Lifelong Learning Programme

Multiculturalism, Migration, Mathematics Education and Language

Project Number: 526333-LLP-1-2012-1-IT-COMENIUS-CMP

FINGER MULTIPLICATION

by Barbro Grevholm*

INTRODUCTION

The area of study in the unit is Multiplication from different approaches (history, culture, traditions, use of tools and books), the use of concrete tools in calculations, the use of early algebra for formulation of rules for multiplication and for proving mathematical results, different ways of proving in mathematics and mathematical reasoning.

Piloting with teachers

Anticipated mathematical topics for development are arithmetic of multiplication, operations and rules, factors, products and factorisation, proof and proving, history of mathematics, metacognitive reflections on learning mathematics, and understanding of mathematics in relation to verbatim learning.

Aims of the unit

The aims of the unit is to make pupils reflect about the process of multiplication, realise the properties of multiplication and see links between multiplication and other areas of mathematics. Pupils may also reflect upon what they need to know by heart in mathematics and what can be reproduced with different tools or aids. Pupils may also notice that mathematics is constructed and used by ordinary people in many parts of the world. By interviewing their own family members they can learn about how multiplication is dealt with in their own countries.

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Main piloting

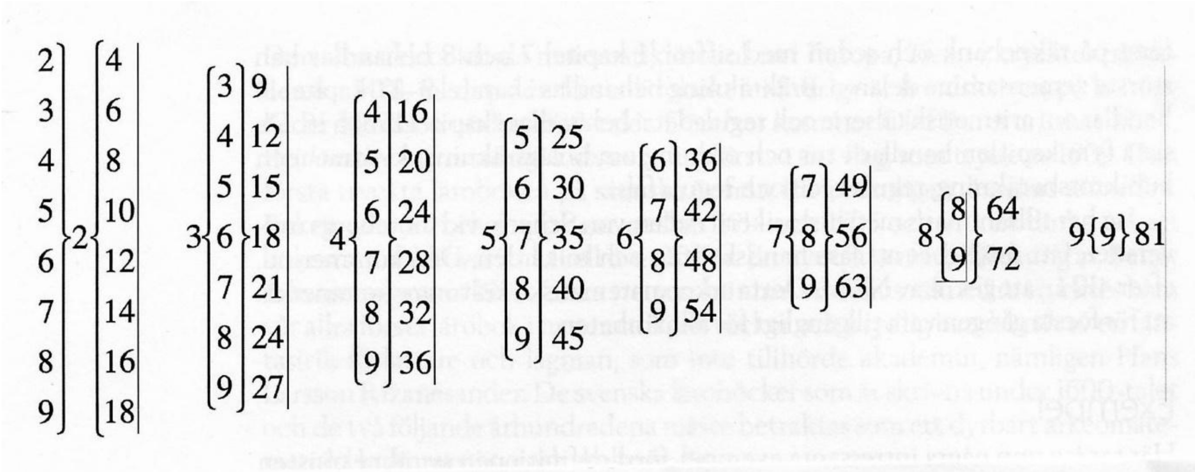
by Barbro Grevholm

The proposal: *Original text of the teaching unit Finger multiplication*

Start of the unit:

Session 1

Pupils are given a picture (see picture below) of a set of numbers in a triangular form from a handwritten book from 1601 (13 years before the first mathematics book was printed in Sweden). The history of the book can be told, see Appendix 2. Group discussion about the picture is initiated by the teacher with the following questions (after pupils had time to study the number pattern):



1. What do you see in this number arrangement? Have you seen something similar before?
2. What could be the reason for presenting this number arrangement in such a way? How did you meet such an arrangement in your mathematics learning?
3. What property for numbers is it that makes the table in the old book possible? What can be the reason for not using such shorter arrangements also today?

Comments for the teacher: We can expect pupils to recognise the row of results in the 2-table and in the 3-table. Maybe they link to the normal multiplication table? Here it is possible for the teacher to ask the class to write down the normal multiplication table with 10 times 10 equalities and in this table colour the products that are present in the triangular pattern. After further investigation and discussion the class may discover that the products that are missing in the triangular pattern are already in some of the present tables. Thus, all copies of calculations are removed in the triangular pattern.

Now the teacher can explain for the pupils that these tables can be used in two ways: to calculate a product of two integers or to find from a number seen as the product what are the factors in that number. That is one can calculate $3 \cdot 4$ or one can ask what are the factors in a given number like 12? Here the teacher may want to repeat the terminology factor and product used in multiplication and make pupils aware of the difference from addition where we talk about terms and sum.

