PHYSICS 111 HOMEWORK SOLUTION #5

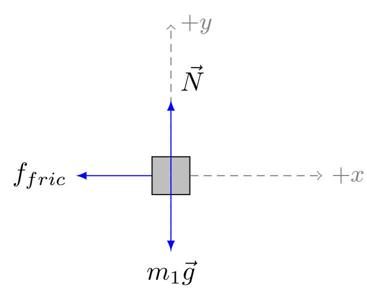
March 3, 2013

0.1

Your 3.80-kg physics book is placed next to you on the horizontal seat of your car. The coefficient of static friction between the book and the seat is 0.650, and the coefficient of kinetic friction is 0.550. You are traveling forward at 72.0 km/h and brake to a stop with constant acceleration over a distance of 30.0 m. Your physics book remains on the seat rather than sliding forward onto the floor. Is this situation possible?

a)

Lets's look at the forces exerted on the physics book:



The acceleration of the car can be calculated using $v_f^2 - v_i^2 = 2a\Delta x$

$$\begin{aligned} a &= \frac{v^2}{2\Delta x} \\ &= \frac{(\frac{72000}{3600})^2}{2 \times 30} \\ &= 6.67\text{m/s}^2 \end{aligned}$$

On the other hand, Projecting 2nd Law on the y-axis gives N=mg;
For the book to slide off the seat, acceleration should overcome frction:

$$\begin{aligned} f_s &< ma \\ \mu_s N &< ma \\ \mu_s mg &< ma \\ \mu_s = 0.650 &< \frac{a}{g} = \frac{6.67}{9.81} = 0.68 \end{aligned}$$

This is valid and the book will definitely slide forward to the floor.

0.2

A 2.70-kg block starts from rest at the top of a 30.0° incline and slides a distance of 1.90 m down the incline in 2.00 s.

- a) Find the magnitude of the acceleration of the block.
- b) Find the coefficient of kinetic friction between block and plane.
- c) Find the friction force acting on the block.
- d) Find the speed of the block after it has slid 1.90 m.

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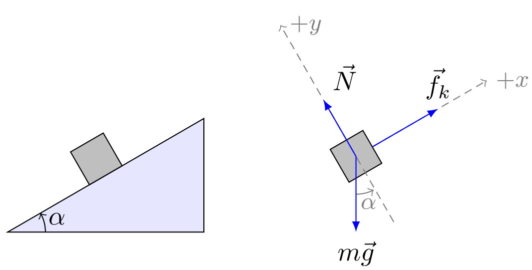
a)



We can use $x = \frac{1}{2}at^2 + v_0 t + x_0$ to find the acceleration of the object as it slides down. with $v_0 =$ and $x_0 = 0$. The object slides 1.90 meters in 2 seconds, this should give us:

$$\begin{aligned} a &= \frac{2x}{t^2} \\ &= \frac{2 \times 1.90}{4} \\ &= 0.95\text{m/s}^2 \end{aligned}$$

b)

Newton Second Law: $\sum \vec{F}_i = m\vec{g} + \vec{F}_{fric} = m\vec{a}$
Projection on the x-axis : $mg \sin\alpha - f_k = ma$ (*)
Projection on the y-axis : $mg \cos\alpha - N = 0$ (**)

We bear in mind that f_k and N are connected : $f_k = \mu_k N$

From (*) and (**) we get :

$$\begin{aligned}
\mu_k &= \frac{f_k}{N} \\
&= \frac{mg\sin\alpha - ma}{mg\cos\alpha} \\
&= \tan\alpha - \frac{a}{g\cos\alpha} \\
&= \tan(30) - \frac{0.95}{9.81 \times \cos(30)} \\
&= 0.465
\end{aligned}$$

c)

The friction force acting on the block:

$$\begin{aligned}
f_k &= \mu_k N \\
&= \mu_k mg\cos\alpha \\
&= 0.465 \times 2.70 \times 9.81 \times \cos(30) \\
&= 10.7\text{N}
\end{aligned}$$

