\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2} \right)^2$

$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho\left(\left(\frac{r_1}{r_2}\right)^4 - 1\right)}}$$

Solve the continuity equation for v_1: $v_1 = \frac{A_2}{A_1} v_2$

Substitute into Bernoulli's equation: $P_1 + \frac{1}{2}\rho\left(\frac{A_2}{A_1}v_2\right)^2 = P_2 + \frac{1}{2}\rho v_2^2$

Solve for v_2: $v_2 = \sqrt{\dfrac{2(P_1 - P_2)}{\rho\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)}}$

The cross sectional area is a circle: $\frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2} = \frac{r_2^2}{r_1^2} = \left(\frac{r_2}{r_1}\right)^2$

$$v_2 = \sqrt{\dfrac{2(P_1 - P_2)}{\rho\left(1 - \left(\frac{r_2}{r_1}\right)^4\right)}}$$

Basic Procedure

1. Measure the air pressure at the inlet and at the constriction.
2. Calculate the velocities at the inlet and the constriction.
3. Compare to the volume flow rate of the air source.

Blood Pressure

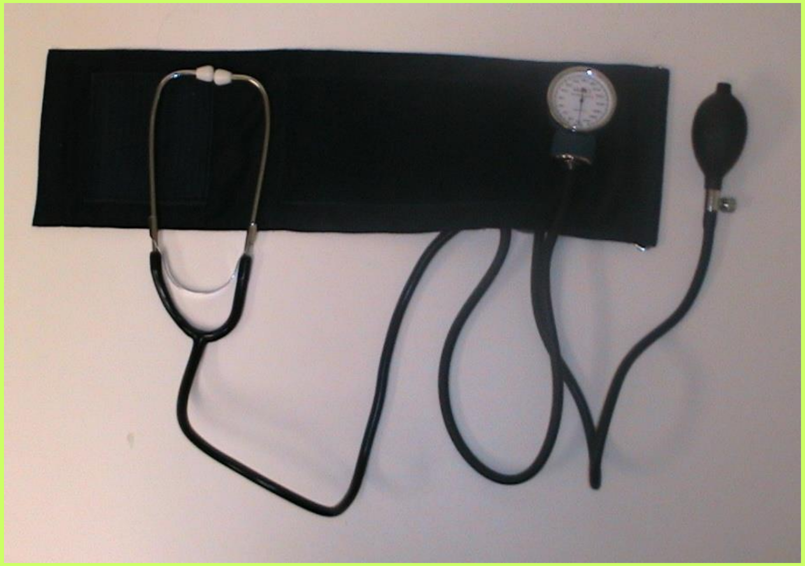
Blood pressure is generally measured on the arm at the level of the heart.

Systolic – Maximum pressure produced by the heart in mm-Hg.

Diastolic – Minimum pressure produced by the heart in mm-Hg.

$$\frac{\text{systolic}}{\text{diastolic}} = \frac{120}{80}$$

Sphygmomanometer



Bernoulli's Equation: $P_1 + \cancel{\tfrac{1}{2}\rho v_1^2} + \rho g y_1 = P_2 + \cancel{\tfrac{1}{2}\rho v_2^2} + \rho g y_2$

Assume: $v_1 = v_2$ $\quad\Rightarrow\quad$ $P_1 + \rho g y_1 = P_2 + \rho g y_2$

Rearrange the terms: $P_2 - P_1 = \rho g y_1 - \rho g y_2$

$$\Delta P = -\rho g \Delta y$$

This tells us that a positive change in height (increase) leads to a negative change in pressure (decrease).

If you measure the blood pressure of a person who is standing how does the pressure at the knee compare to the pressure at the elbow? Assume all other factors are equal.

1) **The pressure at the knee is lower.**
2) **The pressure at the knee is higher.**
3) **The pressure at the knee is the same.**

Δy is negative, so

ΔP is positive

How much will your blood pressure change if your arm is raised by 20 cm?

$$\frac{\text{systolic}}{\text{diastolic}} = \frac{120}{80}$$

$$\Delta P = -\rho g \Delta y$$

$$\Delta y = 0.20 \text{ m}$$

$$\text{Blood: } \rho = 1060 \text{ kg/m}^3$$

$$\Delta P = -\left(1060 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)\left(0.20 \text{ m}\right)$$

$$\Delta P = -2077.6 \text{ Pa}\left(\frac{1 \text{ mm-Hg}}{133.3 \text{ Pa}}\right) = -15.59 \text{ mm-Hg}$$

$$\frac{\text{systolic}}{\text{diastolic}} = \frac{120 \text{ mm-Hg} - 16 \text{ mm-Hg}}{80 \text{ mm-Hg} - 16 \text{ mm-Hg}} = \frac{104}{64}$$