Exercises

1. Use either table to calculate: a) 2·9 and 9·2 b) 8·7 and 7·8 c) 5·8 and 8·5. What do you notice? What is the name for this rule that multiplication of numbers follow? Can you find an easy way to illustrate this law? How about a rectangle with 2 rows and 9 columns? What happens if you look at it from different angles?
2. Find the possible factors in the numbers 18, 27, 42 by using either of the tables. Are there more than one option?
3. In how many ways can you split 48 in integer factors? Is there a simplest way of writing an integer as a product with simple factors? Do you consider 2·24 and 24·2 as two different ways? Or do you see 2·3·8 and 8·3·2 as two different ways? Why are the two last products equal? What do you name the rules that confirm this to us?
4. Can you think of a situation where it is important to be able to find the factors of an integer?
5. With what number do you need to multiply a) 12 in order to get 36? B) 9 to reach 72? c) 15 to get 90?
6. The number 4 can be factorised as 2·2 and 2+2 is 4 so the sum of the factors is equal to the number. Can you find another number with this property?
7. Now you construct some similar problems and ask you friends in the class to solve them.

Session 2

The teacher introduces the work with a conversation along these lines: Mathematicians often call themselves lazy persons and prefer to do things in the simplest possible way and work as little as possible. One of the things pupils struggle with in school for many years is to drill the multiplication table. How can this work be minimised? What did you find most difficult about the multiplication table? A friend of mine found 7·7 very easy as it was in the year 1949 she learnt the seven-table. Do you have a favourite product that you always know for sure?

Today we have digital tools such as the mobile phone and the calculator or the computer to do the multiplications for us. But what happens if you need to multiply and there be no tools around? Well, people thought about that also in old days and found solutions. One way is to use your fingers for the multiplication.

This is a way of multiplying any numbers between 5 and 10 with your fingers:

Figure source: Grevholm (1988), p. 19
For a translation of the task to English see Appendix 1

Try it and see if it works for you!

This method was once shown to a teacher educator by a mathematics teacher who learnt it from some Romany people who went to adult education in Malmö in Sweden. They asked their teacher to explain to them why this method always works. He was not able to prove that to them so he asked the teacher educator. Can you help the teacher explain why the method always works?

In the Nordic countries the method was also known as the peasant's or farmer's multiplication as it was convenient for anyone who does not have immediate access to paper and pencil.

[Comment for the teacher: The proof can be done by early algebra (see Appendix 3) or by convincing oneself that all possible cases are true by a systematic try, as there are only a limited number of cases here.]

<u>Exercises</u>

1. Convince yourself and your worst enemy of the fact that finger multiplication is always giving the correct result if properly done. Explain to your teacher how you did it.
2. Try to make up a mathematical story where you can solve the problem with multiplication. Ask you friend to solve it. Compare each other's methods of thinking. Do you prefer a special one of the methods? Why?

Session 3

There are many different ways of finger multiplication and most of them are known since historical time and come from traditions in different countries. Go to the internet and search for finger multiplication. Explore some other ways of finger multiplication. What is different and what is similar to the way shown here? Is there a proof available? Do the authors explain why the method works? Do you find one method you prefer to the others? Explain why.

Now when you can multiply numbers between 5 and 10 with your fingers how can you reduce the triangular multiplication table to just the cases you need to know by heart? How many different multiplications is that? Write down the reduced table.

With a calculator you can easily find out $12 \cdot 14$. Would it be possible to do that by finger multiplication also? Try to create a way to do that. Is there anything on the internet about multiplying numbers between 11 and 15? Do you find something for even bigger numbers?

What do books about history of mathematics tell you about finger multiplication? See, for example, D. E. Smith, History of mathematics.

In what countries do you find the different methods for multiplication? Think about the way this skill was learnt from one person to another. In old days it was oral communication and you probably showed how to do in a very practical way. You may even describe it for a person who does not speak your language? You can explain by just using gestures and signs. Today if you want to describe the method in written form, as in the internet for example, it takes much more effort than to just show it with your fingers. The skill of finger multiplications seems to be forgotten in many countries today. Why is it so? And why are we asking children in school to learn a table with $10 \cdot 10$ multiplications when we can do with much fewer operations?

Final session

The teacher and the class can have a summarising discussion about what has been learnt.

Here are just some examples of what can be treated:

- What properties of multiplication have we found?
- Can we explain why the table from 1601 looks like it does?
- Can we find the same properties for division of numbers? Why? Find examples.
- How about addition and subtraction? What do you think? Find examples.
- What could be the reason why finger multiplication is not well known any longer?

A fruitful way to handle the summary is to start by letting pupils write down their own view of the learning that took place and then discuss it in the whole class.

Comments for the teacher

Pupils will probably know the commutative law, but they might not have a name for it. The exercises give an opportunity to introduce this terminology and also maybe for pupils to discover other laws for multiplication. We have given some additional material for the teacher, as for example the history of the book from 1601 (in Appendix 2), the algebraic proof (see Appendix 3), the systematic proof, links to good sites about finger multiplication, ways of multiplying numbers larger than 10 (Appendix 4), some texts from history books about finger multiplication (Appendix 2) and so on.