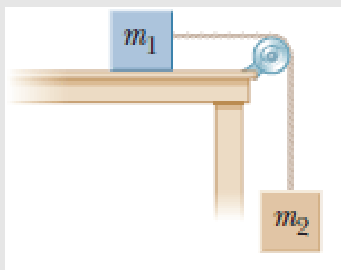
d)

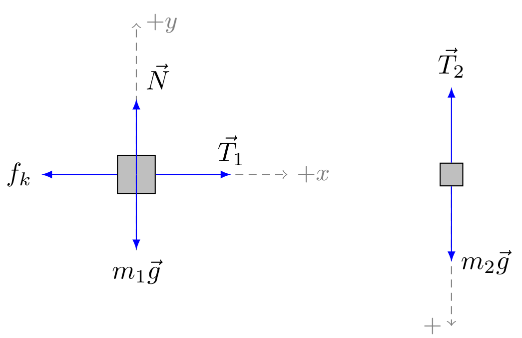
Speed of the block after sliding 1.90m
We can use the time independent equation :

$$\begin{aligned}
v^2 - v_0^2 &= 2a\Delta x \\
v &= \sqrt{2 \times 0.95 \times 1.90} \\
v &= 1.9\text{m/s}
\end{aligned}$$

0.3

A 9.80-kg hanging object is connected by a light, inextensible cord over a light, frictionless pulley to a 5.00-kg block that is sliding on a flat table. Taking the coefficient of kinetic friction as 0.185, find the tension in the string.





$$T_1 = T_2 = T;\ a_1 = a_2 = a \quad and \quad f_k = \mu_k N$$

- Projecting Newton's second law for object m_1 gives us

$$-f_k + T = m_1 a \qquad and \qquad N - m_1 g = 0$$

- Projecting Newtons's Law for object m_2 on the y-axis gives us $m_2 g - T = m_2 a$ thereby, $\quad a = \frac{m_2 g - T}{m_2}$

$$
\begin{aligned}
T &= m_1 a + f_k \\
&= m_1 a + \mu_k N \\
&= m_1 g - \frac{m_1}{m_2} T + \mu_k m_1 g \\
\frac{m_1 + m_2}{m_2} T &= g m_1 (1 + \mu_k)
\end{aligned}
$$

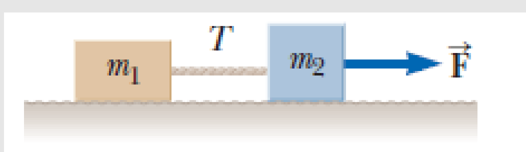
Finally,

$$
\begin{aligned}
T &= \frac{m_1 m_2 g (1 + \mu_k)}{m_1 + m_2} \\
&= \frac{9.80 \times 5 \times 9.81 \times (1 + 0.185)}{9.80 + 5} \\
&= 38.48\text{N}
\end{aligned}
$$

