Analysis of the temporal variability of runoff in Ivory Coast: statistical approach and phenomena characterization

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Abstract The runoffs at four Ivory Coast hydrometric stations (monitoring flows from an area covering between 5930 and 66500 km^2) were analysed with a set of statistical methods for the detection of breaks in the time series. The variables studied were the annual mean discharge and some characteristic discharges. From a quantitative standpoint, the existence of a clear break in the series of annual mean discharges at the beginning of the decade from 1970, the date from which the runoffs decrease significantly, was noted. A more qualitative study of the results showed that low flows were more affected than high flows by this variability of the regime. This fluctuation appears to be in accord with the drought phenomena observed during the same period in the Sahel, to the north of Ivory Coast.

Analyse de la variabilité temporelle des écoulements en Côte d'Ivoire: approche statistique et caractérisation des phénomènes

Résumé Les écoulements de quatre stations hydrométriques de Côte d'Ivoire (superficies comprise entres 5930 et 66500 km^2) ont été analysés à l'aide d'un ensemble de méthodes statistiques de détection de ruptures dans les séries chronologiques. Les variables étudiées sont les modules et différents types de débits caractéristiques. D'un point de vue quantitatif, on note l'existence d'une rupture nette dans les séries de modules au début de la décennie 1970, date à partir de laquelle les écoulements diminuent sensiblement. Un examen plus qualitatif des résultats montre que les basses eaux sont plus affectées que les hautes eaux par cette variabilité du régime. Cette fluctuation apparaît comme étant en phase avec les phénomènes de sécheresse observés à la même époque plus au nord, au Sahel.

INTRODUCTION

In the Sahelian part of West Africa, a significant decrease in the runoff of streams has generally been observed since the end of the 1960s (Sircoulon, 1987;

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Olivry, 1987). The consequences of such a decrease in surface water resources could be extremely damaging, from the environmental standpoint as well as from the standpoint of the human activities that are directly or indirectly linked to the utilization of such resources. Mention can be made, for example, of the failures recorded in daily production figures for existing water utilities, or even for those proposed, during the most humid periods. This diminishing trend of runoff also appears to affect the regions located to the south of the Sahel, and, more particularly, the countries that border the Gulf of Guinea. The consequences are generally less severe here, which explains the lesser interest paid up to now to the study of this phenomenon in those regions considered as humid. The importance of water resources for the countries of that zone is, however, clear. In Ivory Coast, for example, industry is supplied with some of 60% of its electricity generated by hydropower (Servat & Sakho, 1995).

On the basis of such observations, the present study analyses the behaviour of Ivorian streams (Aka, 1994). The methodology employed is aimed at detecting and characterizing the possible changes in some features of the runoff. In this regard, the correlation test on rank (Kendall & Stuart, 1943; WMO, 1966) is used. The alternative hypothesis is the existence of a trend. A set of statistical methods is also used in order to identify any peculiarities of these series such as the existence of breaks in the series which is supposed to be likely.

