Teaching Guidance

For

Multiplication and Division

(Overcoming Barriers
Moving Levels 1 to 2, 2 to 3, 3 to 4, 4 to 5)

Can I solve problems that involve multiplication as repeated addition?

Teaching guidance

Key vocabulary

multiply, multiplied by, lots of, groups of, repeated addition, add, total, equals

Models and images, resources and equipment

Practical equipment that promotes counting in equal groups

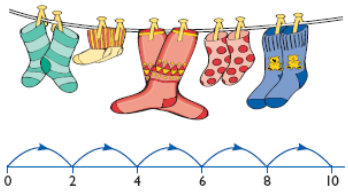
10 + 10 + 10 = 30

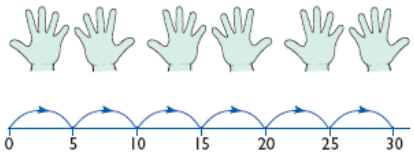
10 x 3 = 30

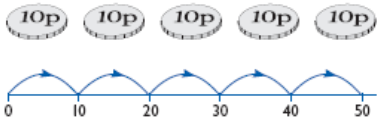
10p + 10p + 10p + 10p + 10p = 50p

10p × 5 = 50p

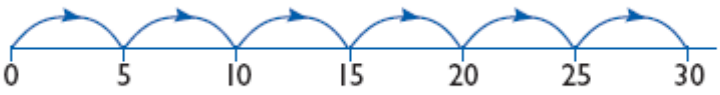
Relating practical equipment to equal steps along a number line







Describing equal jumps along a number line using a repeated addition number sentence, multiplication number sentence and appropriate vocabulary



5 + 5 + 5 + 5 + 5 + 5 = 30

5 × 6 = 30

5 multiplied by 6

6 groups of 5

6 hops of 5

Teaching tips

- In Key Stage 1, children need to begin to gain an understanding of multiplication as repeated addition and as describing an array (see the 'Can I describe an array and write number sentences about it?' section of this resource).
- To encourage children to use repeated addition (and not find the total by counting each individual item) use practical equipment and resources where each individual item cannot be seen, e.g. 2p, 5p or 10p coins or objects that are sold in packs of a given number.
- Use a number line to model repeated addition, recording the equal jumps on the line. Model how to record the repeated addition number sentence this represents and then the associated multiplication number sentence.

5 + 5 + 5 + 5 + 5 + 5 = 30
5 × 6 = 30

Help children to develop the necessary vocabulary so that they can describe the number sentences that have been modelled on the number line in a variety of ways, for example, 6 jumps of 5 equals 30, 6 groups of 5 equals 30, 5 multiplied by 6 equals 30. Tell the children the words and phrases you want to hear them make use of.

When using an image such as the number line it is important that the multiplication number sentence and the associated vocabulary match the image. For the image above children need to understand that 'multiplied by 6' or '× 6' means 'add the number six times'.

- Show children how to draw pictures to help them visualise a repeated addition problem that is set in a context. With a problem such as, 'Sam has eight packs of stickers, each pack has ten stickers. How many stickers has he got altogether?' spend time discussing and modelling what could be drawn (e.g. eight rectangles, each with the number 10 written on it).
- Provide children with problems involving repeated addition that are set in a range of different contexts, for example:
  - There are 10 pairs of socks in the washing pile. How many socks altogether?
  - I have eight 5p coins. How much money do I have altogether?
  - There are 7 boxes of pencils. Each box has 10 pencils. How many pencils altogether?
  - Sophie collects 10 shells on the first day of her holiday, 10 on the next day, 10 on the next and 10 on the next. How many shells has she collected altogether?

Rather than rushing through many problems, spend time discussing a few in depth to make sure that children understand the context, can visualise and represent the problem and can select the relevant mathematics to answer the problem.

Can I describe an array and write number sentences about it?

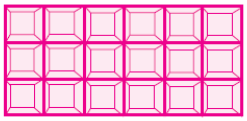
Teaching guidance

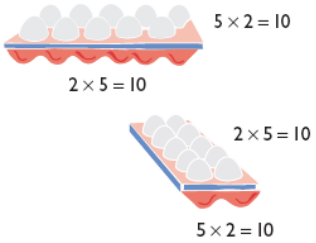
Key vocabulary

array, repeated addition, multiply, multiplied by, groups of, lots of, times, row, column, pattern

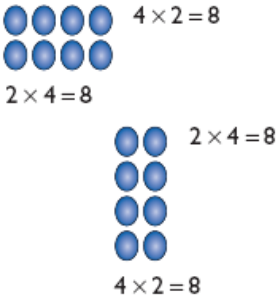
Models and images, resources and equipment

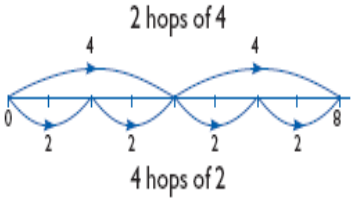
Make use of practical equipment and arrays used in everyday life, e.g. egg boxes, chocolate bars, displays of food in supermarkets, etc.



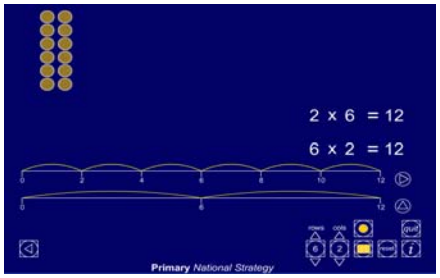


Link arrays to counting in equal groups along a number line



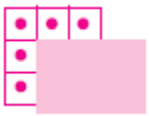


Use the Multiplication facts ITP, alongside practical equipment, to help children link arrays to counting in equal steps

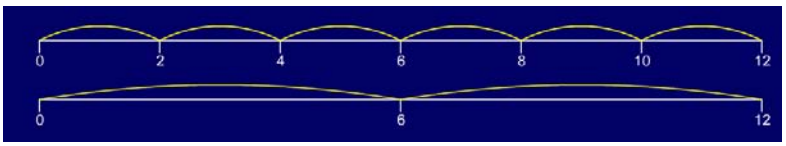


Teaching tips

- Provide opportunities for children to create their own arrays from practical resources, for example ask them to count out 10 counters and then arrange them in rows of 2. Then encourage them to count the counters in twos and in fives.
- Encourage children to describe what they see when you show them an array. Use this opportunity to assess whether they are focusing on the rows and columns or whether they only see an array as a collection of individual objects. Sentence starters could be provided to help model the correct use of mathematical vocabulary. Ask questions such as:
    - What do you see?
    - How many rows are there? How many in each row? How many columns are there?
    - How many can you see altogether?
- As children suggest different ways of describing an array, model how these can be recorded mathematically as number sentences. For example, 'there are five counters in each row, there are three rows and there are 15 counters altogether' could be recorded as 5 x 3 = 15 (or 5 + 5 + 5 = 15).
- To help children move away from counting the number of objects in an array in ones, create an array and then cover everything except the first row and first column. Ask the children to work out how many rows and columns are in the array and how many objects there are altogether.



- After showing children an array in one orientation, rotate it by 90° so that children see the columns and rows in their other orientation.
- Relate arrays to counting in equal steps along a number line, for example show how the following array can be recorded as 2 + 2 + 2 + 2 + 2 + 2 (2 x 6) or 6 + 6 (6 x 2).



- Give children a multiplication sentence such as 4 × 2 = 8 and ask them to draw an array to go with it. Then ask them whether there are any other number sentences they can write to go with it.

Can I count on in twos, fives and tens and use this to begin to say multiplication facts?