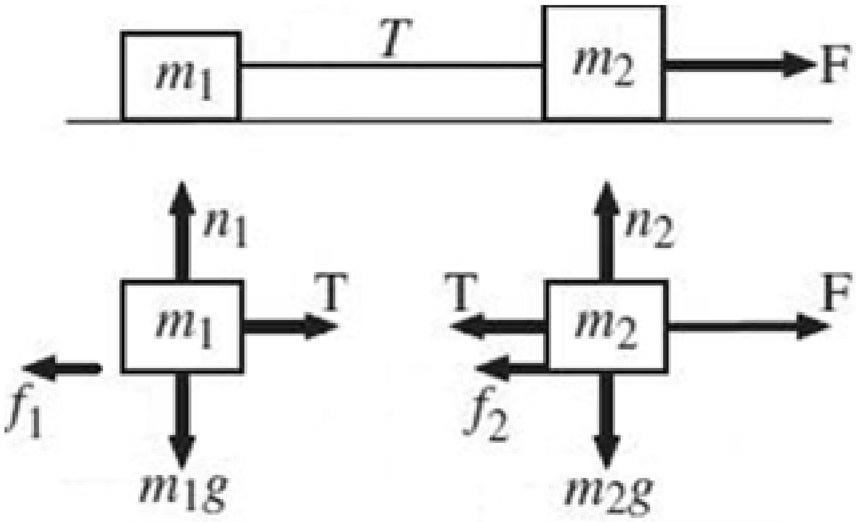
0.4

Two blocks connected by a rope of negligible mass are being dragged by a horizontal force (see figure below). Suppose F = 73.0 N, m1 = 14.0 kg, m2 = 26.0 kg, and the coefficient of kinetic friction between each block and the surface is 0.090.

- a) Draw a free-body diagram for each block.
- b) Determine the acceleration of the system.
- c) Determine the tension T in the rope.



a)Free body diagram for each block



b)

Projecting Newton's 2nd Law of the two blocks on the y-axis gives:

$$n_1 = m_1 g \;\; and \;\; n_2 = m_2 g$$

On the x-axis for m_1:
$T - f_1 = m_1 a$; with $f_1 = \mu_k n_1$ we get

$$\begin{aligned} T &= \mu_k n_1 + m_1 a \\ &= \mu_k n_1 g + m_1 a \end{aligned}$$

On the x-axis for m_2 and with $f_2 = \mu_k n_2$

$$\begin{aligned} m_2 a &= F - T - f_2 \\ &= F - (f_1 + m_1 a) - f_2 \\ &= F - \mu_k n_1 - m_1 a - \mu_k n_2 \end{aligned}$$

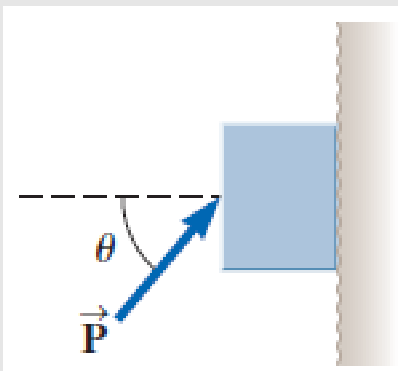
Bearing in mind that $n_1 = m_1 g$ and $n_2 = m_2 g$ we should finally get:

$$\begin{aligned} a &= \frac{F - \mu_k g(m_1 + m_2)}{m_1 + m_2} \\ &= \frac{73 - 0.090 \times 9.81(14 + 26)}{14 + 26} \\ &= 0.94 \text{m/s}^2 \end{aligned}$$

c) Tension in the rope

$$
\begin{aligned}
T &= \mu_k m_1 g + m_1 a \\
&= m_1(\mu_k g + a) \\
&= 14(0.090 \times 9.81 + 0.94) \\
&= 25.52\text{N}
\end{aligned}
$$

0.5

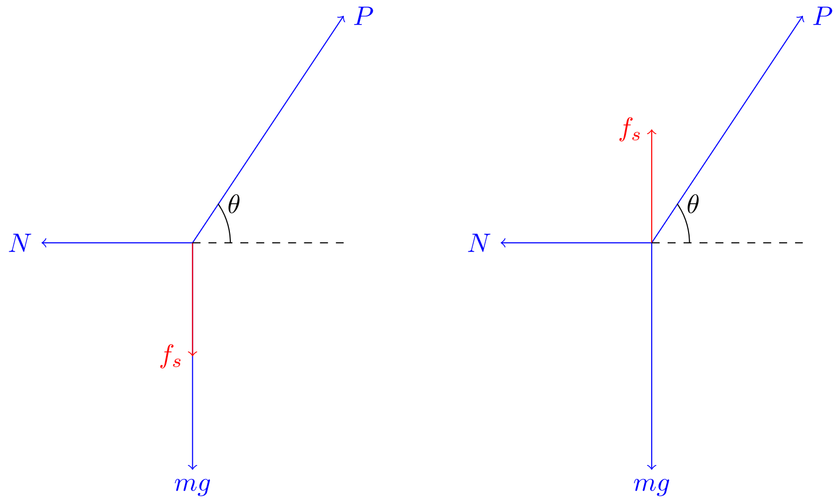


A block of mass 1.75 kg is pushed up against a wall by a force $\vec{P}$ that makes an angle of $\theta = 50.0°$ angle with the horizontal as shown below. The coefficient of static friction between the block and the wall is 0.260.