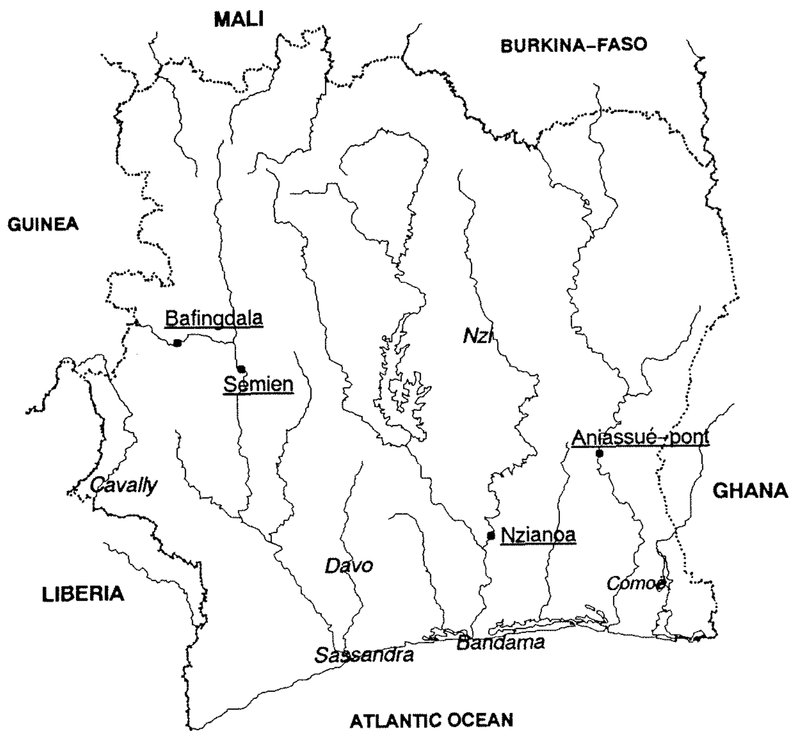

Fig. 1 Hydrographic network of Ivory Coast and stations studied (underlined).

METHODOLOGY AND DATA USED

Description of the study area

Ivory Coast is located along the Gulf of Guinea, bordering the Atlantic Ocean, and south of Sahelian Africa (Fig. 1). Its hydroclimatic characteristics vary progressively along a north-south axis. The climate, which is influenced by the Monsoon and the Harmattan, ranges from a transitional equatorial climate in the south of the country (with two rainy seasons) to a transitional tropical climate in the north (with one rainy season). In terms of vegetation, a dense forest in the southern part of the country replaces the savannah in the north. Rainfall increases from the north to the south and considerably influences the regimes of streams.

The hydrographical network (Fig. 1) is distributed into four main basins (Cavally, Sassandra, Bandama and Comoé). These major rivers flow from the north to the south. They are consequently subjected to the influence of several pluviometric regimes, which renders the determination of their regimes difficult.

Presentation of river basins

The study was made on three major Ivorian river basins: the Sassandra, the Bandama and the Comoé. The approach for the choice of the stations selected was based on the period of time covered by the available observations and by the quality of the data. Four stations were finally selected (Fig. 1). Table 1 provides some characteristics of the basins studied.

Table 1 Characteristics of the catchments studied

Station	Stream	Area (km^2)	Observing period	Main catchment basin
Bafingdala	Bafing	5930	1962-1991	Sassandra
Semien	Sassandra	29300	1955-1985	Sassandra
Nzianoa	Nzi	35000	1954-1985	Bandama
Aniassué-pont	Comoé	66500	1954-1985	Comoé

"Break" detection methods

A "break" can be defined as a change in the law of probability of a time series at a given point in time (Lubès *et al.*, 1994).

The choice of the methods selected was based on the soundness of their foundations and on the conclusions of a simulation study conducted on artificially disturbed random series (Bonneaud, 1994). They enable the detection of a change in the mean of the variable treated in the series. All methods imply that the variance is not affected by the break. These methods are not adapted to the search for several breaks in a given series.

Pettitt test (Pettitt, 1979) To describe the Pettitt test, large parts of Pettitt's paper itself will be quoted. Pettitt considered a sequence of independent random variables $X_1, X_2, \ldots, X_N$. "The sequence is said to have a change point at τ if X_t for $t = 1, \ldots, \tau$ have a common distribution function, $F_1(X)$, and X_t for $t = \tau+1, \ldots, N$ have a common distribution, $F_2(X)$, and $F_1(X) \neq F_2(X)$. We consider the problem of testing the null hypothesis of 'no-change', H: $\tau = N$ against the alternative of 'change', A: $1 \leq \tau < N$, using a non-parametric statistic. We make no assumptions about the functional forms of F_1 and F_2 except that they are continuous". Pettitt describes how "a version of the Mann-Whitney statistic can be utilized for the problem in question and derives approximate significance probabilities for testing 'no change' against 'change'".

"Let $D_{ij} = \mathrm{sgn}(X_i - X_j)$ where $\mathrm{sgn}(X) = 1$ if $X > 0$; 0 if $X = 0$ and -1 if $X < 0$, then consider

$$U_{t,N} = \sum_{i=1}^{t} \sum_{j=t+1}^{N} D_{ij} \tag{1}$$

The statistic $U_{t,N}$ is equivalent to a Mann-Whitney statistic for testing that the two samples $X_1, \ldots, X_t$ and $X_{t+1}, \ldots, X_N$ come from the same population. The statistic $U_{t,N}$ is then considered for values of t with $1 \leq t < N$". Pettitt proposed, "for the test of H: 'no change' against A: 'change', the use of the statistic:

$$K_N = \max_{1 \leq t < N} |U_{t,N}| \tag{2}$$

Using the theory of ranks, Pettitt gave "the significance probability associated with the value k of K_N approximately by:"

$$\mathrm{Prob}(K_N > k) \approx 2\exp(-6k^2/(N^3 + N^2)) \tag{3}$$

Given a risk, α, of first kind, the null hypothesis is rejected, if the estimated probability of exceeding the value k is inferior to α. If so, the series includes a break that takes place at the time τ when K_N is observed.

