

Multiplication Table Patterns Solutions

1. Sample table:

Starting number	Sum of the four numbers
8 (from 2×4)	$4 + 6 + 10 + 12 = 32$
9 (from 3×3)	$6 + 6 + 12 + 12 = 36$
18 (from 6×3)	$12 + 15 + 21 + 24 = 72$
40 (from 5×8)	$32 + 35 + 45 + 48 = 160$
54 (from 9×6)	$45 + 48 + 60 + 63 = 216$

2. The sum of the four numbers is 4 times greater than the starting number (or what you would get by adding the starting number four times). Students may notice other patterns as well.

3. Sample table:

Starting number	Sum of the four numbers
4 (from 2×2)	$2 + 2 + 6 + 6 = 16$
21 (from 3×7)	$14 + 18 + 24 + 28 = 84$
10 (from 5×2)	$5 + 8 + 12 + 15 = 40$

Yes, the patterns are still there!

$$\begin{array}{lll}
 4 + 4 + 4 + 4 = 16 & (\text{or } 4 \times 4 = 16) & 2 + 2 + 6 + 6 = 16 \\
 21 + 21 + 21 + 21 = 84 & (\text{or } 21 \times 4 = 84) & 14 + 18 + 24 + 28 = 84 \\
 10 + 10 + 10 + 10 = 40 & (\text{or } 10 \times 4 = 40) & 5 + 8 + 12 + 15 = 40
 \end{array}$$

4. You can make all four of the surrounding numbers equal to the number in the middle by *compensating*. For example: Choose the starting number 10 from row five, column two. Subtract 5 from 15 and add it to the 5. They will both be 10. Then subtract 2 from 12 and add it to the 8. They will also become 10. Now you have four 10s added together!

5. Sample recording table:

4 (from 2 x 2)	$4 + 4 + 4 + 4 = 16$	$1 + 3 + 3 + 9 = 16$
10 (from 2 x 5)	$10 + 10 + 10 + 10 = 40$	$4 + 6 + 12 + 18 = 40$
9 (from 3 x 3)	$9 + 9 + 9 + 9 = 36$	$4 + 8 + 8 + 16 = 36$

Sample observations:

- The sum is still four times greater than the middle number!
- The sum of the upper-right (UR) and lower-left (LL) numbers is 2 *less* than double the middle number. In symbols:

$$UR + LL = M + M - 2$$

- The sum of the upper-left (UL) and lower-right (LR) numbers is 2 *greater* than double the middle number. In symbols:

$$UL + LR = M + M + 2$$

- This explains the first observation.

$$\begin{aligned} (UR + LL) + (UL + LR) &= (M + M - 2) + (M + M + 2) = \\ M + M + M + M + 2 - 2 &= \\ M + M + M + M &= \\ 4 \times M & \end{aligned}$$

Sample questions:

- Why is $UR + LL$ always two less than double the middle number?
- Why is $UL + LR$ always two greater than double the middle number?

6. Sample prediction:

The sum of the eight surrounding numbers is 8 times greater than the starting number, because the corners are 4 times greater, and the other four numbers are also 4 times greater. (It will be as if you have added eight of the middle number.)

7. Testing the prediction:

Starting Number	Sum of the eight surrounding numbers
6 (from 2 x 3)	$2 + 3 + 4 + 8 + 12 + 9 + 6 + 4 = 48$
8 (from 2 x 4)	$3 + 6 + 9 + 12 + 15 + 10 + 5 + 4 = 64$
20 (from 4 x 5)	$12 + 15 + 18 + 24 + 30 + 25 + 20 + 16 = 160$
21 (from 7 x 3)	$12 + 14 + 16 + 24 + 32 + 28 + 24 + 18 = 168$
45 (from 9 x 5)	$32 + 36 + 40 + 50 + 60 + 54 + 48 + 40 = 360$

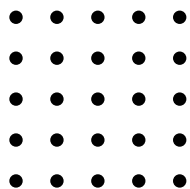
Each of the sums is 8 times greater than the starting number.

Part 2

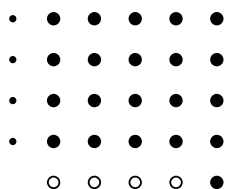
8. Sample observations:

- They grey numbers are always equal to a number times itself. (They are *square* numbers.)
- The grey numbers increase by odd amounts starting at 3 (3, 5, 7, etc.).
- The yellow numbers increase the same way, but starting at 5.
- The diagonal numbers are always 1 less than the square number.

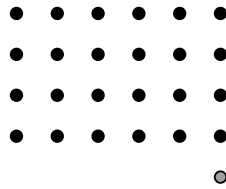
9. A 5 by 5 array:



Move four dots (open dots on the bottom) over to one side (small dots on the left).

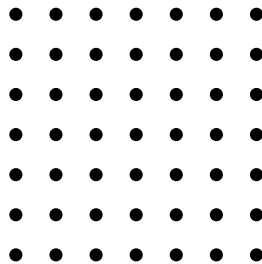


Now the picture looks like this:

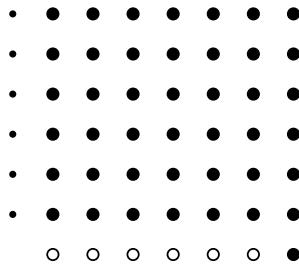


It is a 4 by 6 array with an extra dot (shown in grey). This shows that 4×6 is one less than 5×5 . (Ensure students understand that in 4×6 , one factor is 1 less than 5 and the other factor is 1 greater than 5.)

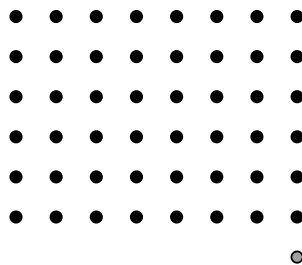
10. Sample response; a 7 by 7 array:



Move six dots from one side (bottom) to another (left):



The final picture:



The 7 by 7 array had 49 dots, so this picture also has 49 dots. Again, there is an extra dot that doesn't belong to the 6 by 8 array, so it has 48 dots.

11. Sample strategy:

Start by finding the value of 20×20 . (For example, skip count by 20s twenty times to get 400. Or count by 20s ten times to get to 200, and then double this to get 400.) Since a 20×20 array has 400 dots, a 19 by 21 array has one less, or 399 dots. Therefore, $19 \times 21 = 399$.

For their own example, some students may be able to reverse the thinking process. For instance: To find the value of 19×19 , begin by calculating 20×18 mentally (perhaps count by 20 eighteen times). Since $20 \times 18 = 360$, a 19 by 19 square array has 361 dots. Therefore, $19 \times 19 = 361$.