- a) Determine the possible values for the magnitude of $|\vec{P}|$ that allow the block to remain stationary. (If there is no maximum, enter NONE in that answer blank.)
- b)What happens if $|\vec{P}|$ has a larger value than $|\vec{P}_{max}|$?
- c))What happens if $|\vec{P}|$ has a smaller value than $|\vec{P}_{min}|$?
- d) Repeat parts (a) and (b) assuming the force makes an angle of $\theta = 12.2°$ with the horizontal.

a)

We will drwas two free body diagrams for this problem, and the reason is that the friction force will either push the object up if it slides down or it will push the object down as it slides up.



For the object to be stationary in the sitution at left, 2nd Law requires:

$$mg + f_s = p\sin\theta$$

and

$$N = p\cos\theta$$

with $f_s = \mu_s N$

$$\begin{aligned} mg + \mu_s N\cos\theta &= p\sin\theta \\ p(\sin\theta - \mu_s\cos\theta) &= mg \\ p &= \frac{mg}{\sin\theta - \mu_s\cos\theta} \\ p &= \frac{1.75\times 9.81}{\sin 50 - 0.260\cos 50} \\ P_{max} &= 28.67\text{N} \end{aligned}$$

For the object to be stationary in the sitution at right, 2nd Law requires:

$$mg - f_s = p\sin\theta$$

and

$$N = p\cos\theta$$

with $f_s = \mu_s N$

$$mg - \mu_s N\cos\theta = p\sin\theta$$

$$
\begin{aligned}
p(\sin\theta + \mu_s \cos\theta) &= mg \\
p &= \frac{mg}{\sin\theta + \mu_s\cos\theta} \\
p &= \frac{1.75 \times 9.81}{\sin 50 + 0.260 \cos 50} \\
P_{min} &= 18.60\text{N}
\end{aligned}
$$

b)

If p has a larger value than p_{max} the object will slide up the wall.

c)

If p has a smaller value than p_{min} the object will slide down the wall.

d)

If the angle changes to 12.2° we will have:
$p_{max} = \frac{1.75 \times 9.81}{\sin 12.2 - 0.260 \cos 12.2} = \textit{negative value}$!!!!!
The block will not slide along the wall.
and $p_{min} = \frac{1.75 \times 9.81}{\sin 12.2 + 0.260 \cos 12.2} = 36.88\text{N}$
The block is capable of sliding down the wall under this value of P_{min}

0.6

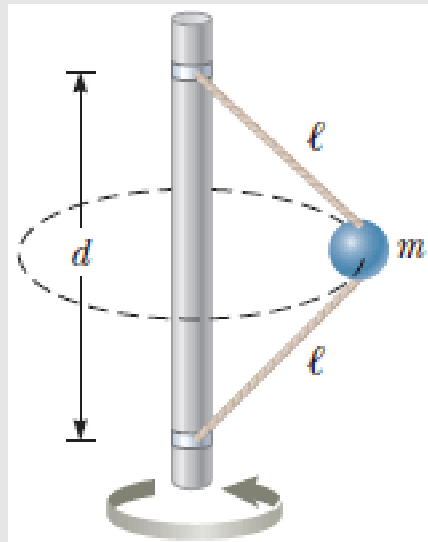
A curve in a road forms part of a horizontal circle. As a car goes around it at constant speed 14.0 m/s, the horizontal total force on the driver has magnitude 149 N. What is the total horizontal force on the driver if the speed on the same curve is 23.9 m/s instead?