Lee and Heghinian Bayesian method (Lee & Heghinian, 1977; Bruneau & Rassam, 1983). The Lee & Heghinian Bayesian method proposes a parametric approach. It requires a normal distribution of the values of the series.

The basic model of the procedure is as follows:

$$X_i = \begin{cases} \mu + \epsilon_i & i = 1, \ldots, \tau \\ \mu + \delta + \epsilon_i & i = \tau + 1, \ldots, N \end{cases} \tag{4}$$

where the ϵ_i are independent and normally distributed, of zero mean and variance σ^2. τ and δ represent respectively the position in time and the magnitude of a possible change of the mean.

The approach is based on the marginal posterior distributions of τ and δ (Lee & Heghinian, 1977). This study was limited to the posterior distribution of τ. When the distribution is unimodal, the date of the break is assessed by the mode of the posterior distribution of τ. This assessment is more accurate when the dispersion of the distribution is low.

The Buishand statistics (Buishand, 1982, 1984) Buishand's statistic comes from an original formulation given by Gardner (1969): "Gardner's statistic for a two-sided test on a shift in the mean at an unknown point can be written as:

$$G = \sum_{k=1}^{N-1} P_k \{S_k/\sigma_k\}^2 \text{ with } S_k = \sum_{i=1}^{N} \left(X_i - \bar{X}\right) \tag{5}$$

where P_k denotes the prior probability that the shift occurs just after the kth observation". This formulation supposes that the variance σ_X^2 is known. If it is unknown, it can be replaced by the sample standard deviation, D_x, and, if a uniform prior distribution is chosen for P_k, one obtains the statistic U defined by:

$$U = \frac{\sum_{k=1}^{N-1} (S_k/D_k)^2}{N(N+1)} \text{ with } D_X^2 = \sum_{i=1}^{N} (X_i - \bar{X})^2/N \tag{6}$$

Critical values of the statistic U were first given by Buishand (1982) from a Monte Carlo procedure. Improved critical values are given in Buishand (1984).

In addition to these different methods, the construction of a control ellipse allows an analysis of the homogeneity of the (X_i) series. The S_K variable, which is defined within the framework of the Buishand Statistic, follows a normal distribution of zero mean and variance $k(N - k)N^{-1}\ \sigma^2$, $k = 0, \ldots, N$ under the null hypothesis of the homogeneity of the X_i series. This control ellipse was used here only to estimate visually the importance of the deviations under the null hypothesis of the homogeneity of the series.

Before using these methods concerned with the homogeneity of the series, the correlation test on rank (Kendall & Stuart, 1943; WMO, 1966) was systematically used, enabling an evaluation of the random character of the series X_i. This test is based on the calculation of the number of pairs P for which $X_j > X_i$ ($j > i$ with $i = 1, \ldots, N - 1$). Under the null hypothesis, the ω variable defined as:

$$\omega = \frac{4P}{N(N-1)} - 1$$

follows a normal distribution with a zero mean and a variance equal to:

$$\frac{2(2N+5)}{9N(N-1)}$$

The alternative hypothesis of this test is a trend.

RESULTS AND DISCUSSION

The conditions for the use of the tests were checked before their application to the series studied.

Annual runoff

Developments in the annual mean discharge A first study was on autocorrelation in order to determine the linear dependence between the successive values of the series. This study was conducted with the Anderson test (Yevjevich, 1972). The use of this test at a 5% level did not show the existence of a persistent effect in the annual mean discharges series, unlike what had been previously described in the Sahelian regions (Hubert & Carbonnel, 1987).