The work is useful from multicultural aspect as we know it has been used in many different groups of people in different countries and over long time spans, and it is a way of working which has been transposed orally or via gestures in old days so it can be made independent of language and is highly concrete. Pupils can interview their parents and grandparents about the table and see if they know the different tables. The triangular table combined with finger multiplication reduces some of the learning by heart of operations in the multiplication table. From research we know that multiplication is an area where learners struggle much.

The main piloting of this teaching unit took place in Norway, in one school in Kristiansand and one in Trondheim. Below are reports and summaries of the results from main the piloting,

General information:

The main piloting was carried out by Kari SofieHolvik and Camilla Normann Justnes and this report builds on the written reports from them and the evaluations made by them. The photos are taken by Camilla Normann Justnes. In a meeting with one of the teachers before the piloting we discussed in what ways the unit would be helpful for the pupils. The teacher saw it as a great help for repeating about multiplication and she also suggested that she could use it to prepare the factorization of numbers which the pupils in her class should learn about later that semester. Thus we included some exercises that deal with factorization. In fact these also make explicit that multiplication and division are inverse operations.

From Karuss school

First half of the class learnt about finger multiplication. The pupils were fascinated and thought it was fun. They were given the task to teach someone else about it, but all of them forgot about that. Thus I, as teacher, had to teach the rest of the class about it later. None of the pupils used the method in the examination as they have the multiplication table in their heads and then that is faster to use. If we repeat the method constantly, it could happen that some of the pupils can use it in a test without aids. There was not time or reason for showing proofs to anyone. But I would like to bring it forward in year 9 when we study algebra.

The final lesson in this project we used to explore other methods for multiplication. The pupils use the internet and searched for other methods, in order to learn at least one of them and then show the class afterwards. They worked in pairs and were very eager. The method that was most fascinating to them was Japanese method, which has been exposed in Facebook lately, where you can count and make crosses (see link).This is some of the methods they found:

- http://vivas.us/i-promise-that-this-japanese-multiplication-technique-will-make-math-way-easier/
- magic math for 7`s
- Multiplication, learning the times table (Mister numbers)
- No.swewe.com
- Guro.sol.no/questions/naturvitenskap/matematikk/hvordan-multiplisere-firesifrede-tall-i-hodet

Reflections on possible changes if I use the unit once more

There was too little time for the pupils to work through all the tasks and make a summary afterwards. Next time I would use a couple of questions, for example 1, 2 and 3 if I was thinking of usefulness for the examination and 5, 6 and 7 if I wanted to focus on investigations. Another option is to differentiate the tasks. Or distribute the tasks randomly and ask pupils to explain to one another in larger groups afterwards. It is in any case important that they have time to talk mathematics together.

I could have used more time (a double lesson) so that more pupils would reach more. The level and type of tasks was well adapted to the year group of pupils. The unit is fine related to repetition of factorization and prime number factorization and other mathematical concepts (factor, multiplication, sum and so on).

The lesson after the final on this unit we had to use for learning of the last subject before examination (volume) and the second lesson this week will be used for exercises on that theme. But later this week we start with repetition and then I will refer to what we worked with in this unit were it fits in and maybe ask if some pupils will use some of the other multiplication tables they got.

I will use finger multiplication when we repeat for examination in week 18 and 19, including task 1. The mathematical proof for why it is correct will be distributed to a few pupils. It is too advanced for most of the pupils in year 8.

The exploration of other multiplication methods must be done after the examination (week 21). They are actually more relevant and suitable in school year 4-6.

From Saupstad School

Here the teacher carried out the teaching unit in a year 5 class and has given us detailed notes of her work. One lesson (45 minutes) was used for the first session. She used the first session starting with telling the story about the old book. She then distributed a copy of the number set to each pupil and gave them 5 minutes to study it. After that the pupils discussed two and two and formulated questions. The teacher took notes on the green board.

Here is the translation of the notes in Photo 1 below:

What do we see?

- *It is times. We can find the answer.*

- *Times tables from 2-9-times.*
- *Times from left to right. Parting from right to left.*
- *It looks like a triangle.*

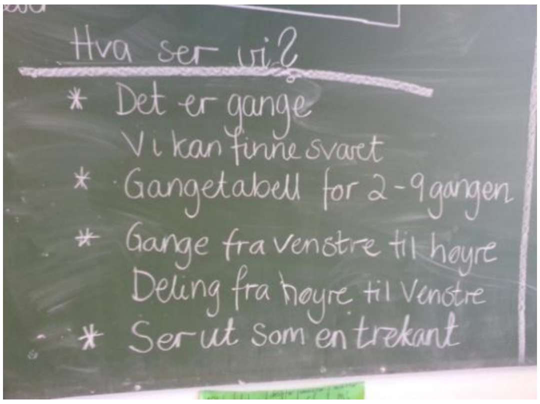


Photo 1: Pupils' answers to "What do we see?"