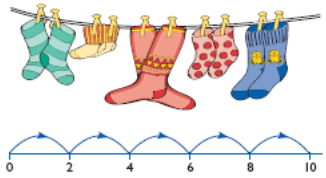
Teaching guidance

Key vocabulary

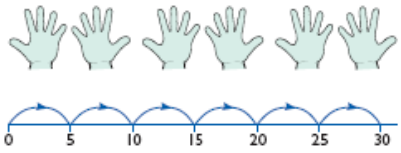
count in twos…fives…tens, multiply, multiplied by…, times, lots of, groups of, multiple of, count forward

Models and images, resources and equipment

Link counting in multiples using practical equipment to counting in multiples on a number line and associated number sentences



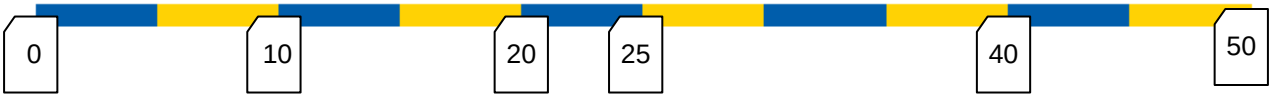
$2+2+2+2+2=10$
$2\times5=10$
2 multiplied by 5
5 pairs
5 hops of 2



$5+5+5+5+5+5=30$
$5\times6=30$
5 multiplied by 6
6 groups of 5
6 hops of 5

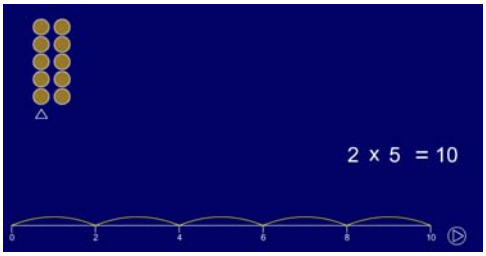
Counting stick

Count forwards and backwards with the multiples marked. Use sticky notes to mark multiples and gradually remove them as children gain confidence and familiarity.



Arrays

Arrays can be created using squared paper, counters, tiles, pegboards, ICT, etc.



Multiplication facts ITP

Teaching tips

- Use visual aids such as a set of multiple cards (don't include zero as this would make the third multiple, the fourth card). Ask questions such as: What is this sequence of numbers? (use the word 'multiple') What's the third multiple of 5? So what are three fives? How many fives are there in 15? Continue to ask several questions of this kind keeping to the pattern in the questions. Occasionally, stop and record the questions they have answered, e.g. 5 x 3 = 15. Turn over/hide the numbers on some of the cards. Can children still count forwards and backwards and respond to a similar sequence of questioning?
- Using a counting stick (initially with the multiples marked) say the facts in order and then point to random divisions on the stick so that children have to work out which fact is indicated.
- Show children how they can use their fingers to keep track of how many multiples they have counted.
- It is important that children are shown how the questions they are responding to orally would look if they were recorded. Show children that a question such as 'What are three fives?' can be written as 5 x 3. The language in this question is challenging for EAL learners – the word 'fives' is heard as a plural as well as a verb that implies the operation between 5 and 3. Children would benefit from continuing to see concrete objects displayed in groups of five and from linking this phrase to other more familiar phrases such as 'groups of'.
- Help children explore, identify and visualise patterns in multiples by, for example, highlighting the multiples of 2, 5 or 10 on a number grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Number grid ITP

- For more guidance on using arrays, see the 'Can I describe an array and write number sentences about it?' section of this resource.

Can I use grouping to solve division problems?

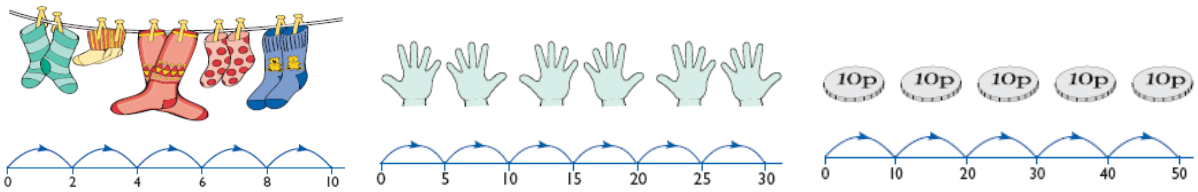
Teaching guidance

Key vocabulary

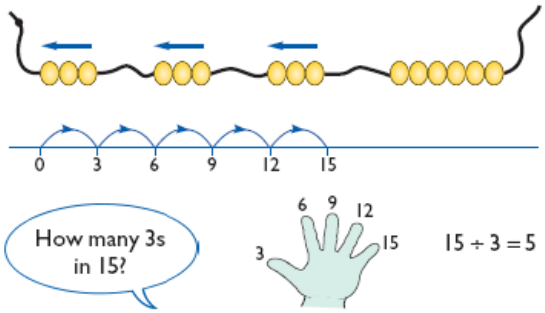
divide, division, groups, equal groups of, count on/back, left over, ÷

Models and images, resources and equipment

Link practical equipment to equal jumps along a number line

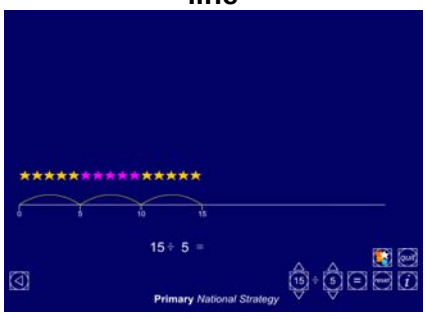


Images to help keep count of equal groups



Images also used for multiplication such as arrays and counting sticks

Grouping ITP to link the grouping of objects to counting in equal jumps along a number line



Teaching tips

- There are two models of division and children need experience of both. Calculations such as 10 ÷ 2 can be interpreted as:
    - Sharing, for example ten stickers are shared between two children; how many will they have each?
    - Grouping, for example if we have ten children, how many pairs can we make? (how many 2s are there in 10?)

  Provide opportunities for children to solve the same division problem using both grouping and sharing. For example for 30 ÷ 10, share 30 between 10 and then show how you could also find how many groups of 10 there are in 30. Establish that the answer is the same and discuss which way was quicker.
- Children often have more everyday experience of sharing and so the concept of grouping usually requires the greatest emphasis when exploring division.
- Provide frequent opportunities for children to gain practical experience of grouping using a wide variety of practical opportunities, equipment and models and images, for example:
    - Count along a counting stick in 5s. Stop when you get to 20 and then ask: How many 5s in 20?
    - Show 14 identical socks. How many pairs do you think we'll be able to make?
    - In PE, ask children to predict how many groups they will be able to make. There are 30 children here today. How many groups of five do you think we'll be able to make? Get yourselves into groups of five and we'll check.
    - Show children how they can count on their fingers to keep track, for example find how many fives are in 20, saying five, ten, fifteen, twenty.
    - Show 30 beads. How many groups of ten are there?
    - Display 25 objects (e.g. coloured circles) arranged randomly on the whiteboard. How many groups of five do you think we could make? Ring the groups of five to confirm. Record 25 ÷ 5 saying that we can read this as 'how many 5s are there in 25?' Repeat, this time arranging the objects in a line. Sketch a number line directly underneath the line of objects, labelling 0 at one end and 25 at the other. Draw hops of 5, labelling where they land on the line.
    - Show an array of four rows of five objects. Ask children to describe this picture. How many altogether? In each row? In each column? How many 5s are in 20? What division number sentence could we write? How many 4s are in 20? What division number sentence could we write?
- Use the Grouping ITP to link grouping objects to counting in equal jumps along a number line.
- Begin to explore, in practical contexts, the concept of remainders. Discuss questions such as, 'How many packs of four buns can we make using 17 buns? How many four-person tents will we need if 17 children are going camping?' Through these contexts explore how sometimes there are either too many or not enough items to make a whole group.

Can I use understanding of multiplication and division to solve problems?

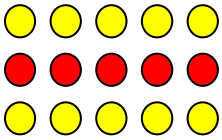
Teaching guidance

Key vocabulary

operation, symbol, number sentence, equation, number line, count on, count back, repeated addition, array, repeated subtraction, multiply, divide, divided by, divided into, groups, equal, grouping, recall, fact

Models and images

Arrays

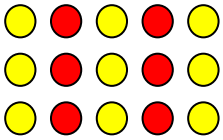


three fives

Arrays provide a visual image for multiplication.

They can help children to understand that multiplication is commutative: 5 × 3 = 3 × 5 = 15

Create arrays using counters, squared paper, tiles, pegboards, Cuisenaire rods, etc.

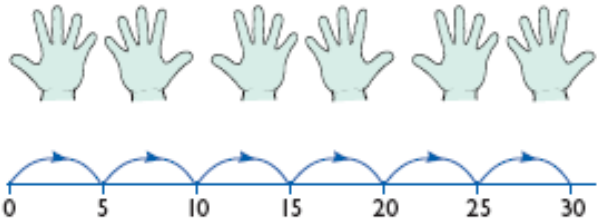


five threes

Number lines

Counting along number lines emphasises that multiplication involves combining equal groups.

