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Characterization of Canonical Semi Symmetric Semi- Metric Connection on Transversal Hypersurfaces Oflorentzian Para Sasakian Manifolds

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***Abstract*-- The present paper deals with transversal hypersurface of Lorentzian para-Sasakian manifold endowed with a canonical semi symmetric semi metric connection. It is shown that transversal hypersurfaces of Lorentzian para-Sasakian manifold with a canonical semi symmetric semi metric connection admits an almost product structure and each transversal hypersurfaces of Lorentzian para-Sasakian manifold with a canonical semi symmetric semi metric connection admits an almost product semi-Riemannian structure. The fundamental 2-form on the transversal hypersurfaces of Lorentzian para-Sasakian manifold with $(\mathbf{f}, \mathbf{g}, \mathbf{u}, \mathbf{v}, \boldsymbol{\lambda})$ -structure are closed. It is also proved that transversal hypersurfaces of Lorentzian para-Sasakian manifold with a canonical semi symmetric semi metric connection admits a product structure. Some properties of transversal hypersurfaces of Lorentzian para-Sasakian manifold are closed.**

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I. Introduction

Matsumoto [8] introduced the notion of Lorentzian para-Sasakian (*LP*-Sasakianfor short) manifold. Mihai and Rosca defined the same notion independently in [9]. This type of manifold is also discussed in [10,11].Almost contact metric manifold with an almost contact metric structureis very well explained by Blair [4]. In [11], S. Tanno gave a classificationfor connected almost contact metric manifolds whose automorphism groupshave the maximum dimension. For such a manifold, the sectional curvatureof plane sections containing ξ is a constant, say c. He showed that they canbe divided into three classes: (1) Homogenous normal contact Riemannianmanifolds with $c > 0$, (2) global Riemannian products of a line or a circle witha Kaehler manifold of constant holomorphic sectional curvature if $c = 0$ and(3) a warped product space $R \times f^{C^n}$ if $c < 0$.

It is known that the manifoldsof class (1) are characterized by some tensorial relations admitting a contactstructure. Kenmotsu [6] characterized the differential geometric propertiesof the third case by tensor equation$(\overline{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$. Thestructure so obtained is now known as Kenmotsu structure. In general, thesestructures are not Sasakian [6].On the other hand (f, g, u, v, λ) structure on a manifold was introduced by Yano and Okumura [7]. Transversal hypersurfaces is a hypersurface which never contain the vector field ξ defining the almost contact structure. It is well known that on a transversal hypersurface of almost contact metric manifold there exist a (f, g, u, v, λ) structure.

Let ∇^* be a linear connection in an n-dimensional differentiable manifold$M$. The torsion tensor $T$of$\nabla^*$is given by

$$T(X,Y) = \nabla^*_X Y - \nabla^*_Y X - [X,Y]$$

For all vector fields X and Y on M and is of type $(1,2)$.The connection ∇^* is symmetric if its torsion tensor T vanishes, otherwiseit is non-symmetric. The connection $\nabla^*$is metric if there is a Riemannianmetric g in M such that $\nabla^* g = 0$, otherwise it is non-metric. In ([1], [5]), A. Friedmann and J. A. Schouten introduced the idea of a semi-symmetric linear connection in a differentiable manifold. A connection is said to be semisymmetricif its torsion tensor T is of the form

$$T(X,Y) = u(Y)X - u(X)Y$$

Where$u$ is a 1-form. In ([2], [3]), some properties of semi-symmetric semi-metricconnections were studied.

The paper is organized as follows: In section 2, we give a brief introduction to Lorentzian para-Sasakian manifolds. In Section 3, It is proved that transversal hypersurfaces of Lorentzian para-Sasakian manifold with a semi symmetric non metric connection admits an almost product structure and each transversal hypersurfaces of Lorentzian para-Sasakian manifold with a semi symmetric non metric connection admits an almost product semi-Riemannian structure.

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The fundamental 2-form on the transversal hypersurfaces of Lorentzian para-Sasakian manifold with Lorentzian para-contact structure(f, g, u, v, λ) structure are closed. It is also proved that transversal hypersurfaces of Lorentzian para-Sasakian manifold with a semi symmetric non metric connection admits a product structure. Some properties of transversal hypersurfaces of Lorentzian para-Sasakian manifold are closed.