Let's call F_1 and F_2 the forces exerted at speed v_1 and speed v_2 respectively. The two are centripetal forces of magnitudes $\frac{mv^2}{r}$. we have:

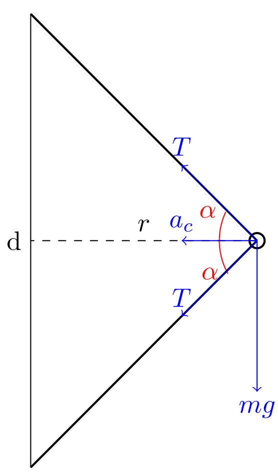
$$
\begin{aligned}
\frac{F_2}{F_1} &= \frac{mv_2^2/r}{mv_1^2/r} \\
F_2 &= F_1 \frac{v_2^2}{v_1^2} \\
&= 149(\frac{23.9}{14})^2
\end{aligned}
$$

$$= \quad 434.23\text{N}$$

0.7



Consider the following figure.
Why is the following situation impossible? The object of mass m = 4.00 kg in the figure above is attached to a vertical rod by two strings of length = 2.00 m. The strings are attached to the rod at points a distance d = 3.00 m apart. The object rotates in a horizontal circle at a constant speed of v = 3.00 m/s, and the strings remain taut. The rod rotates along with the object so that the strings do not wrap onto the rod.



a)

A projection of Newton's 2nd Law on the centripetal direction gives:

$$T_1 \cos\alpha + T_2 \cos\alpha = ma_c = \frac{mv^2}{r}$$

A projection on the vertical direction gives:

$$T_1 \sin\alpha - T_2 \sin\alpha - mg = 0$$

The geometry of the problem requires

$$\begin{aligned} r &= \sqrt{l^2 - (\frac{d}{2})^2} \\ &= \sqrt{2^2 - 1.5^2} \\ &= 1.32\text{m} \end{aligned}$$

$$\begin{aligned} \cos\alpha &= \frac{r}{l} \\ &= \frac{1.32}{2} \\ &= 0.66 \end{aligned}$$

$$\begin{aligned} \sin\alpha &= \frac{d/2}{l} \\ &= \frac{1.5}{2} \\ &= 0.75 \end{aligned}$$

Back to our firts equations we solve for T_1 and T_2

$$0.66T_1 + 0.66T_2 = \frac{mv^2}{r} = \frac{4 \times 3^2}{1.32} = 27.27$$

$$0.75T_1 - 0.75T_2 = mg = 4 \times 9.81 = 39.24$$

We should get $T_1 = 46.81$N and $T_2 = -5.5N$!!!
This value of T_2 is not possible in our situation and it implies that the lower string is pushing the object rather than pulling it.

b)

The variables in this problem that can make this situation physically acceptable are the speed of the object and the g value of gravity. In Mars for instance $g = 3.711\mathrm{m/s}^2$, our eqations become