Table 2 presents some of the features of the annual mean discharges for the four stations studied. The high coefficient of variation values show a strong dispersion of runoff. The lowest coefficients of variation in Table 2 are those of the two scarcely developed basins, part of the Sassandra River basin. It is furthermore noted that the year of occurrence of the minimum annual mean discharge was the same for the four stations studied, namely 1983, an extreme and generalized drought year in Ivory Coast. As for the maximum annual mean discharge, it was observed in 1968 for the stations studied except on the Sassandra River in Semien. Figures 2 and 3, which are for the Comoé in Aniassué-pont and the Nzi River in Nzianoa, are two examples which allow the visualization of the temporal trends of the annual mean discharge.

Table 2 Characteristics of the interannual mean discharge

Station	Interannual mean discharge ($m^3 s^{-1}$)	Coefficient of variation	Maximum of the interannual mean discharge ($m^3 s^{-1}$)	Year of occurrence of the maximum	Minimum of the interannual mean discharge ($m^3 s^{-1}$)	Year of occurrence of the minimum
Aniassué-pont	201.4	0.60	465.6	1968	17.2	1983
Nzianoa	74.9	0.75	261.2	1968	5.8	1983
Semien	220.7	0.29	366.3	1957	96.9	1983
Bafingdala	56.5	0.26	84.2	1968	29.2	1983

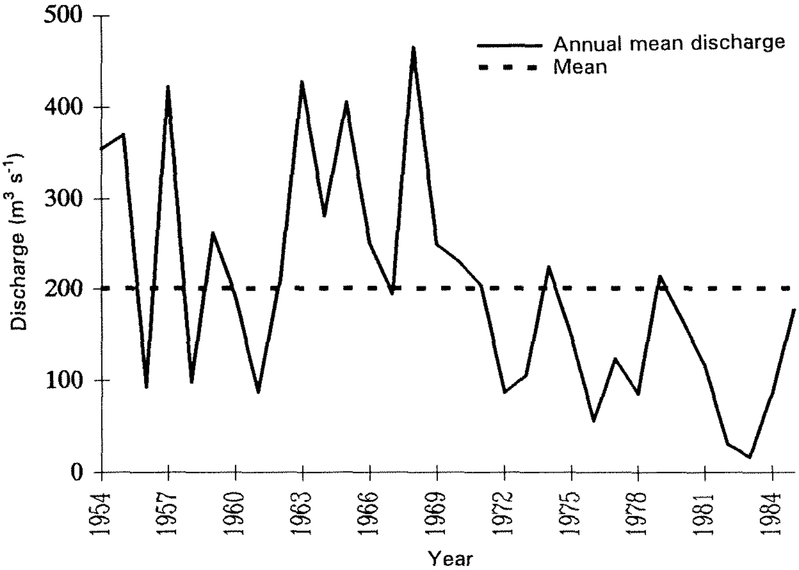

Fig. 2 Development of the annual mean discharges for the Comoé station at Aniassué-pont.

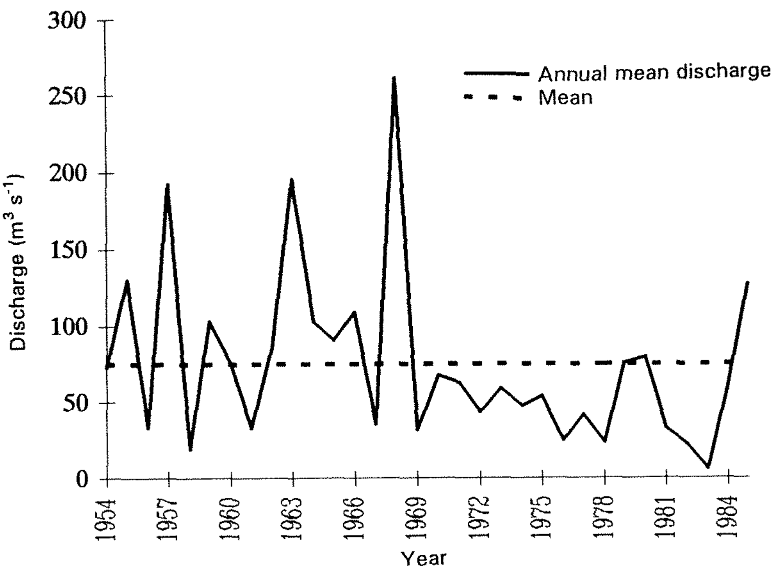

Fig. 3 Development of the annual mean discharges for the Nzi station at Nzianoa.