II. Lorentzian Para-Sasakian Manifolds

Let $\bar{M}$ be a $2n+1$ dimensional almost contact metric manifold with a metric tensor g, a tensor field $\phi$of type (1, 1), a vector field $\xi$and a 1-form η which satisfy

(2.1) $\phi^2 = I + \eta\otimes\xi, \eta(\xi) = -1,$

(2.2) $g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y),$

(2.3) $g(X, \xi) = \eta(X)$

(2.4)$g(\phi X, Y) = g(X, \phi Y)$

For vector fieldsX, Ytangent to $\bar{M}$. Such a manifold is termed as Lorentzian para-contact manifold and the structure(ϕ, ξ, η, g)a Lorentzianpara-contact structure.

Also in a Lorentzianpara-contact structure the following relations hold:

$\phi\xi = 0, \quad \eta(\phi X) = 0$, and $rank\ (\phi) = n - 1$

A Lorentzianpara-contact manifold $\bar{M}$ is called Lorentzian para-Sasakian (LP-Sasakian) manifold if

(2.5) $(\bar{\bar{\nabla}}_X\phi)Y = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi,$

(2.6) $\bar{\bar{\nabla}}_X\xi = \phi X,$

For all vector fields X, Y tangentto $\bar{M}$ where $\bar{\bar{\nabla}}$ is the Riemannian connection with respect to g.

We remark that owing to the existence of the $1-$form η, we can definethe canonical semi-symmetric semi-metric connection $\bar{\nabla}$ in any almost contact metric manifold $(\bar{M}, \phi, \xi, \eta, g)$ by [10]

(2.7) $\bar{\nabla}_X Y = \bar{\bar{\nabla}}_X Y - \eta(X)Y + g(X, Y)\xi$

such that $\bar{\nabla}_X g = 2\eta(X)g(Y, Z) - \eta(Y)g(X, Z) - \eta(Z)g(X, Y)$ for any $X, Y, Z \in TM$.

In particular, if $\bar{M}$is an LP-Sasakian manifold then from (2.5), (2.6) and (2.7) we have

(2.8)$(\bar{\nabla}_X\phi)Y = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi + g(X, \phi Y)\xi,$

(2.9)$\bar{\nabla}_X\xi = \phi X.$

III. Main Results

Let M be a hypersurface of Lorentzian para-Sasakian manifold $\bar{M}$ equipped with aLorentzian para-contact structure(ϕ, ξ, η). We assumethat the structure vector field ξ never belongs to tangent space of the hypersurface$M$, such that a hypersurface is called a transversal hypersurface of a Lorentzian para-Sasakian manifold.

In this case the structure vector field ξ can betaken as an affine normal to the hypersurface. Vector field X on M and ξ arelinearly independent, therefore we may write

(3.1)$\phi X = F(X) + \omega(X)\xi$

where$F$ is a (1, 1) tensor field and ω is a 1-form on M.

From (3.1)

$$\phi\xi = F\xi + \omega(\xi)\xi$$

or,$0 = F\xi + \omega(\xi)\xi$

(3.2)$\phi^2 X = F(\phi X) + \omega(\phi X)\xi$

$$X + \eta(X)\xi = F(FX + \omega(X)\xi) + \omega(FX + \omega(X)\xi)\xi$$

(3.3) $X + \eta(X)\xi = F^2 X + (\omega o F)(X)\xi$

Taking account of equation (3.3), we get

(3.4)$F^2 X = X$

(3.5)$F^2 = I$

$$\eta = \omega o F$$

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Thus we have

Theorem 3.1. Each transversal hypersurface of an almost Lorentzian para-Sasakian manifoldendowed with a canonical semi symmetric semi metric connectionadmits an almost product structure and a 1-form ω.

From (3.4) and (3.5), it follows that

$$\eta = \omega o F$$
$$\eta(FX) = (\omega o F)FX$$
$$\eta(FX) = \omega(F^2X)$$
$$(\omega o F)X = \omega(X)$$

(3.6) $\omega = \eta o F$

Now, we assume that $\bar{M}$ admits aLorentzian para-contact structure(ϕ, ξ, η, g). We denote by g the induced metric on M also. Then for all$X, Y \epsilon TM$, we obtain

(3.7) $g(FX, FY) = g(X, Y) + \eta(X)\eta(Y) - \omega(X)\omega(Y)$

We define a new metric G on the transversal hypersurface given by

(3.8)$G(X, Y) = g(\phi X, \phi Y) = -g(X, Y) - \eta(X)\eta(Y)$

So,

$$G(FX, FY) = g(FX, FY) + \eta(FX)\eta(FY)$$
$$= g(X, Y) + \eta(X)\eta(Y) - \omega(X)\omega(Y) + (\eta o F)(X)(\eta o F)(Y)$$
$$= g(X, Y) + \eta(X)\eta(Y) - \omega(X)\omega(Y) + \omega(X)\omega(Y)$$
$$= g(X, Y) + \eta(X)\eta(Y) = G(X, Y)$$

Then, we get

(3.9) $G(FX, FY) = G(X, Y)$

Where equation (3.4), (3.6), (3.7) and (3.8) are used.