$$0.66T_1 + 0.66T_2 = \frac{mv^2}{r} = \frac{4 \times 3^2}{1.32} = 27.27$$

$$0.75T_1 - 0.75T_2 = mg = 4 \times 3.711 = 14.84$$

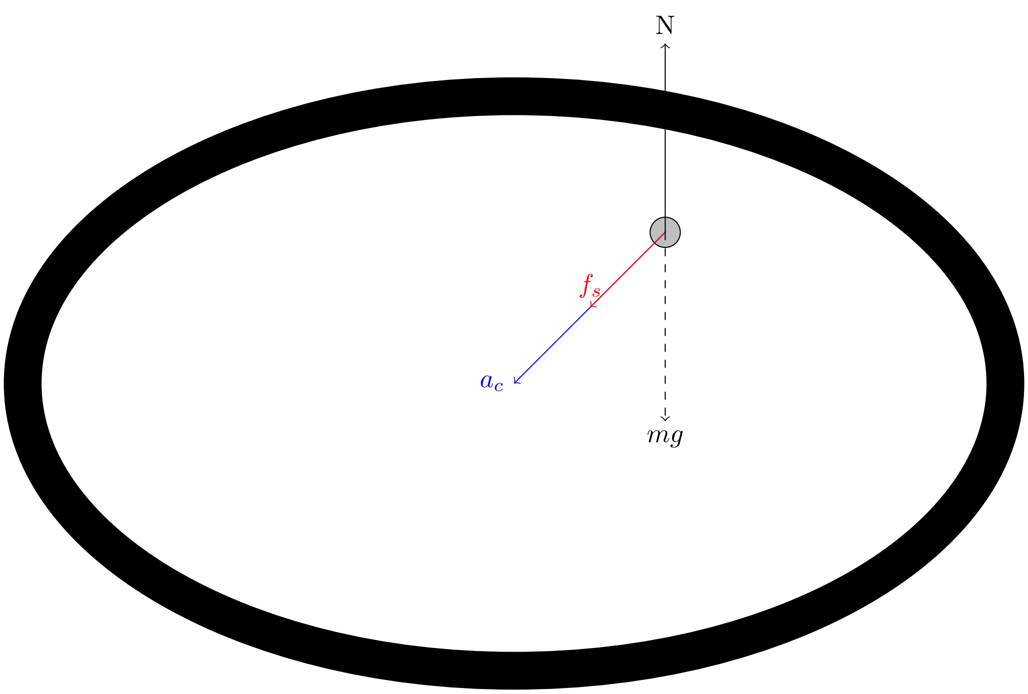
This gives $T_1 = 30.55\mathrm{N}$ and $T_2 = 10.76N$.

0.8

A coin placed 30.2 cm from the center of a rotating, horizontal turntable slips when its speed is 50.2 cm/s.

- a) What force causes the centripetal acceleration when the coin is stationary relative to the turntable?
- b) What is the coefficient of static friction between coin and turntable?

a)



a)

The centripetal force is provided by the static friction $\vec{f_s}$

b)

Projecting Newton's 2nd Law on the vertical gives $mg = N$
On the centripatal direction :

$$\begin{aligned} f_s = \mu_s N &= m\frac{v^2}{r} \\ \mu_s mg &= m\frac{v^2}{r} \\ \mu_s &= \frac{v^2}{gr} \\ &= \frac{(50.2 \times 10^{-2})^2}{9.81 \times 0.302} \\ &= 0.085 \end{aligned}$$

0.9

A hawk flies in a horizontal arc of radius 14.2 m at a constant speed 4.10 m/s.

- a) Find its centripetal acceleration.
- b) It continues to fly along the same horizontal arc, but increases its speed at the rate of 1.00 m/s2. Find the acceleration in this situation at the moment the hawk's speed is 4.10 m/s.

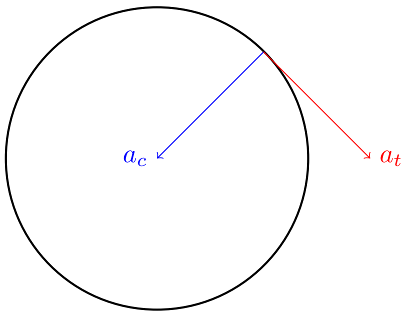
---

a)

The centripetal acceleration is :

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{4.10^2}{14.2} \\ &= 1.18 \mathrm{m/s}^2 \end{aligned}$$

b)



The hawk is now starting to accelerate along the horizontal with a tangential acceleration $a_t = 1.00 m/s^2$.
The overall vector aceleration of the hawk the moment its speed was $4.10 m/s$ becomes $\vec{a} = \vec{a_c} + \vec{a_t}$ having a magniutde of:

$$\begin{aligned} a &= \sqrt{a_c^2 + a_t^2} \\ &= \sqrt{1.18^2 + 1^2} \\ &= 1.54 \mathrm{m/s}^2 \end{aligned}$$

0.10

A 43.0-kg child swings in a swing supported by two chains, each 2.98 m long. The tension in each chain at the lowest point is

- a)Find the child's speed at the lowest point.
- b)Find the force exerted by the seat on the child at the lowest point. (Ignore the mass of the seat.)