- 5 + 5 + 5 + 5 + 5 + 5 = 30
- 5 × 6 = 30
- 5 multiplied by 6 = 30
- 6 groups of 5
- 6 hops of 5

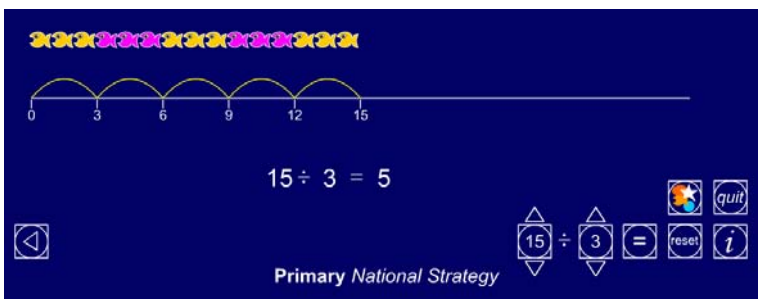


Scaling

One important context for multiplication and division is scaling.

The big tree is three times taller than the small one. The small tree is three times shorter than the tall one.

Grouping ITP



This ITP models division as counting in equal groups.

It provides a good link between grouping objects and jumping in groups along a number line.

It can be used to provide a visual image for remainders.

Teaching tips

- Many children continue to use repeated addition to solve problems even when it is inefficient. For example, take the problem: Find the cost of 12 books that cost £5 each. Children might represent the problem as 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5. Where children do this, explore more efficient strategies that are less likely to lead to mistakes. Move them into using multiplication by asking appropriate questions such as:
  - How many lots of £5 are needed?
  - What multiplication sentence describes this?
  - How can we find the answer quickly?

- Give children experience of different practical situations and contexts that involve multiplication.

  Arrays:
  - How many buns can you cook in this bun tray? Describe the array using multiplication.
  - For a concert there are five rows of seats and each row contains eight seats. How many seats are there? How do you know?

  Combining equal groups/measures/costs:
  - There are six packs of pencils and each pack contains ten pencils. How many pencils is this altogether?
  - A £2 coin weighs 12 g. How much will three of these coins weigh?
  - Pens cost 20p each. I buy four pens. How much will this cost?

  Scaling:
  - Amy gets 20p pocket money a week. Jack gets three times as much. How much pocket money does Jack get?

- Give children experience of division problems that involve sharing and others that involve grouping. They need to appreciate that both types of problems can be represented by division.

  Sharing:
  - 15 biscuits are arranged onto three plates so that there is the same number of biscuits on each plate. How many biscuits are on each plate?
  - Four children share 20 cherries equally. How many cherries does each child get?

  Grouping:
  - How many teams of four can be made from 28 people?
  - Notebooks come in packs of five. How many packs do I need to get 30 notebooks?

- Ask children to write their own multiplication and division problems. This is an effective way of assessing whether children understand what contexts and situations can be represented by multiplication and division. Use the children's problems to create a bank of resources for others to solve.

Can I multiply a two-digit number by a one-digit number?

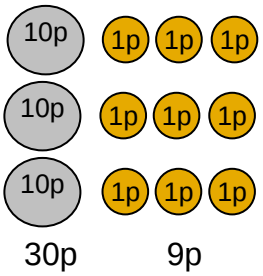
Teaching guidance

Key vocabulary

calculate, calculation, inverse, answer, method, explain, predict, estimate, reason, partition, ones, tens, one-digit number, two-digit number, mental calculation, informal method, jottings, recording, number line, count on, count back, double, halve, inverse, multiply, times, multiplied by, product, multiple

Models and images

Money

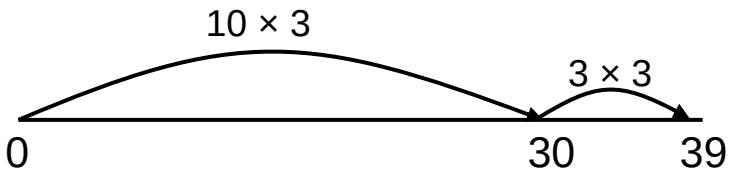


13p × 3
= 10p × 3 + 3p × 3
= 30p + 9p
= 39p

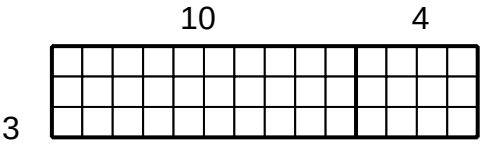
Number lines

Children can begin to use number lines efficiently by making bigger jumps.

13 threes can be made from 10 threes and 3 threes.



Arrays

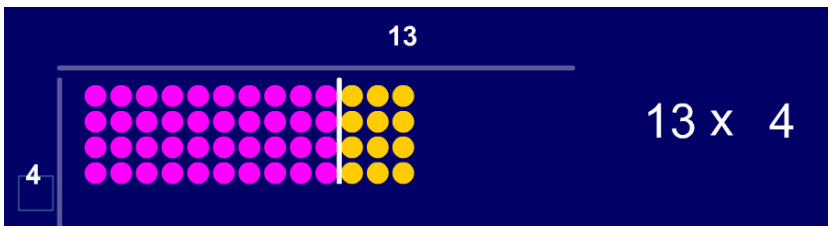


10 × 3 = 30 4 × 3 = 12
so 14 × 3 = 42

Arrays can provide a visual image for multiplication using partitioning.

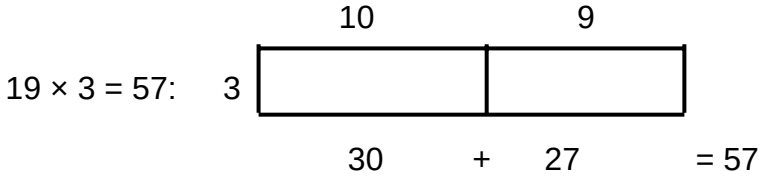
***Multiplication array* ITP**

This ITP creates arrays quickly. Marker lines allow you to partition the array to use known facts.



Teaching tips

- Plan regular opportunities for children to explain and/or record their answers to calculations, showing each step in a way that makes sense to them. Initially, you may need to model possible ways of recording a child's method using arrays, number sentences or number lines:



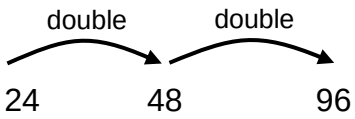
- Ensure that children can use known facts and place value to multiply and divide multiples of 10, for example 20 × 4 = 80 (using 2 × 4 = 8). See the section: Can I multiply one-digit and two-digit numbers by 10 and 100?

- Encourage children to use known facts in order to multiply efficiently rather than using repeated addition.
  - For example, find 14 × 3 by partitioning 14 into 10 and 4 and multiplying each part by 3:

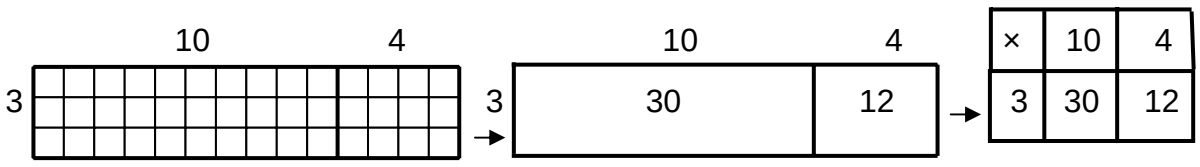
    10 × 3 = 30 and 4 × 3 = 12, so 14 × 3 = 42

  Number lines or arrays are useful vehicles for recording such partitioning.

- Teach children 'special case' mental strategies, including doubling twice to multiply by four, e.g. 24 × 4:



- Provide children with opportunities to create arrays for two-digit × one-digit multiplication as this leads them into recording the 'grid method' effectively.



- Discuss sensible estimates before working through each calculation. It is vital that children have an idea of the approximate size of their expected answer, so that they are able to spot errors. Ask questions such as:
  - Will the answer be bigger or smaller than 50? How do you know?

Can I divide a two-digit number by a one-digit number and explain any remainders?

Teaching guidance

Key vocabulary

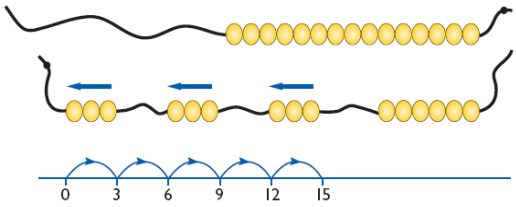
calculate, calculation, inverse, answer, method, explain, predict, estimate, reason, partition, ones, tens, one-digit number, two-digit number, mental calculation, informal method, jottings, number line, count on, count back, halve, inverse, multiply, times, multiplied by, product, multiple, share, divide, divided by, divided into, left, left over, remainder

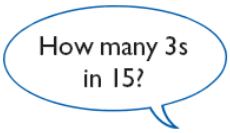
Models and images

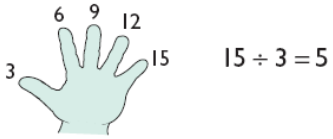
***Grouping* ITP**

The *Grouping* ITP can be used to explore remainders.