Then G is semi Riemannian metric on M that is (F, G) is an almostproduct semi-Riemannian structure on the transversal hypersurface $M$of $\bar{M}$.

Thus, we are able to state the following.

Theorem 3.2. Each transversal hypersurface of an almost Lorentzian para-Sasakian manifoldendowed with a canonical semi symmetric semi metric connectionadmits an almost product semi-Riemannian structure. We nowassume that M is orientable and choose a unit vector field $N$of $\bar{M}$, normal to M. Then Gauss and Weingarten formulaeof canonical semi symmetric semi metric connectionare given respectively by

(3.10)$\bar{\nabla}_X Y = \nabla_X Y + [h(X, Y) + g(X, Y)\lambda]N, \ (X, \ Y \epsilon TM)$

(3.11) $\bar{\nabla}_X N = -HX - \eta(X)N$

Where$\bar{\nabla}$ and ∇ are respectively the Levi-civita and induced Levi-civita connections in $\bar{M}$, M and h is the second fundamental form related to H by

(3.12)$h(X, Y) = g(HX, Y)$,

for any vector field X tangent to M, defining

(3.13) $\phi X = fX + u(X)N$,

(3.14) $\phi N = -U$,

(3.15)$\xi = V + \lambda N$,

$\eta(X) = v(X)$,

(3.16) $\lambda = \eta(N) = g(\xi, N)$,

for $X \epsilon TM$ we get an induced Lorentzian para-contact structure(f, g, u, v, λ) on thetransversal hypersurface such that

(3.17)$f^2 = I + u \otimes U + v \otimes V$

(3.18) $fU = -\lambda V$, $fV = \lambda U$

(3.19)$u o f = \lambda v$,$v o f = -\lambda U$

(3.20) $u(U) = -1 - \lambda^2$,$u(V) = v(U) = 0$, $v(V) = -1 - \lambda^2$

(3.21)$g(fX, fY) = -g(X, Y) - u(X)u(Y) - v(X)v(Y)$

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(3.22)$g(X,fY) = -g(fX,Y)$,$g(X,U) = u(X)$, $g(X,V) = v(X)$,

for all for $X,Y \epsilon TM$ where

(3.23)$\lambda = \eta(N)$

Thus, we see that every transversal hypersurface of analmost Lorentzian para-contact metric manifold endowed with a canonical semi symmetric semi metric connectionalso admits a Lorentzian para-contact structure (f,g,u,v,λ). Nextwe find relation between the induced almost product structure (F,G) and theinduced Lorentzian para-contact structure (f,g,u,v,λ) on the transversal hypersurface ofan almostLorentzian para-Sasakian metric manifoldendowed with a canonical semi symmetric semi metric connection. In fact, we have the following

Theorem 3.3. Let M be a transversal hypersurface of analmost Lorentzian para-contact metric manifold$\bar{M}$endowed with a canonical semi symmetric semi metric connectionequipped with Lorentzian para-contact metric structure (ϕ,ξ,η,g) and induced almost product structure(F,G).

Then we have

(3.24)$\lambda\omega = u$,

(3.25) $F = f - \frac{1}{\lambda}u \otimes V$,

(3.26) $FU = \frac{1}{\lambda}V$,

(3.27)$uoF = uof = \lambda v$,

(3.28)$FV = fV = \lambda U$,

(3.29) $uoF = \frac{1}{\lambda}u$.

Proof.

$$\phi X = FX + \omega(X)\xi$$
$$\xi = V + \lambda N$$

(3.30)$\phi X = FX + \omega(X)V + \lambda\omega(X)N$,

(3.31)$\phi X = fX + u(X)N$.

From equation (3.30) and (3.31) we have

$\lambda\omega X = u(X)$, $\omega(X) = \frac{1}{\lambda}u(X)$,

$FX = fX - \omega(X)V$,

$$FX = fX - \frac{1}{\lambda}u(X)V$$
$$F = f - \frac{1}{\lambda}u \otimes V$$

which is equation (3.26).

$(uoF)(X) = (uof) - \frac{1}{\lambda}u(X)u(V)$, $u(V) = 0$,

$uoF = uof = \lambda v$,

which is equation (3.27).