Basic Procedure

1. **Measure your blood pressure in a normal sitting position.**
2. **Measure your blood pressure with your arm resting on a raised platform.**
3. **Compare the actual change in pressure to the expected change.**
4. **Measure your blood pressure with your arm raised over your head and compare to the expected result.**

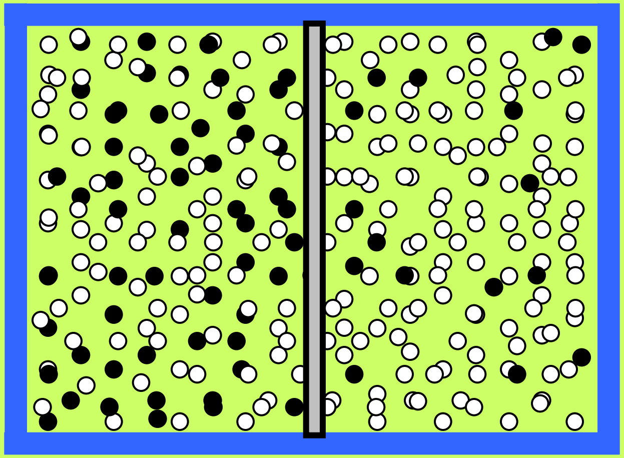
Diffusion and Osmosis

Solution – A fluid mixture of a solute dissolved in a solvent.

Example: Saltwater is a solution of a solute (salt) dissolved in a solvent (water).

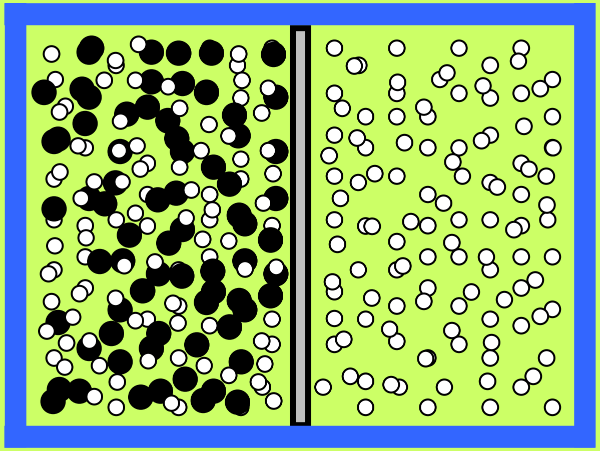
Diffusion is the process where the solute spreads out randomly in a solvent until it is evenly distributed. Think of a drop of ink spreading out in a glass of water.

Diffusion



Osmosis is a special case of diffusion where high and low concentration solutions are separated by a selectively permeable membrane.

Selectively permeable means that certain molecules can pass through but not others.



For instance, a selectively permeable membrane may allow water molecules to pass through but not sugar molecules.

Fick's Law of Diffusion

$$R_D = \frac{\Delta m}{\Delta t} = D\left(\frac{A\Delta C}{L}\right)$$

R_D = mass flow rate (kg/s)

A = cross sectional area (m^2)

ΔC = concentration difference (kg/m^3)

L = membrane thickness (m)

D = diffusion constant (m^2/s)

$$C = \frac{\text{mass of solute}}{\text{volume of solution}} = \frac{m_s}{m_T/\rho} = \rho\left(\frac{m_s}{m_T}\right)$$

ρ = density of the solution (kg/m^3)

m_s = mass of the solute (kg)

m_T = mass of the solution (kg)

A 30% sugar solution means that 30% of the solution is sugar and 70% is water.

If we have pure water on one side and a 30% sugar solution on the other, then:

Pure water: $C_1 = \left(1000 \ \text{kg/m}^3\right)\left(\dfrac{0.0 m_T}{m_T}\right) = 0 \ \text{kg/m}^3$

Sugar solution: $C_2 = \left(1000 \ \text{kg/m}^3\right)\left(\dfrac{0.30 m_T}{m_T}\right) = 300 \ \text{kg/m}^3$

$$\Delta C = C_2 - C_1 = 300 \ \text{kg/m}^3$$

In this experiment you will find the diffusion constant for a selectively permeable membrane. So we will need to rearrange Fick's law and solve for *D*.