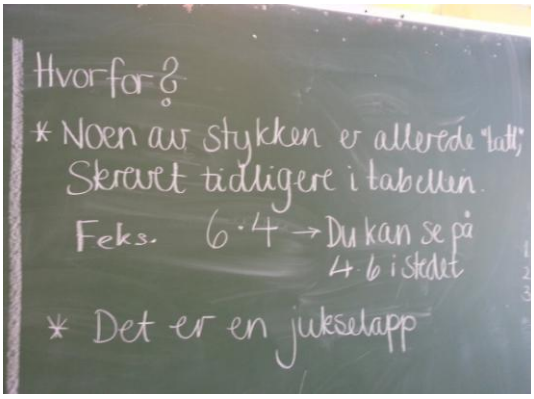


Photo 2: Pupils' answers to "Why?"

Translation of the text in Photo 2:

Why?

- *Some of the tasks are already done. Written earlier in the table. For example 6·4 →You can look at 4·6 instead.*
- *It is a cheating note.*

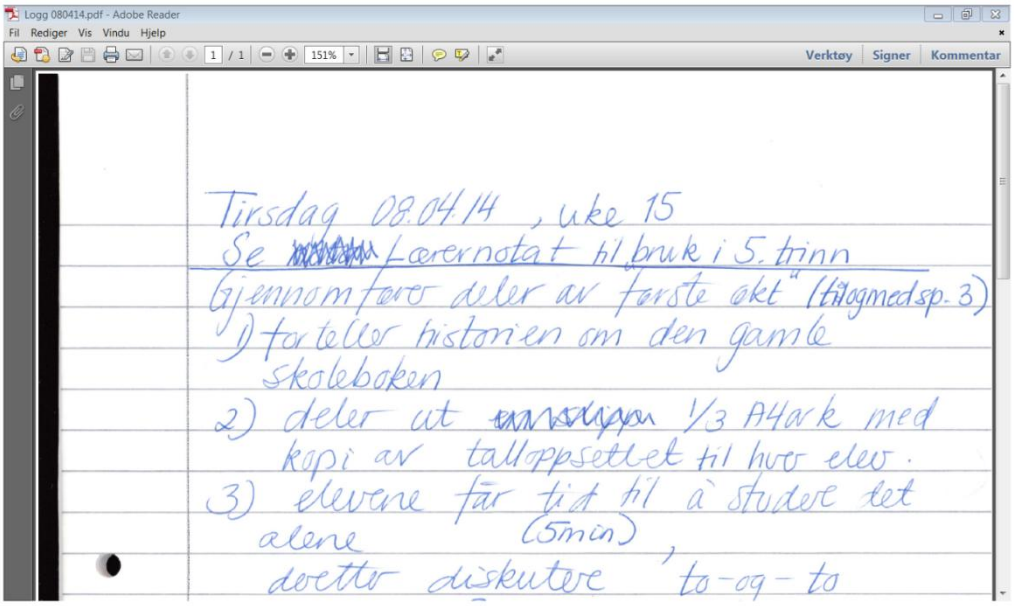


Photo 3. Part of the teacher's notes for her planning of the unit.

The pupils were eager in the discussion and some of them wanted to keep the number set on their desk and use is as a tool. Some of them glued it in their mathematics note book. Pupils then made exercise 1 and used the number set for the calculations.

Conclusions

In Norway teachers are very aware of the need to go through the curriculum and include all parts of it. The examinations are highly important and much time is used to prepare for them. One effect of this is that teachers often feel that there is little freedom and they dare not do other things than what is directly and explicitly in the curriculum. Thus is it is not unexpected that the teachers who are piloting here experience that they cannot use much time on a teaching unit where they do not immediately see how it supports the pupils' learning. This explains why the teacher in Karuss school could not do all parts of the unit in a sequence but has to come back to it later. In the case of Saupstad school we have unfortunately only got the report from the first session and the rest is missing. We hope to be able to take part of it later.

Both teachers are reporting that pupils were eager to learn and enthusiastic about the triangular shaped number set. Pupils wanted to keep the triangular number set as a tool in the calculations. It is interesting to note that when the pupils see something that can support them and ease the learning they see it as a cheating note. It is as if mathematics has to be hard.

When pupils searched for complementary methods of multiplication they seem to have discovered mathematics from different parts of the world and different cultures.

From the pupils' answers to the questions it is obvious that even in year 8 they use a rather immature terminology like times instead of multiplication and parting instead of division. In teaching sequences like this one there are natural opportunities to learn more about terminology also in different languages.

The teachers are teaching in different age group but still they both found ways to use the unit in fruitful plans. Thus the level of difficulty and the level of choice of subject can be seen as appropriate.