$FU = fV - \frac{1}{\lambda}u(v)V,$

$FU = -\lambda V - \frac{1}{\lambda}(-1-\lambda^2)V = \frac{1}{\lambda}V,$

$FU = \frac{1}{\lambda}V,$

which is equation (3.26).

$$(uoF)(X) = (uof)(X) - \frac{1}{\lambda}u(X)u(V)$$

$$= (uof)(X) - \frac{1}{\lambda}u(X)(-1-\lambda^2)$$

$$= -\lambda u(X) + \frac{1}{\lambda}u(X) + \lambda u(X)$$

$= \frac{1}{\lambda}u(X),$

$$uoF = \frac{1}{\lambda}u$$

$FV = fV - \frac{1}{\lambda}u(V)V = fV = \lambda U,$

Which is equation (3.28) here equations (3.18), (3.19), (3.20), (3.21), (3.22), (3.23) are used.

Lemma 3.4. Let M be a transversal hypersurface with Lorentzian para-contact structure (f, g, u, v, λ) of an almostLorentzian para-Sasakian metric manifold $\overline{M}$ endowed with a canonical semi symmetric semi metric connection. Then

(3.32) $(\overline{\nabla}_X\phi)Y = \big((\nabla_X f)Y - u(Y)HX + h(X,Y)U + g(X,Y)\lambda U\big)$

$$+\big((\nabla_X u)Y + h(X, fY) - \eta(X)u(Y) + g(X, fY)\lambda\big)N$$

(3.33) $\overline{\nabla}_X\xi = \nabla_X V - \lambda HX + (h(X,V) + Y(\lambda))N$

(3.34) $(\overline{\nabla}_X\phi)N = -\nabla_X U - \eta(X)U + fHX - [h(X,U) + g(X,U)\lambda - \mu(HX)]N$

(3.35) $(\overline{\nabla}_X\eta)Y = (\nabla_X v)Y + h(X,v)NY + g(X,v)\lambda NY$

for all $X, Y \epsilon TM$. The proof is straight forward and hence omitted.

Theorem 3.5. Let M be a transversal hypersurfaces with Lorentzian para-contact structure (f, g, u, v, λ) of a Lorentzian para-Sasakian manifold $\overline{M}$ endowed with a canonical semi symmetric semi metric connection. Then

(3.36) $(\nabla_X f)Y = u(Y)HX - [h(X,Y) + g(X,Y)\lambda]U,$

(3.37) $(\nabla_X u)Y = -h(X, fY) + u(X)u(Y) - g(X, fY)\lambda,$

(3.38) $\nabla_X V = \lambda HX,$

(3.39) $h(X,V) = -Y(\lambda)$

(3.40) $\nabla_X U = fHX - \eta(X)U$ and $h(X,U) = u(HX) - g(X,U)\lambda$

(3.41) $(\nabla_X v)Y = 0$ and $h(X,v)Y = -g(X,v)\lambda Y$

for all $X, Y \epsilon TM$.

Proof. Using (2.8), (3.12), (3.15) in (3.31), we obtain

$$\big((\nabla_X f)Y - u(Y)HX + h(X,Y)U + g(X,Y)\lambda U\big)$$

$$+\big((\nabla_X u)Y + h(X,fY) - \eta(X)u(Y) + g(X,fY)\lambda\big)N = 0$$

Equating tangential and normal parts in the above equation, we get (3.36) and(3.37) respectively. Using (2.9) and (3.15) in (3.33), we have

$$\nabla_X V - \lambda HX + (h(X,V) + Y(\lambda))N = 0$$

Equating tangential and normal parts we get (3.38) and (3.39) respectively. Using (2.8), (3.14) and (3.15) in (3.34)

Using (2.8), (3.14) and (3.15) in (3.33) and equating tangential, we get (3.40). Inthe last (3.41) follows from (3.35).

Theorem 3.6. If M be a transversal hypersurface with Lorentzian para-contact structure (f,g,u,v,λ) of a Lorentzian para-Sasakian manifold endowed with a canonical semi symmetric semi metric connection, then the 2-formΦon M isgiven by

$$\Phi(X,Y) = g(X,fY)$$

is closed.

Proof. From (3.36) we get

$(\nabla_X\Phi)(Y,Z) = h(X,Y)u(Z) - h(X,Z)u(Y),$

$(\nabla_X\Phi)(Y,Z) + (\nabla_Y\Phi)(Z,X) + (\nabla_Z\Phi)(X,Y) = 0.$

Hence the theorem is proved.