On the basis of these graphs, it appears that, after 1969, most of the annual mean discharges remained lower compared to the interannual mean discharge calculated for the whole observation period. During ten consecutive years (1969-1978), the annual discharges were lower than the average on the

Nzi River in Nzianoa. Similar observations can be made for the other stations after 1970.

The weight of the successive annual mean discharges observed in the estimation of the interannual mean discharge can be assessed on the basis of a simple cumulation. The simple C_k cumulation of the year k is defined as the mean of the annual discharge coefficients (which are expressed as $Q_i/\overline{Q}$) observed from the beginning of the observations up to the year k:

$$C_k = \frac{1}{k}\sum_{i=1}^{k}\frac{Q_i}{\overline{Q}} \quad (5)$$

where Q_i is the annual mean discharge observed in the year i and $\overline{Q}$ the interannual mean discharge.

It follows from equation (5) that a positive slope of the development in time of the simple cumulation between the years i and $i + 1$ implies that the annual discharge coefficient of the year $i + 1$ is higher than the mean of the annual discharge coefficient of the first i years. Conversely, a decrease means an annual discharge coefficient of the year $i + 1$ that is inferior to the mean of the first i annual discharge coefficients.

The graphical representation of the simple C_k cumulation of the annual discharge coefficient (Fig. 4) for the Nzi station at Nzianoa and the Comoé station at Aniassué-pont confirms the configuration of the annual mean discharge. Indeed, one notes a significantly regular decrease from 1968 onwards for these two stations. It can be deduced that the mean of the annual mean

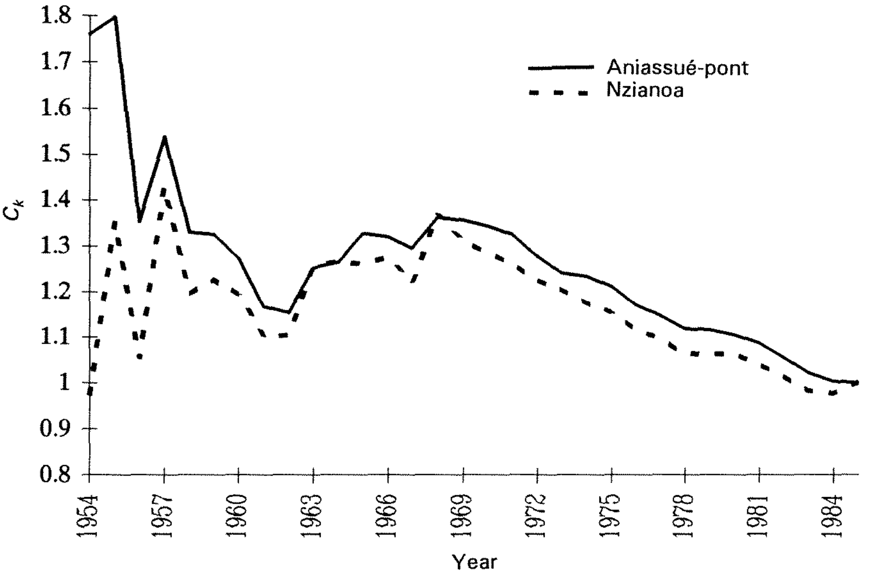

Fig. 4 Development of the simple cumulation of the annual discharge coefficient for the Nzi station at Nzianoa and the Comoé station at Aniassué-pont.

discharges observed since 1968 is lower than the mean of the same variable observed beforehand. The same conclusions can be drawn for the time series of the two other stations.

Detection of breaks A simple comparison test of the means of the annual mean discharges during the two sub-periods, as defined by the previous graphs, made it possible to reject the hypothesis of the equality of means at the 5% level.

Figure 5 illustrates the result of the Lee & Heghinian procedure for the Bafing station at Bafingdala. For the given station, all the break detection methods used here provided identical results not only for the existence of a break but also for its situation in time (Table 3). The breaks in the time series are clearly shown at the end of the sixties. The relative variations $(\bar{X}_2 - \bar{X}_1)/\bar{X}_1$ of means under the two sub-series 1 and 2, circumscribed on each side of the date of the break, were significant and negative (Table 3).