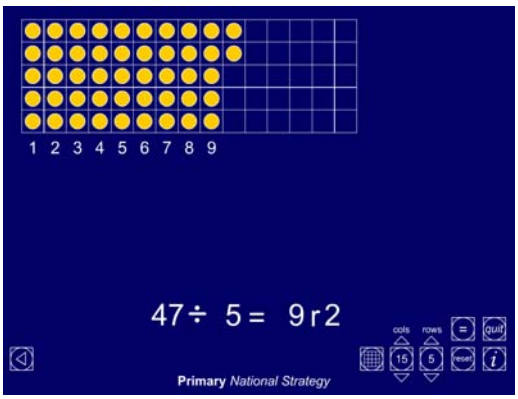
Concept of grouping







***Remainders after division* ITP**

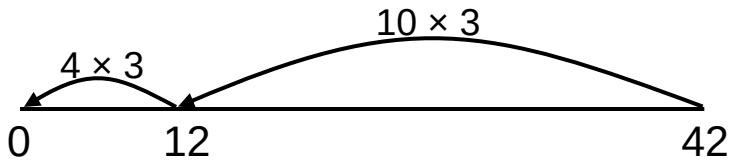


Teaching tips

- Plan regular opportunities for children to explain and/or record their approach to calculations, showing each step in a way that makes sense to them. Initially, you may need to model possible ways of recording a child's method using, for example, images, number sentences or number lines.

- Encourage children to use known facts to divide efficiently. For example, in demonstrating how to work out 42 ÷ 3, subtract a 'chunk' of ten 3s leaving 12, which is a further four 3s.The answer to the question 'How many threes make 42?' is therefore 14, so 42 ÷ 3 = 14.

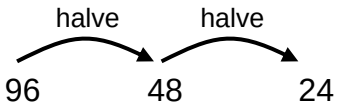
  Number lines or arrays can be useful vehicles for recording such partitioning.



- Help children to use known facts and place value to multiply and divide multiples of 10 (e.g. 120 ÷ 3 = 40, using 12 ÷ 3 = 4).

- When dividing large numbers it can be inefficient to use a sharing method. Instead, a grouping method will often be more appropriate. Check that children can interpret, for example, 42 ÷ 3 as 'How many groups of three are there in 42?' or 'How many threes are there in 42?'

- Teach children 'special case' mental strategies, including halving twice to divide by four, e.g. 96 ÷ 4:



- Discuss sensible estimates before working through each calculation. Children need to have an idea of the approximate size of their expected answer, so that they are able to spot errors. Ask questions such as:
  - Will the answer be bigger or smaller than 10? How do you know?
  - Roughly how big do you expect the answer to 57 ÷ 3 to be? Why?

- Use practical resources, models and images, such as the *Grouping* ITP, to help children appreciate that you cannot have a remainder that is larger than the divisor.

Further support for progression in the teaching and learning of division can be in the Calculation guidance paper on the Primary Framework:
www.standards.dfes.gov.uk/primaryframeworks/library/Mathematics/guidance/resources/

Can I identify the calculation needed to solve a word problem?

Teaching guidance

Key vocabulary

problem, solution, calculate, calculation, operation, answer, method, explain, reasoning, count on/back, add, subtract, multiply, times, divide, share, group, sum, total, difference, plus, minus, How many more …?, How many fewer …?, How much less …?

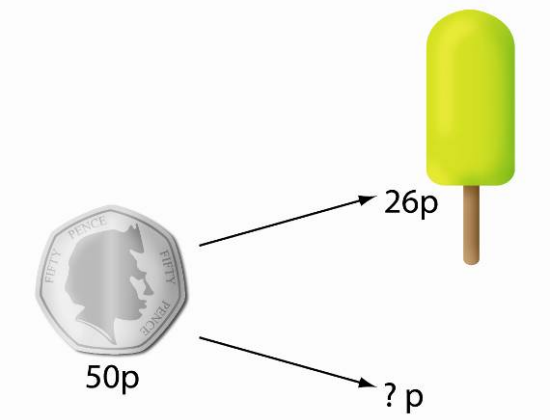
Models and strategies

Identify important words and numbers

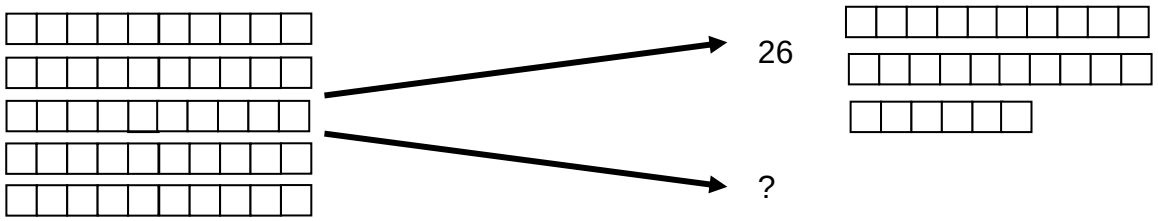
Ella buys a 26p lolly. She pays with a 50p piece. How much change does she get?

Ask children to highlight important words and numbers, to restate the situation in their own words, then to tell you the calculation needed to solve the problem.

Draw a sketch



Represent the problem using equipment



Check that your answer sounds reasonable

Tom is 114 cm tall.
Josh is 12 cm taller.
How tall is Josh?

Sam writes:
114 – 12 = 102 Josh is 102 cm tall.
Does Sam's answer make sense? Explain your answer.

Teaching tips

- Give children sets of mixed problems so that they have to read carefully to select the appropriate operation.
  - Ask them to identify the appropriate operation for each problem;

  If you want children to practise a particular operation, ask them to solve only those problems that involve your chosen operation.

- A good activity is to ask children to classify a set of one-step word problems into those involving the same operation (they do not need to actually solve them). Together, talk about what is the same about the problems in each set. Use this to identify the key features of addition/subtraction/multiplication/division problems.

- Incorporate discussion time into problem-solving sessions so that children have to explain to each other how they identified the appropriate operation. It is better to spend time unpicking one or two problems rather than rushing through many problems.

- Use strategies that help children to focus on the language used in a word problem:
  - encourage children to underline/highlight important words and numbers;
  - ask children to explain the problem in their own words;
  - ask children to clarify what they have to find.

- The vocabulary in a word problem can give useful clues about the appropriate operation but this is not always straightforward.
  - The phrases ‘more than’/‘less than’ occur in addition **and** subtraction problems.
  - The word ‘altogether’ occurs in addition **and** multiplication problems.

- Ask questions to help children identify the appropriate operation.
  - What is the biggest number in the problem? Will the answer be bigger or smaller than this?
  - In the problem, are there lots of things that are the same in some way? (Multiplication and division scenarios involve equal sets, for example several objects that cost the same or a large set that is shared out equally.)

- If children find it hard to identify the appropriate operation for a problem, encourage them to represent the problem with equipment or to draw a sketch of it.

- Once children have got an answer to the problem, ask them to give that answer as a whole sentence. They should then reflect on whether their answer sounds about right. (Often, if they have chosen the wrong operation, the answer will be of an inappropriate size.)

- Reinforce children’s understanding by asking them to write their own word problems, for example ask them to write a problem that matches a particular calculation.

Can I recall multiplication and division facts for the 2, 3, 4, 5 and 10 times-tables?

Teaching guidance

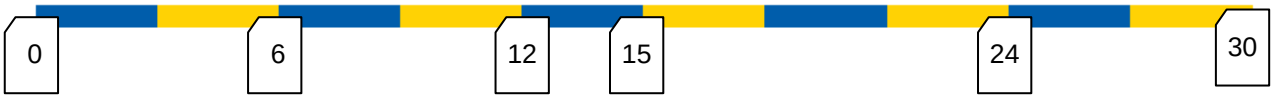
Key vocabulary

pattern, relationship, number sentence, equation, number line, count on, count back, double, halve, inverse, multiply, times, multiplied by, product, multiple, share, share equally, divide, divided by, divided into

Models and images

Counting stick

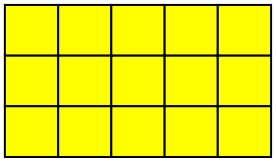
Build up each table starting from simple facts. Use strategies such as doubling 2 × 3 to get 4 × 3. Count forwards and backwards through the tables. Use sticky notes to mark multiples and gradually remove them as children remember the facts.