Second piloting

by Andreas Ulovec[**] and Therese Tomiska

General Information

The teaching unit was piloted by a female mathematics teacher with five years teaching experience working in an upper secondary school near Vienna. The Austrian project team sent the material to the teacher approximately 3 weeks before the planned piloting activity. The teacher had a 5th (age 14-15 years), 6th (15-16) and 8th (17-18) grade available for piloting. After a meeting with the project team, she chose to conduct the piloting during a regular mathematics class (50 minutes) in the

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6th grade. Eight students (age 17-18), three of which are migrant students, attended the class, which was video recorded and observed by a member of the Austrian project team. After the piloting, an interview was conducted with the teacher.

Classroom piloting

The teacher conducted *Session 1* as described in the Norwegian material by handing out sheets containing a multiplication table from the year 1601 and started a group discussion about it. This discussion lasted about 12 minutes.

Group discussion about multiplication table

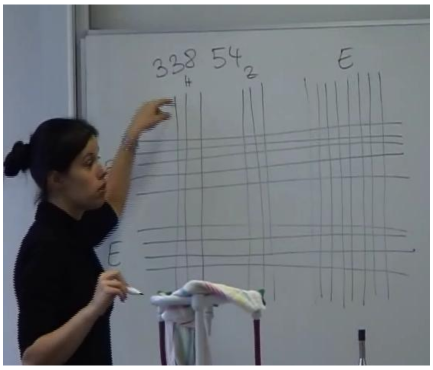
Teacher listening to the students' arguments

The students were particularly interested in the aspects of why there was a need for such tables, whether such tables existed in their own cultures' history, and (mathematically) why these (shortened) tables were sufficient and contain the same basic data as the traditional, square-matrix shaped full multiplication tables they know. The information about the various aspects was partly given by the teacher, partly the students used internet resources to retrieve additional information.

Session 2 started with the introduction of the method of finger multiplication by the teacher.

Teacher demonstrating and students trying out finger multiplication

Students were then asked to try the method out and – together with the teacher – find an explanation why it works (15 minutes). Students came up with several explanations and wanted to find out whether the method can be extended for larger numbers.



Teacher supporting students' generalisation efforts

They also were interested in whether this or other "unusual" multiplications were used historically. Two of the migrant students (from Turkey) mentioned a geometric-based multiplication method from their own culture, which the teacher then explained further.

As can be seen, the four sessions that were foreseen in the Finger Multiplication material were fitted by the teacher into one lesson. Since there was no timeframe given in the materials, and since the students already knew about multiplication, number systems, and various algebraic methods, a longer duration of the unit was not deemed necessary.

Interview with the teacher

An interview was conducted with the teacher in the afternoon after the piloting took place. The teacher reported that during the break immediately following the class, students were asked by the teacher about their experience with this teaching unit. Both the migrant and non-migrant students responded very positively. The migrant students particularly mentioned the chance of giving background information about their own culture that the other students did not know before. The non-migrant students commented positively on the various historical and cultural references that they not usually get during regular mathematics lessons. The teacher particularly welcomed the possibility of having various anchor points for cultural references, and the opportunity to have the migrant students not only participate, but being a source of information for the other students.

Conclusions

The piloting clearly showed that students are interested in mathematics content from different cultures, and that the active participation of migrant students and the introduction of their cultural backgrounds can enrich the learning situation.

Third piloting

by Hana Moraová*** and JarmilaNovotná***

Place: ZŠ Fr. Plamínkové s RVJ, Prague

Time: 9th September 2014

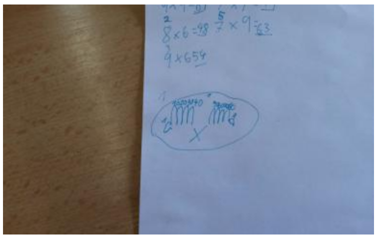
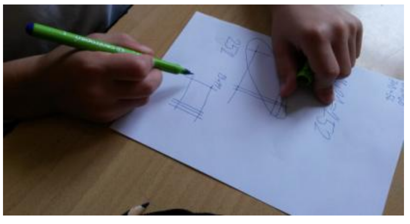
Class: 3rd grade (2 different classes)

Knowledge: knowledge of multiplication tables to 5, one of the classes already started multiplication by 6-9 before holidays, some children remembered it

The lessons were taught in English, videos were prepared to present how to use finger multiplication, but the projector failed to work

The course of both lessons

- Revision of numbers from one to one hundred
- Revision of multiplication 1-5
- Demonstration of finger multiplication (the teacher) on two examples.Using fingers and whiteboard.
- Children asked to do it, very few understood, i.e. two other problems solved in front of the whiteboards, one pupil asked to come, done in cooperation of the teacher, the pupil and the rest of the class who were telling the numbers.
- Then asked to work on their own, the teacher monitoring, helping individually to those, who needed it.
- Then the teacher shows another magic trick, multiplication of two-digit numbers using lines, the children liked this one very much and understood more easily, less counting, no multiplication, i.e. easier.