Table 3 Situation of the break and characteristics of the sub-series of annual mean discharges

	Comoé	Bafing	Nzi	Sassandra
Date of the break (year)	1971	1969	1968	1969
Sub-series 1. Mean ($m^3\ s^{-1}$)	266.8	72.1	102.6	259.0
Sub-series 2. Mean ($m^3\ s^{-1}$)	117.4	50.9	50.4	184.7
Relative variations of means (%)	−56	−29	−51	−29

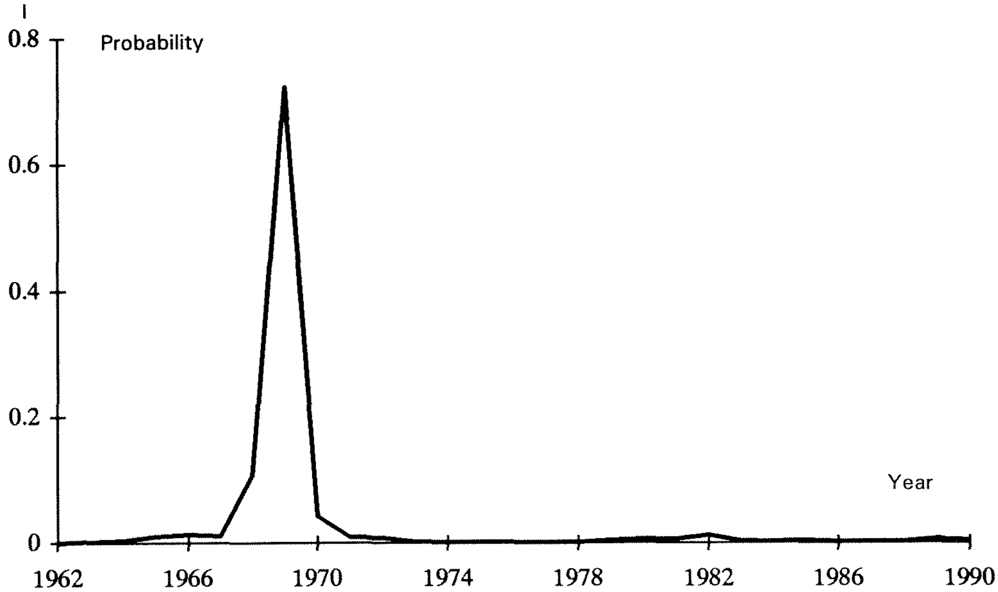

Fig. 5 Probability density of the situation in time of the break in the series of annual mean discharges for the Bafing station at Bafingdala.

Extreme values of mean discharge calculated for ten consecutive days

Developments The extreme values concerned are the maxima (VCX10) and the minima (VCN10) of the mean daily discharge assessed during ten consecutive days during the year. Table 4 presents a few characteristics of these variables for the stations treated. Two observations can be made. First the two basins (Bafing at Bafingdala and Sassandra at Semien) belonging to the Sassandra River basin, which are scarcely developed, have low coefficient of variation values for a given variable. Furthermore, the coefficients of variation estimated on the VCN10 series are the highest. The characteristics of the low flows consequently appear to be less stable compared to those of high flows.

The curve representing the temporal trends of these variables for the Sassandra River in Semien shows a downward trend (Fig. 6). It is quite clear as early as 1972 for the VCN10, and less obvious for the VCX10.

Table 4 Characteristics of the VCX10 and VCN10 series

	Aniassué-pont		Bafingdala		Nzianoa		Semien	
	VCX10	VCN10	VCX10	VCN10	VCX10	VCN10	VCX10	VCN10
Mean ($m^3 s^{-1}$)	1095	4.03	228	5.03	354	0.8	1003	19.05
Std. deviation ($m^3 s^{-1}$)	632	4.30	67.5	4.05	189	1.21	289	13.6
Coeff. of variation	0.58	1.07	0.29	0.81	0.53	1.51	0.29	0.71