Arrays

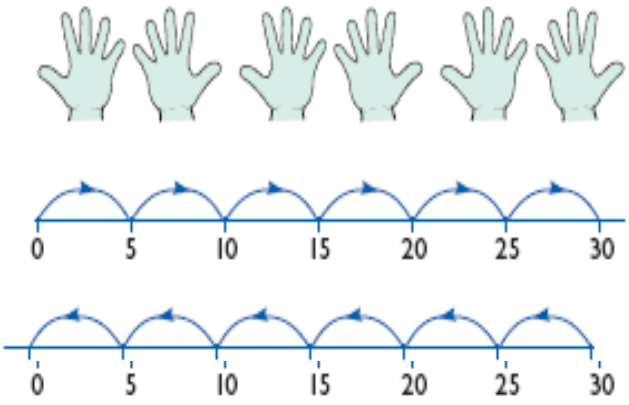
An array can be used to model the four related multiplication and division facts. Arrays can be created using ICT, squared paper, Cuisenaire rods, counters, tiles, pegboards, etc.

5 × 3 = 15
3 × 5 = 15
15 ÷ 3 = 5
15 ÷ 5 = 3



Number lines

Counting along number lines emphasises that multiplication and division involve equal groups.



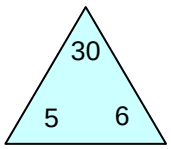
5 + 5 + 5 + 5 + 5 + 5 = 30
5 × 6 = 30
5 multiplied by 6 = 30
6 groups of 5
6 hops of 5

30 ÷ 5 = 6
6 fives make 30
30 divided by 5 = 6

Number trio cards

Children can state the four possible calculations.

Alternatively, they can suggest the hidden number when one corner is covered.



5 × 6 = 30
6 × 5 = 30
30 ÷ 6 = 5
30 ÷ 5 = 6

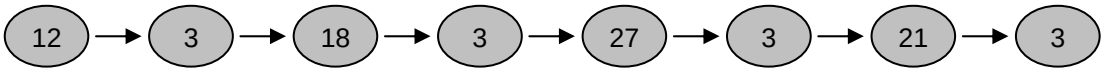
Teaching tips

- Plan regular activities for children to learn, rehearse and use multiplication and division facts rather than simply testing their recall.

- Use a variety of strategies and activities to help children to learn facts, including:
  - kinaesthetic – for example use fingers, manipulate objects, use actions;
  - visual – use images and models such as flashcards, arrays;
  - oral – make up a rap for a table; all say the 'fact of the day' at the start and end of every teaching session;
  - written – children make their own flashcards;
  - patterns – for example highlight the multiples of 2, 3, 4, 5 or 10 on a 100-grid. Ask: What do you notice?

Number grid ITP

- Games provide a good vehicle for learning facts. For example:
  - Stepping stones – Children key the start number into a calculator then work out which operation (× or ÷) to key in to make the number in the next stepping stone. They continue until they reach the end of the line. Repeat the activity several times using the same numbers. Children try to improve how far they can get in a set time.



  - Calculator crunch – Children work in pairs to rehearse a chosen table (e.g. the 3 times-table). One child calls out a problem (e.g. 24 ÷ 3) before keying this into the calculator. Their partner tries to write the answer on a whiteboard before it appears on the calculator display. They swap roles.

- Encourage children to keep track of the facts they have learned, for example through colouring known facts on a tables square.

- Discourage children from relying on counting from zero. Teach them strategies that build on the facts they already know, for example double the answer to 2 × 6 to get the answer to 4 × 6, double this to find 8 × 6. Make sure they know and use key 'benchmark' facts such as the 5 times-table, for example use 5 × 6 = 30 to find 7 × 6 by adding on 12.

- Remind children of BOGOF (Buy One, Get Others Free), for example the fact 6 × 5 = 30 gives you three 'free' facts: 5 × 6 = 30, 30 ÷ 6 = 5 and 30 ÷ 5 = 6. Rehearse multiplication and division facts together to reinforce the link.

- Ensure that children meet calculations written in different ways:
  ■ × 6 = 24 10 = 30 ÷ ● 3 × 10 = 5 × ★

- Teach children how to derive new facts from known ones. For example, point out that knowing 7 × 2 = 14 helps you to work out that 7 × 4 = 28 and 7 × 20 = 140.

Can I multiply one-digit and two-digit numbers by 10 and 100?

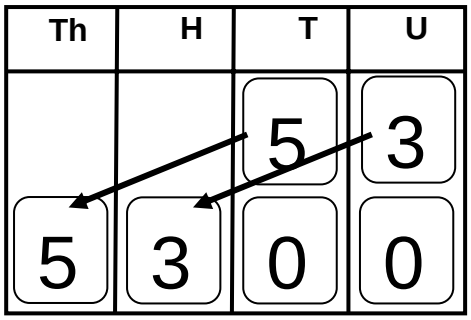
Teaching guidance

Key vocabulary

place value, digit, units, ones, tens, hundreds, thousands, one-digit number, two-digit number, multiply, place holder

Models and images

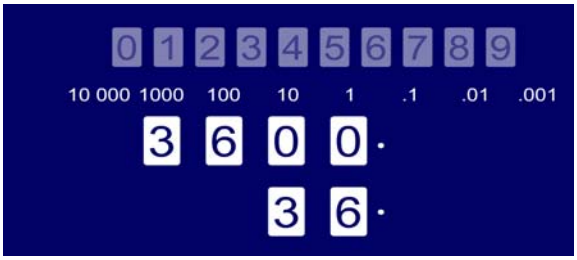
Place value grid



Use digit cards to make numbers in the grid. Show how each digit in a number moves one column to the left when a number is multiplied by 10 and two columns to the left when it is multiplied by 100.

***Moving digits* ITP**

Use the *Moving digits* ITP to model the effect of multiplying and dividing a number by 10 or 100.



Calculators

Use an overhead calculator to demonstrate to children how each digit in a number moves to the left as you multiply a number by 10 or 100.

Show children how to set a constant function so that they can enter different start numbers, predict and then check the answer when the number is multiplied by 10 or 100.

Teaching tips

- Before teaching children how to multiply numbers by 10 and 100, ensure that they can confidently read and write numbers to 10 000. They should also be able to partition numbers and state the value of each digit in a given number.

- Build from what children already know. Write:

1 × 10 = 10	1 × 100 = 100
2 × 10 = 20	2 × 100 = 200
3 × 10 = 30…	3 × 100 = 300

  Continue the pattern.

  Ask: What do you notice?

- Use practical equipment, such as straws grouped in ones, tens and hundreds or £1, 10p and 1p coins, to reinforce children's understanding of the relative values of hundreds, tens and ones. Children need to understand and be able to use the fact that 10 ones make one ten, 10 tens make one hundred and 100 ones make one hundred. Ask questions such as:
  - How many 10p coins are worth the same as these three £1 coins?
  - How many tens do you need to make 280? How many ones do you need?

- Avoid using the explanation that you 'add a zero' when you multiply a number by 10 or 'add two zeros' when you multiply a number by 100. Instead, stress the following.
  - Multiplying by 10 gives an answer that is bigger than the original number and all the digits move one place to the left.
  - Multiplying a whole number by 10 will leave a gap in the units column which you need to fill with a zero. Similarly, multiplying a whole number by 100 leaves gaps in the units and tens columns which you need to fill with zeros.
  - These zeros are called 'place holders'. Without the zero place holder, the number 560, for example, would say 56. The zero is important, because it holds the other digits in the right place so that you know their correct value.

- Ensure that children understand that multiplying by 10 and then by 10 again is equivalent to multiplying by 100. One way to reinforce this is for children to work in pairs using calculators. Both key in the same start number. One child multiplies by 10, then multiplies the answer by 10 again. The other multiplies the number by 100. They compare answers.

- It is important that children fully understand how to multiply numbers by 10 as this is a key strategy in multiplication. For example, to work out:
  - 13 × 30, children could multiply 13 by 10 then multiply the answer by 3;
  - 6 × 16, children could find 6 × 10 and 6 × 6, then combine the answers.

Can I use my tables to multiply and divide?