Lesson 2: November 2014

One of the two **third grade**s, CLIL lesson.

Warm up – number lines, saying bang instead of numbers divisible by 3, then by 4.

*** Faculty of Education - Charles University in Prague, Czech Republic.

Lead in – revision of finger multiplication (the pupils now have learned multiplication tables, just motivation and fun).

Main activity: *I'll show you a trick –* two digit line multiplication, the pupils were introduced to it in the previous CLIL lesson.

Material – square grid paper (to make it easier to draw the lines and also to make it easier to practice work with units, tens and hundreds that are greater than ten).

The first line multiplication teacher controlled, the teacher draws the model, the children count together the number of intersecting points, numbers that do not exceed ten (in units, tens, hundreds).

Then similar multiplication carried out individually (the same numbers set to everybody).

Then two digit numbers where children will have to put up with addition of units, tens, hundreds exceeding 10, first example done together on the blackboard.

Then pupils work on another pair of numbers on their own, the teacher monitors and checks understanding, works with possible mistakes (the children draw the lines too close and fail to see the border between the first and the second digit, the children forget to add the units etc. higher than ten).

The result is checked and possible mistakes corrected.

Lesson 3: 13th February 2015

Short video recording

Warm up – a song with numbers (only for motivation and as an ice-breaker), number lines as in previous lesson.

Lead-in: revision of multiplication of one-digit numbers; learning the word "divisible"; the children get a card with a number and say I am a number divisible by … and by …, what number am I?", who answers correctly is the next one to go and "be" a number.

Revision of finger multiplication.

Main activity:

1. one multiplication of two two-two digit numbers on a sheet of paper, the teacher monitors and helps where needed.
2. the results is taken and described as units, tens, hundreds.
3. a larger number is written on the blackboard and labelled as tens of thousands, thousands, hundreds, tens, units (starting from units from the right hand side).
4. the teacher instructs the children: *Take 5 pencils with different colours from your pencil case.* The teacher takes 5 different chalks. *Underline tens with one colour, e.g. blue.* The teacher does it on the blackboard. *Take another colour. Underline*

hundreds. The teacher underlines hundreds on the blackboard … Until they get to tens of thousands.

5. the teacher instructs: *Now, let us multiply three-digit numbers.* And demonstrates that on the blackboard, slowly, step by step, using the different colours to show units, tens, hundreds and also drawing attention to the difficulty of having more than ten units, tens, hundreds etc. and how to cope.
6. the teacher elicits other two three-digit numbers and asks pupils to work on their own, carefully monitoring the progress, helping out individually; the pupils understand fast, some of them solve the multiplication very quickly, so a new pair of three digit numbers is presented, careful monitoring helps majority of pupils to get into the system.
7. checking the numbers together, language obstacle – some of the children can only say numbers to one hundred in English, the teacher writes a model number in words on the whiteboard, the class practices.

warm up (revision of finger multiplication]

Another piloting: 24th February 2014

4th grade, pictures, use of interactive board, the same school

The teacher decided to test the same teaching unit with one year older pupils (slightly higher level of English and more practice in adding higher numbers). The class already know multiplication tables, thus the lesson was not based on finger multiplication but on line multiplications, both two and three digit numbers, use of IT.

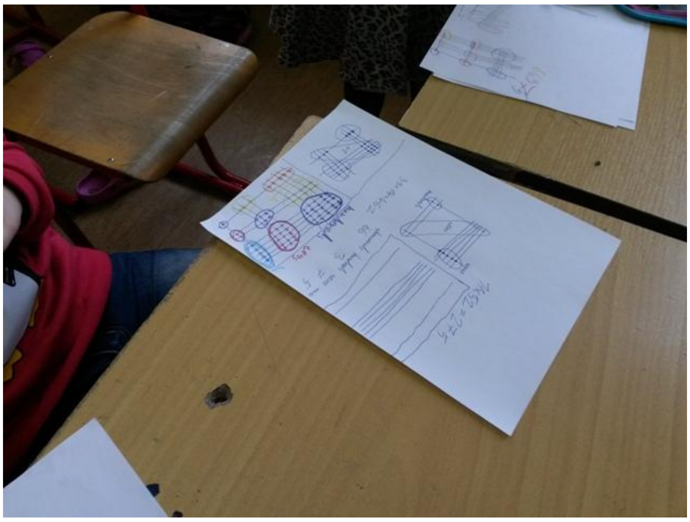
Goal of the lesson: practicing recording numbers as units, tens, hundreds thousands, multiplication and addition, motivation, doing mathematics in English.