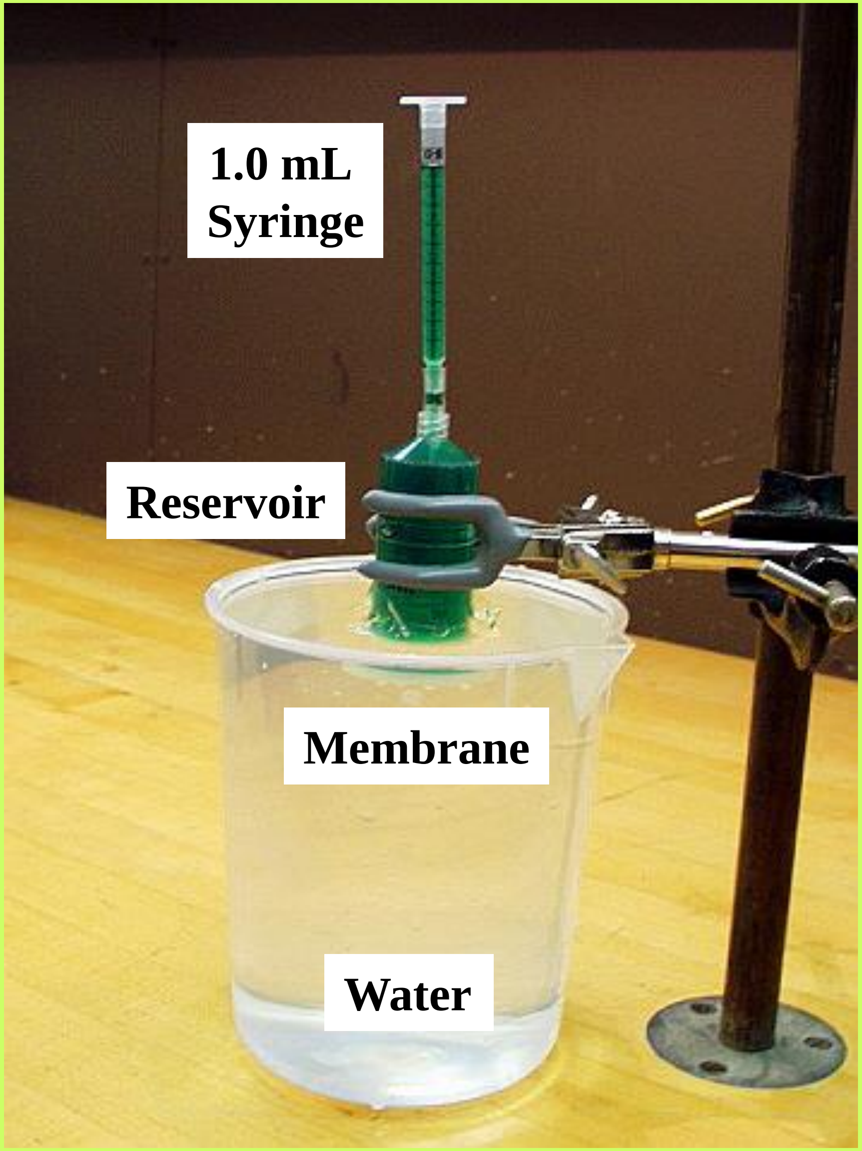
$$D = R_D\left(\frac{L}{A\Delta C}\right)$$

Osmometer

The water will flow across the membrane into the reservoir causing the volume of the solution to increase.

The change in volume will be measured with the 1.0 mL syringe.

You will use the change in volume over time to determine the mass flow rate which you will then use to calculate the diffusion constant.



Basic Procedure

1. Measure the thickness L and the cross sectional area A of the membrane.
2. Place the sugar solution in the reservoir and cap with the membrane.
3. Set the osimeter upright with the membrane just below the surface of the water.
4. Start the stop watch.
5. Record the volume of water in the 1.0 mL syringe at regular time intervals.
6. Convert the volume to mass and make a plot of mass versus time. The slope will be the mass flow rate R_D.
7. Calculate the diffusion constant D.

Osmotic Pressure

Ideal gas law

$$P_{osm} = \left(\frac{n}{V}\right)RT \Rightarrow \frac{n}{V} = \frac{\Delta C}{M} \Rightarrow P_{osm} = \frac{\Delta CRT}{M}$$

$n = \text{number of moles}$

$R = 8.31 \ \text{J/mol}\cdot\text{K} \ \text{Ideal gas constant}$

$T = \text{temperatue (K)}$

$V = \text{volume } (\text{m}^3)$

$M = \text{mass per mole } (\text{kg/mol})$

$$P_{osm} = P_0 + \rho g h$$

$$\frac{\Delta CRT}{M} = P_0 + \rho g h$$

$$h = \frac{1}{\rho g}\left(\frac{\Delta CRT}{M} - P_0\right)$$

Assume 5% sugar solution, room temperature (*T* = 295 K) and standard pressure.

$$h = \frac{1}{\left(1000 \ \text{kg/m}^3\right)\left(9.80 \ \text{m/s}^2\right)}\left(\frac{\left(50 \ \text{kg/m}^3\right)\left(8.31 \ \text{J/mol}\cdot\text{K}\right)\left(295 \ \text{K}\right)}{0.342 \ \text{kg/mol}} - 1.013\times10^5 \ \text{Pa}\right)$$

$$h = 26.2 \ \text{m}$$

Any Questions?

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