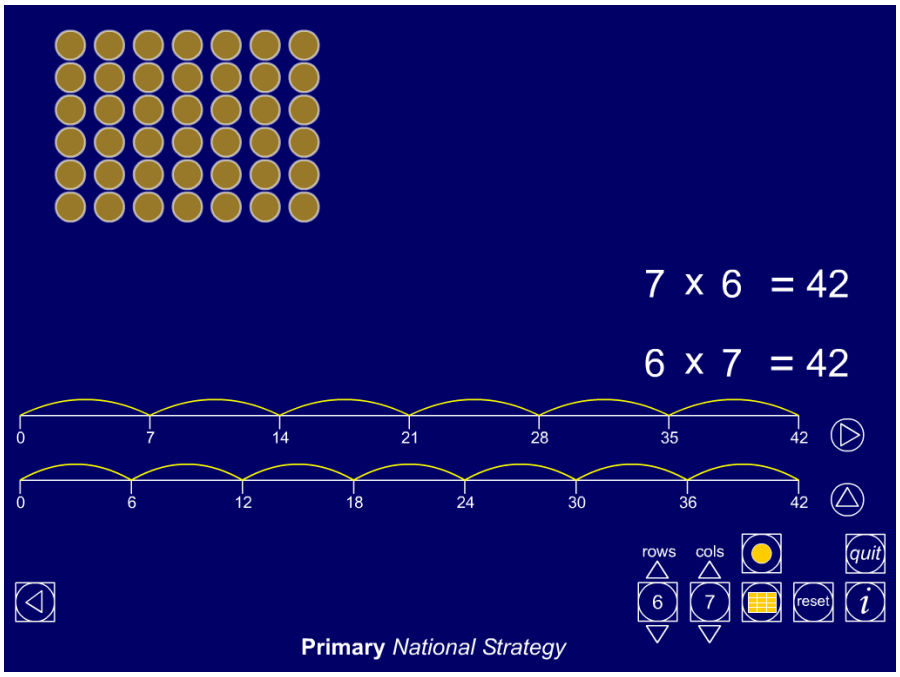
Teaching guidance

Key vocabulary

multiply, multiplied by, multiple of, times, array
divide, divided by, divisible by
factor, product, inverse

Models and images

Use the *Multiplication facts* ITP to make the link between arrays and multiplication facts.



Multiplication facts ITP

Teaching tips

- Plan regular activities for children to *learn*, *rehearse* and *use* multiplication and division facts rather than simply test their recall.

- Reinforce multiplication facts and the corresponding division facts, for example:
  $8 \times 7 = 56$ $7 \times 8 = 56$ $56 \div 7 = 8$ $56 \div 8 = 7$
  When solving a missing number question, it is helpful to write down the other three number sentences and then decide which one to use to find the missing number.

Can I use my tables to multiply and divide?

- Children need to understand and use the language of multiples and factors.

- Help children develop strategies for quickly deriving multiplication facts. For example, knowing 7 × 2 = 14 helps you to work out that 7 × 4 = 28 and 7 × 20 = 140.

- Ensure that children meet calculations written in different ways:
  ■ × 8 = 56 9 = 54 ÷ ● 3 × 8 = 6 × ★

Can I use my tables to work out multiplication and division facts with decimals?

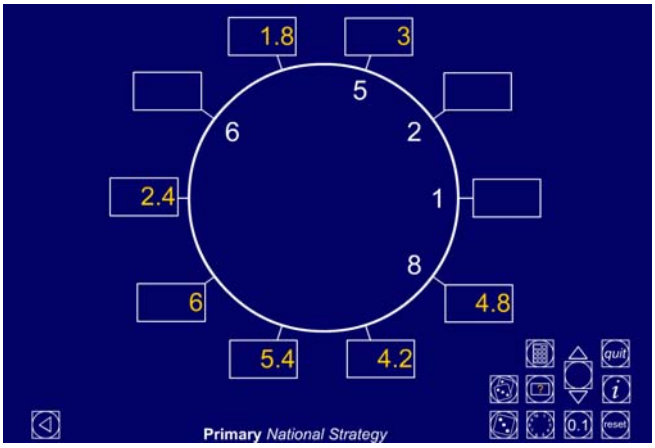
Teaching guidance

Key vocabulary

times, multiply, multiplied by, product, multiple of, divide, divided by, divisible by, quotient, factor, inverse
decimal, decimal point, tenths, hundredths, thousandths

Models and images

Use the *Number dials* ITP to explore multiples of decimal numbers.
Explore known multiplication facts, such as multiples of 6, before exploring related facts such as multiples of 0.6.



Number dials ITP

What multiplication table does this diagram represent? How do you know? What are the missing numbers? What division facts do you know by using this diagram?

Teaching tips

- Ensure children can confidently multiply and divide by 10 and 100 and that they understand that multiplying by 10 gives an answer that is bigger than the original number and all the digits move one place to the left, while dividing by 10 gives an answer that is smaller than the original number and all the digits move one place to the right. (See the teaching guidance 'Can I multiply and divide by 10 and 100 and 1000?', in the Calculating strand).

- Start with known multiplication facts before relating these to decimal multiplication facts; for example, count on and back in steps of 3 before relating this to counting on and back in steps of 0.3. Encourage children to explain the relationship between the two sets of numbers.

Can I use my tables to work out multiplication and division facts with decimals?

- Ensure that children meet and can interpret multiplication and division calculations that are written in a variety of different ways, for example:
  ■ × 0.8 = 5.6 9 = 5.4 ÷ ● 0.3 × 8 = 6 × ★

- Reinforce the division facts corresponding to multiplication facts; for example:
  8 × 0.7 = 5.6 0.7 × 8 = 5.6 5.6 ÷ 0.7 = 8 5.6 ÷ 8 = 0.7
  When solving a missing number question, it is helpful to write down the other three number sentences and then decide which one is most useful to use to help find the missing number.

- Model the use of jottings and encourage children to use jottings to help keep track of the stages within a mental calculation.

Can I multiply and divide by 10 and 100 and 1000?

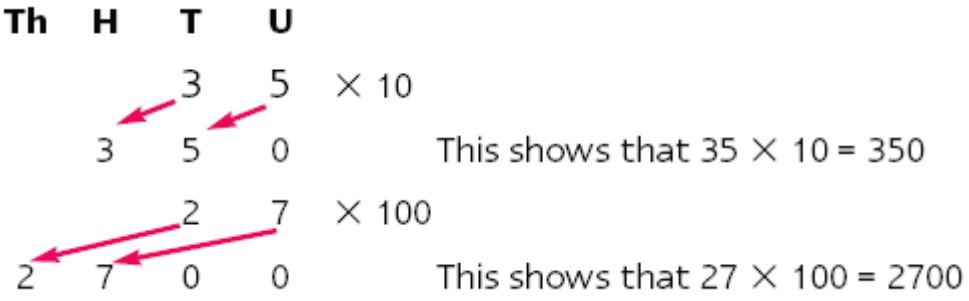
Teaching guidance

Key vocabulary

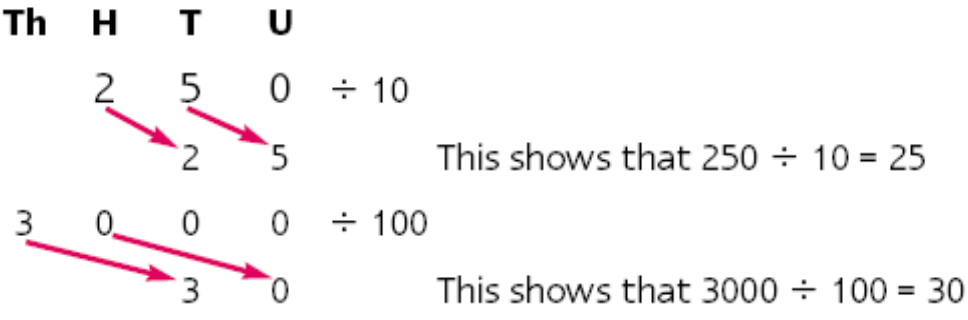
digit, decimal, multiply, times, divide, share, scale up, scale down, increase, decrease, factor, how many 100s in …?, tens of thousands, thousands, hundreds, tens, units, ones, tenths, hundredths, thousandths

Models and images

Show children how multiplying a number by 10 moves the digits one place to the left, and multiplying by 100 moves the digits two places to the left.