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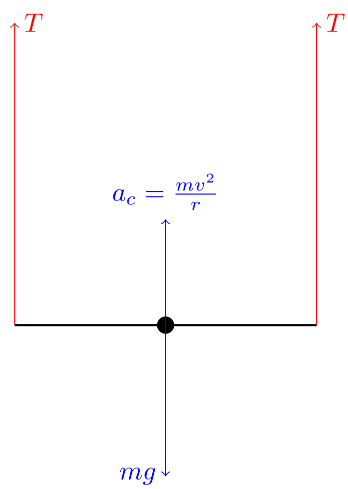
a)

At some time of this motion the child on his seat will be on a vertical position, this will help us derive equations easily: The seat and the child as one object are moving with same speed.



We should have : $T + T - mg = \frac{mv^2}{r}$ and

$$
\begin{aligned}
v &= \sqrt{\frac{2Tr}{m} - gr} \\
&= \sqrt{\frac{2 \times 352 \times 2.98}{43} - 9.81 \times 2.98} \\
&= 4.42\text{N}
\end{aligned}
$$

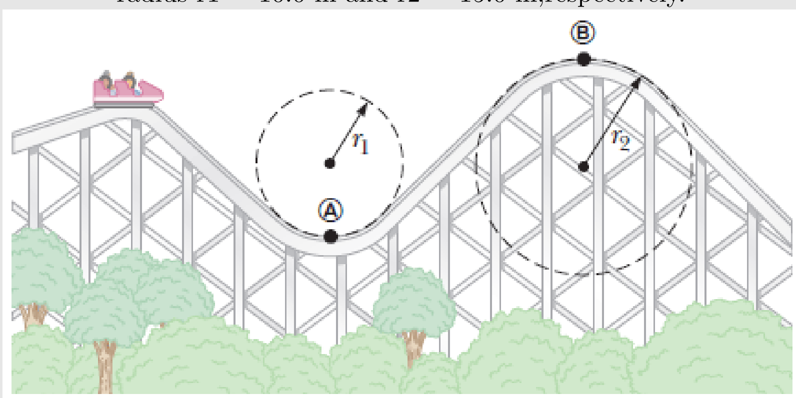
b)

The child is now our sole object, experiencing only gravity mg and the normal force N exerted by the seat: we should have:

$$
\begin{aligned}
N - mg &= \frac{mv^2}{r} \\
N &= mg + \frac{mv^2}{r} \\
&= 2T \\
& 704\text{N}
\end{aligned}
$$

0.11

A roller-coaster car has a mass of 499 kg when fully loaded with passengers. The path of the coaster from its initial point shown in the figure to point B involves only up-and-down motion (as seen by the riders), with no motion to the left or right. Assume the roller-coaster tracks at points A and b are parts of vertical circles of radius r1 = 10.0 m and r2 = 15.0 m,respectively.



- a) If the vehicle has a speed of 19.1 m/s at point A , what is the force exerted by the track on the car at this point?
- b) What is the maximum speed the vehicle can have at B and still remain on the track?

a)

On point A, the force exertad by the track is the normal which is also the centripetal force and we have:

$$\begin{aligned} N - mg &= \frac{mv^2}{r_1} \\ N &= m(g + \frac{v^2}{r_1}) \\ &= 499(\frac{19.1^2}{10} + 9.81 \\ &= 23099\text{N} \end{aligned}$$

b

On B, the normal force pointing downwars along with gravity should go to zero to allow the roller-coaster to remain in track, gravity is thus the only centripetal force:

$$
\begin{aligned}
mg &= \frac{mv_{max}^2}{r_2} \\
v_{max} &= \sqrt{r_2 g} \\
v_{max} &= \sqrt{15 \times 9.81} \\
&= 12.13\text{m/s}
\end{aligned}
$$