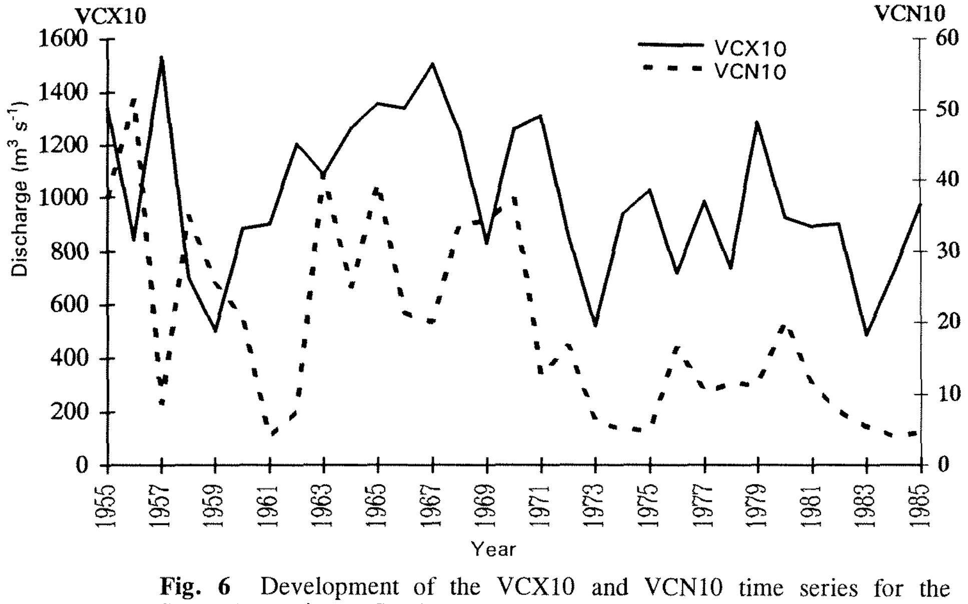

Fig. 6 Development of the VCX10 and VCN10 time series for the Sassandra station at Semien.

Detection of peculiarities The results of the Pettitt non-parametric approach enable a comparison of the magnitude of the variations that have occurred on the VCX10 and VCN10 series respectively. It is only the VCX10 series of Aniassué-pont, which corresponds to the basin with the largest area, that has a peculiarity, *viz.* a break found between 1971 and 1972. The other series of VCX10 were homogeneous at the 10% level.

As for the VCN10 series, they showed a significant break in 1971 in Aniassué-pont and in 1972 in Bafingdala, Semien and Nzianoa. It should also be noted that the relative variations of the means on each side of the break were superior to 60% whereas they remained lower than 50% for the VCX10. The changes that occurred for the VCN10 were more important and more clear-cut than for the VCX10. This change in flow regimes seemed to have much more obvious consequences on low flows than on high flows.

CONCLUSION

The analysis presented above was based on a set of diverse and complementary methods. It involved the study of time series of annual mean discharges and of the maximum and the minimum mean daily discharges calculated for 10 consecutive days. From a quantitative standpoint, the results obtained clearly showed a strong decline of annual runoffs since the beginning of the 1970s decade. The different methods used all converged towards the identification of a break in the time series which varied between 1968 and 1971 according to the hydrometric station studied. The differences between the observations made before and after this date of the break are important. Indeed, the means of the annual mean discharges were lower by 30% after 1970 in the stations of the Sassandra basin and by more than 50% in the other stations.

From a more qualitative standpoint, the results reached for the maximum (VCX10) and the minimum (VCN10) of the mean daily discharge for 10 consecutive days showed that low flows were much more affected than high flows by this change in the runoff regime. The decrease noted in the VCN10 series was much more clear-cut than that of the VCX10 series.

The conclusions drawn from these four stations are similar and enable one to formulate the hypothesis whereby Ivory Coast could have been subjected to an important climatic fluctuation since the end of the 1960s. This phenomenon which demonstrated, among other things, a clear decline in the volumes of flow should, however, be confirmed by an ongoing study on rainfall (Lubès *et al.*, 1995). In any case, this climatic variability is in keeping with what has been observed more towards the north, in the Sahel (Hubert & Carbonnel, 1987; Olivry, 1987; Sircoulon, 1987).

Some regions of humid Africa could also be concerned by this drought, the serious consequences of which are already known in the Sahelian regions. The difference in behaviour perceived between the Sassandra basin and the other basins might be explained by the fact that it is undeveloped to any large extent. The other basins, Bandama and Comoé, have several developments. This extensive activity could be the reason behind the increase in the perturbations linked to drought because of the intensive use of water resources.

If this study has made it possible to take note of specific facts and to define orientations, it would be desirable to extend it to Ivory Coast in general during a first stage, and then to the countries bordering the Gulf of Guinea during a second phase. An analysis within a regional framework would indeed allow a better understanding and a better interpretation of the phenomena, which constitute the keys to an optimum integrated management of water resources.

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