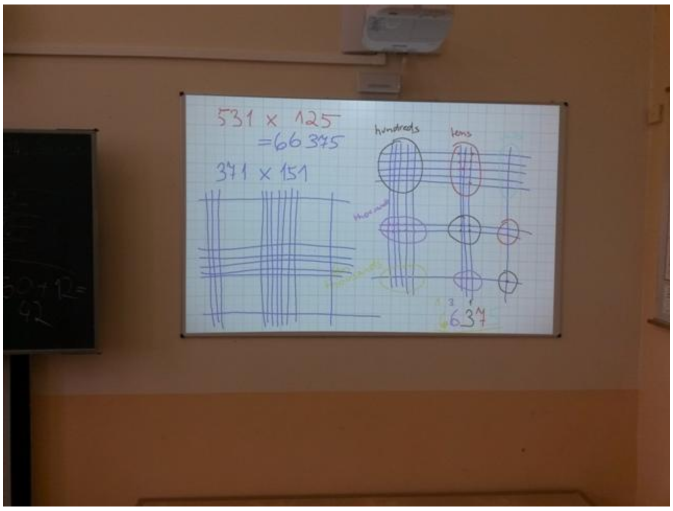
Warm up – motivation and ice-breaker, showing finger multiplication, trying it out on few numbers.

Main activity – learning new ways of multiplying larger numbers.

1. introduction to basic terminology – units, tens, hundreds …., add, multiply etc.

2. eliciting two 2 digit numbers, showing the principle of line multiplication on the digital whiteboard.
3. eliciting other 2 two-digit numbers, individual work, the teacher monitoring carefully; about half of the pupils discovered how to work with more than 10 units, tens, hundreds and got the right solution, the rest of the pupils were helped individually by the teacher and all was demonstrated on the board – a pupil who finished earlier drew the lines and the rest was done together with attention paid to what to do with units, tens… if there are more then 10.
4. transition to three digit numbers; again making use of 5 different colours for units, tens …; the first example shown on the digital whiteboard using the different colours.
5. other two three-digit numbers elicited, pupils work on their own, the teacher monitors and helps individually, checks that pupils do not mix up units, tens, hundreds; about half of the pupils found it very easy, the rest needed help and support.





No specific conclusions are reported from the Czech teachers.

Conclusions from the three piloting

by Barbro Grevholm

All three piloting reports show that the teaching unit worked well and it could be used in different age groups from year 3 to 8. Pupils are reported to have worked eagerly and with interest and fascination. The teachers seem to have been inspired to carry out both the suggested activities and similar activities found by them about multiplication. The teachers also used the unit to work on language and terminology and they found links and connections between multiplication and other areas of mathematics. The historical aspects were well used and in some cases pupils contributed with experiences from their own culture. The teaching unit seemed to offer something new and unexplored from earlier instruction. Many of the pupils wanted to include the triangular number set as a tool in their mathematics.

A critical aspect could be how much time to use for a unit like this. The answer depends very much on which age group the teacher is working with and from what perspective the unit is used, as repetition, consolidation or exploration.

Another aspect that can be discussed is for which age group a unit like *Finger multiplication* is better used. It probably depends on the level of difficulty that the teacher sets to the questions. It is obviously possible to use it from grade 3 to 8. Some teacher have even used it in algebra I upper secondary school.

The unit can be used to let pupils contribute with experiences and with the creation of tasks. When it comes to multiplication it is generally more difficult to construct your own problems than when it is the operations addition and subtraction, which are intended to be used. Construction of problems can illuminate the different kinds of multiplication tasks that we normally use in school (Verschaffel & De Corte, 1996).

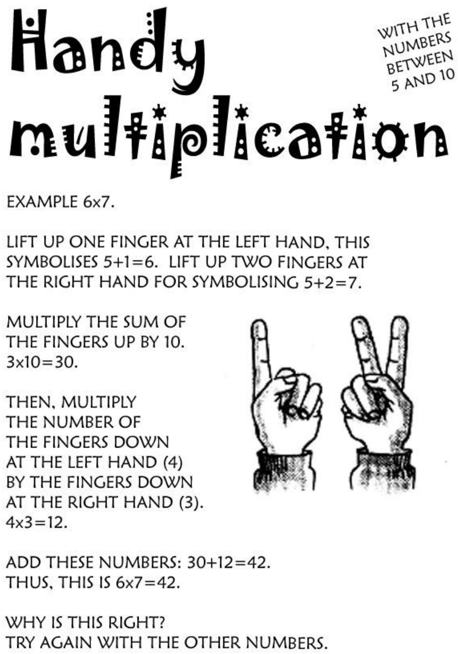
References

Grevholm, B. (1988). *Utmaningen. Problem och tankenötter i matematik*. [The challenge. Problems and mindnuts in Mathematics]. Malmö: Liber.