Theorem 3.7. IfM is a transversal hypersurface with almost product semi Riemannian structure (F,G) of a Lorentzian para-Sasakian manifoldendowed with a canonical semi symmetric semi metric connection. Then the2-form Ω on M is given by

$$\Omega(X,Y) = G(X,fY)$$

is closed.

Using (3.36), we calculate the Nijenhuis tensor

$$[F,F] = (\nabla_{FX}F)Y - (\nabla_{FY}F)X - F(\nabla_X F)Y + F(\nabla_Y F)X$$

and find that $[F,F] = 0$.

Therefore, in view of theorem (3.6), we have

Theorem 3.8. Every transversal hypersurface of a Lorentzian para-Sasakian manifoldendowed with a canonical semi symmetric semi metric connection, admits product structure.

Theorem 3.9. Let M be a transversal hypersurface with Lorentzian para-contact structure (f,g,u,v,λ) of a Lorentzian para-Sasakian manifold $\overline{M}$ endowed with a canonical semi symmetric semi metric connection. Then

(3.42) $(\nabla_X f)Y = g(X,Y)V + \eta(Y)X + 2\eta(X)\eta(Y)V$

$$+g(X,fY)V + u(Y)HX - [h(X,Y) + g(X,Y)\lambda]U$$

(3.43)$(\nabla_X u)Y = g(X,Y)\lambda + 2\eta(X)\eta(Y)\lambda + g(X,fY)\lambda$

$$-h(X,fY) + u(X)u(Y) - g(X,fY)\lambda$$

(3.44)$\nabla_X V = \lambda HX + fX$

(3.45)$h(X,V) = u(X) - Y(\lambda)$

(3.46)$\nabla_X U = -\lambda X - 2\eta(X)\lambda V + g(X,U)V - \eta(X)U + fHX$

(3.47)$h(X,U) = -2\eta(X)\lambda^2 + u(HX)$

for all $X,Y \epsilon TM$.

Proof. Using (2.8), (3.13), (3.15) in (3.32), we obtain

$$g(X,Y)V + g(X,Y)\lambda N + \eta(Y)X + 2\eta(X)\eta(Y)V + 2\eta(X)\eta(Y)\lambda N + g(X,\phi Y)\xi$$

$$= \big((\nabla_X f)Y - u(Y)HX + h(X,Y)U + g(X,Y)\lambda U\big)$$

$$+\big((\nabla_X u)Y + h(X,fY) - \eta(X)u(Y) + g(X,fY)\lambda\big)N$$

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Equating tangential and normal parts in the above equation, we get (3.42) and (3.43) respectively. Using (2.9) and (3.15) in (3.32), we have

$$\nabla_X V - \lambda HX + (h(X,V) + Y(\lambda))N = fX + u(X)N$$

Equating tangential and normal parts we get (3.44) and (3.45) respectively.

Using (2.8), (3.14) and (3.15) in (3.33) and equating tangential parts, we get (3.46) in the last (3.47) follows from (3.34).

Theorem 3.10. If M be a transversal hypersurface with Lorentzian para-contact structure (f, g, u, v, λ) of aLorentzian para-Sasakian manifold endowed with a canonical semi symmetric semi metric connection, then the 2-form Φ on$M$ is given by

$$\Phi(X,Y) = g(X, fY)$$

is closed.

Proof. From (3.42) we get

$$(\nabla_X \Phi)(Y,Z) = -g(fX,Y)v(Z) = g(fX,Z)v(Y) = h(X < Y)u(Z) - h(X,Z)u(Y)$$

which gives

$$(\nabla_X \Phi)(Y,Z) + (\nabla_Y \Phi)(Z,X) + (\nabla_Z \Phi)(X,Y) = 0$$

that is
$d\Phi = 0.$
Hence the theorem is proved.

Theorem 3.11. If M is a transversal hypersurface with almost product semi Riemannian structure (F, G) of a Lorentzian para-Sasakian manifold endowed with a canonical semi symmetric semi metric connection. Then 2-form Ω on M is given by

$$\Omega(X,Y) = G(X, FY)$$

is closed.

Using (3.42), we calculate the Nijenhuis tensor

$$[F,F] = (\nabla_{FX}\mathrm{F})Y - (\nabla_{FY}\mathrm{F})X - F(\nabla_X \mathrm{F})Y + F(\nabla_Y \mathrm{F})X$$

and find that $[F,F] = 0$.
Therefore, in view of theorem 3.10, we have

Theorem 4.4. Every transversal hypersurface of a Lorentzian para-Sasakian manifoldendowed with a canonical semi symmetric semi metric connectionadmits a product structure.

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