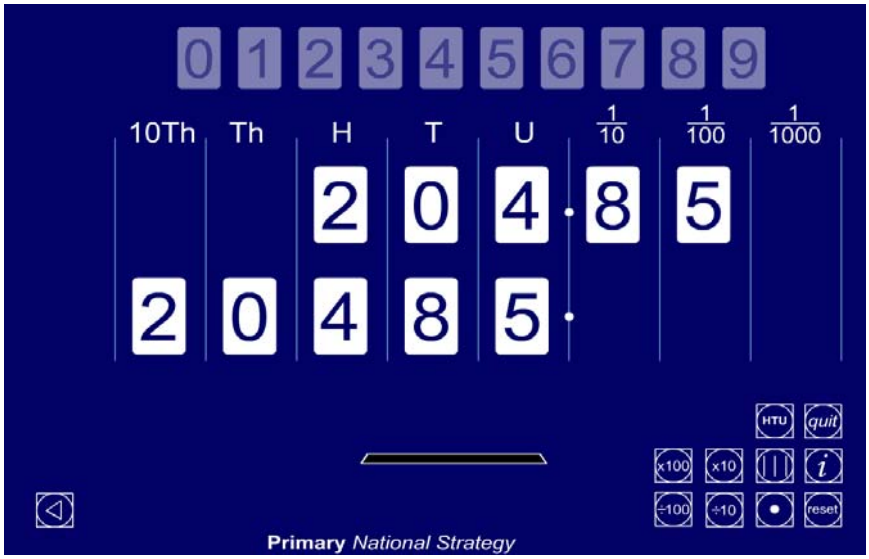


Demonstrate the effect of dividing a number by 10. Show children how the digits move one place to the right, and when dividing by 100 the digits move two places to the right.



Can I multiply and divide by 10 and 100 and 1000?

Use a calculator or the *Moving digits* ITP to model how the digits move when we multiply or divide by powers of 10.



Moving digits ITP

Teaching tips

- Help children to generalise correctly so that they can cope with decimals. Multiplying by 10 gives an answer that is bigger than the original number and all the digits move one place to the left. Dividing by 10 gives an answer that is smaller than the original number and all the digits move one place to the right.

- Discuss why 4.6 × 10 is not the same as 4.60 and 40.3 ÷ 10 is not the same as 4.3.

- Explore with children the relationships between the operations and how to simplify combinations of operations. For example, multiplying by 10 then dividing by 100 is the same as dividing by 10. Help children to recognise that dividing by 200 is the same as dividing by 10, dividing by 10 again and then halving, by using a calculator to explore different examples.

- Emphasise that a multiplication and division by 10, 100 and 1000 should be a mental calculation.

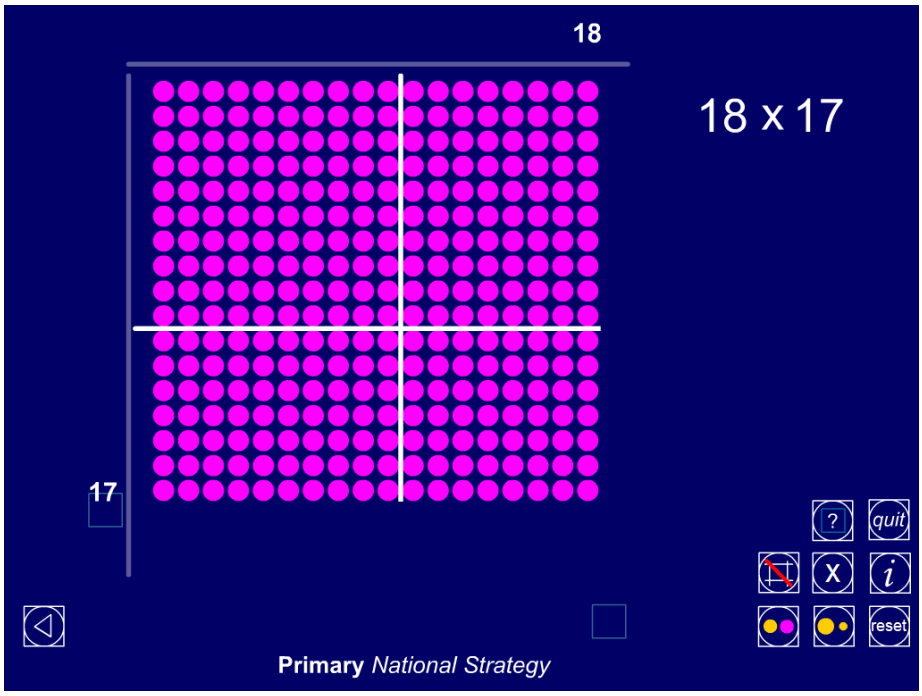
Can I use a written method to multiply?

Teaching guidance

Key vocabulary
array, partition, multiply, multiplied by
calculate, calculation, strategy, method, equation

Models and images

Create arrays and then partition them to provide a visual representation of the grid method of multiplication. The *Multi array* ITP can be used to help model this.



Multi array ITP

Can I use a written method to multiply?

Teaching tips

- Children should use mental methods of calculation, where appropriate, but for those calculations they cannot solve mentally they need to be able to use an efficient method, such as the grid method of multiplication, accurately and with confidence.

- To multiply successfully, children need to:
  – know or quickly recall multiplication facts up to 10 × 10;
  – understand the effect of multiplying numbers by 10, 100 or 1000;
  – multiply multiples of 10, for example, 20 × 40;
  – approximate; for example, recognise that 72 × 38 is approximately 70 × 40 = 2800 and use this information to check whether their answer appears sensible.

- Give children opportunities to work with numbers for which they could use mental or written calculation strategies, in order to build up their confidence in the written method.

- Check which methods have been taught earlier on in the school, to ensure children receive consistent modelling and demonstration.

Can I make use of my understanding of place value to explain how to mentally multiply or divide a decimal number by an integer?

Teaching guidance

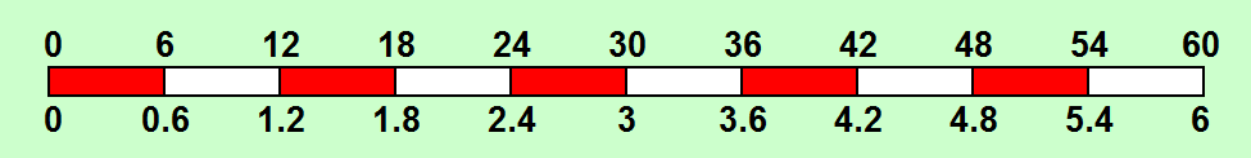
Key vocabulary

place value, digit, column, decimal point, tenth, hundredth, thousandth, partition, integer

Models and images and resources

Counting stick or Counting stick with further options spreadsheet

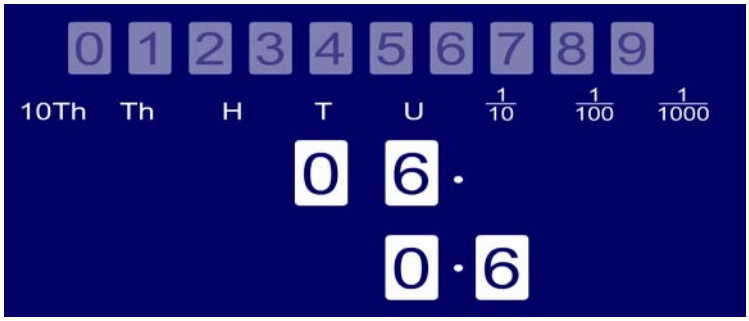
Use a counting stick to create related sequences, for example count in sixes and then in 0.6s. Ask children to comment on the patterns and relationships between the numbers in the two counts and to use this to predict and explain future terms. Alternatively use the spreadsheet *Counting stick with further options*, showing one sequence above the counting stick and a related one below.



Moving digits ITP

Ensure that children are confident in understanding the effect of multiplying and dividing numbers by 10, 100 and 1000.

Children need to appreciate that 0.6 is six divided by ten, in other words that it is ten times smaller than six.



Place-value cards

X 3 0.3 0.15

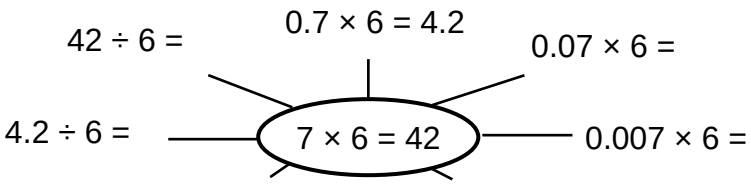
Carry out multiplication calculations involving decimals by partitioning the decimal number using place-value cards and then multiplying each part separately before recombining to get the answer.