Verschaffel, L. & De Corte, E. (1996). Number and arithmetic. In *International handbook of mathematics education*, (pp. 99-137). Dordrecht: Kluwer academic Publishers.

Appendices for the teaching unit “Finger multiplication”

Appendix 1: Translation of the task Fingerfärdig multiplikation (Grevholm, 1988)



Appendix 2

About the book by Rizanesanders from 1601 (source SharezaHatami, 2014)

Rizanesanders' book was called Recknekonsten, The art of calculation

About 412 years ago (in 1601) Hans Larsson Rizanesander wrote the first Swedish textbook in Arithmetic. One single copy of this handwritten book, *Recknekonsten*, is kept in Uppsala universitets bibliotek (Uppsala University library).

An interesting question is how much space the multiplication table should be given in the teaching.

Rizanesanders' multiplication-table rests on the commutative law for multiplication; the table demonstrates in all its simplicity use of both mathematical and didactical knowledge.

Rizanesanders' multiplication-table in a simpler version

Below we show Rizanesanders' table in a way which is maybe more adapted to today's teaching. It is clearly visible that the worst table is the simplest one!

Tvåanstabell **Table 2**	**Treanstabell** **Table 3**	**Fyranstabell** **Table 4**	**Femmanstabell** **Table 5**	**Sexanstabell** **Table 6**	**Sjuanstabell** **Table 7**	**Åttans Tabell** **Table 8**	**Nianstabell** **Table 9**
2 · 2 = 4							
2 · 3 = 6	3 · 3 = 9						
2 · 4 = 8	3 · 4 = 12	4 · 4 = 16					
2 · 5 = 10	3 · 5 = 15	4 · 5 = 20	5 · 5 = 25				
2 · 6 = 12	3 · 6 = 18	4 · 6 = 24	5 · 6 = 30	6 · 6 = 36			
2 · 7 = 14	3 · 7 = 21	4 · 7 = 28	5 · 7 = 35	6 · 7 = 42	7 · 7 = 49		
2 · 8 = 16	3 · 8 = 24	4 · 8 = 32	5 · 8 = 40	6 · 8 = 48	7 · 8 = 56	8 · 8 = 64	
2 · 9 = 18	3 · 9 = 27	4 · 9 = 36	5 · 9 = 45	6 · 9 = 54	7 · 9 = 63	8 · 9 = 72	9 · 9 = 81

In Rizanesanders' table the one and ten tables are not used. One can suppose that he wanted the teacher and the pupils to reflect together upon that. He simply uses the commutative law for multiplication to remove all multiplications that exist more than once in the 10 times 10-table.

A multiplication-table, similar to Rizanesanders' multiplication-table, can be found in the first printed textbooks in Sweden in Arithmetic, the one by Aurelius from 1614. Another similar multiplication-table exists in Nils Buddaeus' (1595-1653) textbook. His multiplication-table reminds us about Rizanesanders' multiplication-table.

1

2 4

3 6 9

4 8 12 16

5 10 15 20 25

6 12 18 24 30 36

7 14 21 28 35 42 49

8 16 24 32 40 48 56 64

9 18 27 36 45 54 63 72 81

Below, marked in red, is the 2-table and the following colours show tables for 3, 4, 5, 6 7, 8 and finally the 9 table.

1

1 4

369

4 81216

51015 2025

612 18 243036

714 21 28354249

81624 3240 48 5664

9 1827364554 637281

It is interesting to ask from where the Swedish textbook authors got the idea to present the table in this triangular form. One could hypothesize that it came from Germany as many scholars at that time studied at German universities. And from where did it in such a case come to Germany? We do not know about this at the moment but it would be interesting to search for more information.

Appendix 3

Let us think that we want to multiply the two numbers a and b, both are between 5 and 10.

Following the instruction the number of fingers we hold up are (a-5)+(b-5)=a+b-10

This number we multiply by 10 to get 10(a+b-10)

The number of fingers we hold down are (10-a) and (10-b) and we do multiply these numbers to get (10-a)·(10-b)=100-10(a+b)+a·b

The sum of these two numbers is 10(a+b)-100+100-10(a+b)+a·b=a·b, which is the product we wanted to calculate.

Thus we have proven that for any numbers a and b between 5 and 10 the instruction gives the product a·b

Appendix 4

Some useful links about finger multiplication:

http://ncm.gu.se/media/namnaren/npn/arkiv_xtra/09_2/mattfolk.pdf

http://gwydir.demon.co.uk/jo/numbers/finger/multiply.htm

http://scimath.unl.edu/MIM/files/MATExamFiles/WestLynn_Final_070411_LA.pdf

http://threesixty360.wordpress.com/2007/12/31/three-finger-tricks-for-multiplying/

http://www.dccc.edu/sites/default/files/faculty/sid_kolpas/mathteacherfingers.pdf