Teaching tips

- Ensure that children understand the effect of multiplying or dividing whole numbers and decimals numbers by 10, 100 and 1000. Reinforce understand by using visual images:
  - Put place value or column labels (..., tens, units, tenths, hundredths, ...) onto cards and stick these onto the wall. Include a decimal point. Ask a group of children to create a given number, for instance 1.35, by taking the necessary digit cards and standing in the appropriate places. Ask what will happen when this number is multiplied by ten. Discuss the effect on each digit – each child should jump a column to the left to show the effect of multiplying by ten.
  - Use the ITP Moving digits. Ask children to predict the answer when a given number is multiplied or divided by ten, 100 or 1000, before using the program to check.
- Check that children fully understand the relationship between the values of digits in different columns in decimal numbers. For example, children need to appreciate that ten hundredths is equivalent to one tenth, so 0.13 can be thought of as one tenth + three hundredths or as 13 hundredths.
- Give children opportunities to identify missing operations and numbers in calculations. Ask them to explain how they identified the missing number(s) or operation, for instance in:

  5 □ □ = 0.05 0.6 × □ = 5.4
- Give children experience in creating whole number then decimal sequences:

  6, 12, 18, 24, 30, 36, ... 0.6, 1.2, 1.8, 2.4, 3, 3.6, ...
- Encourage them to explain the relationship between corresponding numbers in the sequences, in other words to recognise that each number in the second sequence is ten times smaller than the equivalent number in the first sequence. They should use this information to answer related calculation questions, for example: If 7 × 6 = 42, what is 7 × 0.6?
- Spider diagrams are useful for rehearsing how to use known facts to work out unknown ones:



- When asked to multiply or divide a decimal by a whole number, children need to identify a linked whole number fact that can help them. For example, for 2.1 ÷ 3, they need to recognise that it is linked to the fact 21 ÷ 3 = 7 and use this to help them. Encourage them to explain their reasoning. For example: 21 ÷ 3 = 7, and 2.1 is ten times smaller than 21, so the answer will be ten times smaller than seven, in other words 2.1 ÷ 3 = 0.7. Or 21 ÷ 3 = 7, so 21 tenths divided by three is seven tenths, that is 0.7
- In order to multiply decimal numbers with two or more digits, partition them and multiply each part before recombining answers. The grid method is a clear way to record this thinking:

×	1	0.3
4	4	1.2

so 1.3 × 4 = 4 + 1.2
= 5.2

- Always ask children to check that the size of their answer sounds reasonable. For example, children should recognise that 0.18 ÷ 2 = 0.9 cannot be correct. Using a number line to locate numbers such as 0.18 and then thinking about dividing by two could help children to check the reasonableness of an answer.

Can I use an appropriate non-calculator method for dividing a three-digit integer by a two-digit integer?

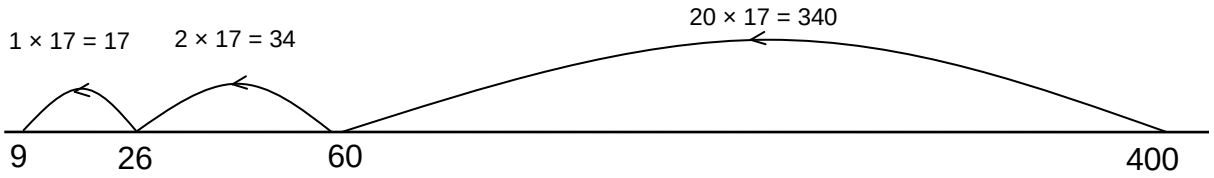
Teaching guidance

Key vocabulary

place value, digit, column, partition, integer, method, strategy, remainder, quotient, round

Models and images and resources

Number lines



400 ÷ 17 = 23 r 9

Number lines provide a visual image for division as counting back in ‘chunks’.

Chunking

Ensure that children have a clear layout and that they understand the process for using chunking to divide by a single-digit number before moving into dividing by a two-digit number.

$$\begin{array}{r|l} 17 & 400 \\ -340 & \quad 17 \times 20 = 340 \\ \hline 60 & \\ -51 & \quad 17 \times \ \ 3 = \ \ 51 \\ \hline 9 & \end{array}$$

Factorising

480 ÷ 15

= 480 ÷ 5 ÷ 3

Teach children to split the divisor into factors, where possible, to make division by a two-digit number easier.
For example to divide by 15, you can divide by five and then three.

Teaching tips

- Ensure that children are able to divide by a single digit number before moving into 'long division' of a three-digit number by a two-digit number. Make sure that they are able to explain each step of their calculation showing understanding of the process. They should refer to the value of each digit as they explain their method.
- Teach children how you can use factors of the divisor to split a division into more manageable calculations. For example:

$$480 \div 15$$
$$= 480 \div 5 \div 3$$
$$= \quad 96 \div 3$$
$$= \quad 32$$

- Written methods of division involve multiplication and subtraction, so ensure that children have good mental and written methods for these operations as well as good knowledge of multiplication and division facts.
- Ensure that children are able to 'read' a division calculation as 'How many xx can we get from xxx?' Remind them that to solve long division questions, we use the inverse operation of multiplication.
- In order to be efficient at long division, children need to be good at estimating, for example, 'How many 17s make 51? Teach children how to make approximations:
  - Use rounding: to find an approximate answer to 51 ÷ 17, round the numbers to give 50 ÷ 20. This tells you that the answer is likely to be two or three 17s.
  - Use activities such as 'Call my bluff' where you give children three possible answers to a calculation and give them 20 seconds to decide which they think is correct. Ask children to explain the reason behind their answer.
- Before starting a division calculation ask children to jot down key multiplication facts for the divisor, such as 10 × ... and 20 × ... . This gives them sensible chunks that they can use in their calculation and helps them to make a sensible estimate for the answer before they start the calculation. For example:

$400 \div 17$

Useful facts: 10 × 17 = 170, 20 × 17 = 340, 30 × 17 = 510
so a sensible estimate for the answer is between 20 and 30

$$\begin{array}{rl} 17\overline{)400} & \\ -\underline{340} & 17 \times 20 \\ 60 & \\ -\underline{51} & 17 \times 3 \\ 9 & \end{array}$$

So the answer to 400 ÷ 17 is 23 r 9

- Children sometimes struggle to give the answer to a calculation once they have completed the written method. Remind them that they are answering the question 'How many xx can we get from xxx?' It may help to suggest that they underline how many lots of the divisor they have taken out in each chunk.
- Remind children that when they have completed a division calculation they should always check that the answer looks sensible. They should also look at the original context if solving a problem so that they can decide whether their answer needs to be rounded up or down.

Can I extend my written methods for multiplying whole numbers to multiplying decimals by whole numbers?

Teaching guidance

Key vocabulary

place value, digit, column, decimal point, tenth, hundredth, thousandth, partition, integer, method, strategy

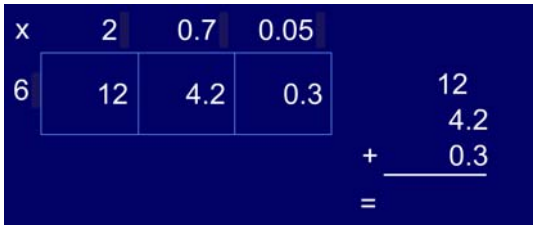
Models, images and resources

Place-value cards

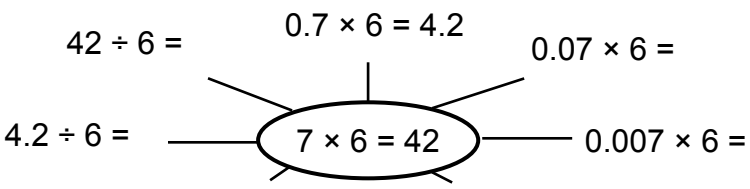
Use place-value cards to model how to partition decimal numbers before multiplying each part and then recombining to get the final product.

Grid method or Multiplication grid ITP

Even where children are working with a compact written method for multiplication, it is sensible when first multiplying decimals to go back to an expanded form such as the grid method, to help them focus on the value of each digit in the calculation.



Spider diagrams



To be successful at multiplying decimal numbers using a written method, children need to be completely secure in using known multiplication facts to derive linked decimal facts. Spider diagrams provide a visual way of recording these facts.

Teaching tips

- Ensure that children know the value of each digit in decimal numbers and can state them as decimals and fractions, for example, the value of the six in 3.69 is 0.6 or 6/10.

- Help children to secure their understanding of place value in decimals. Use resources such as base ten apparatus or money to represent wholes, tenths and hundredths to provide them with a visual image. They need to know that ten thousandths = one hundredth, ten hundredths = one tenth and ten tenths = one whole. Teach them to use this understanding to appreciate that, for example, 0.06 × 4 = 24 hundredths = two tenths + four hundredths, and so is written as 0.24.

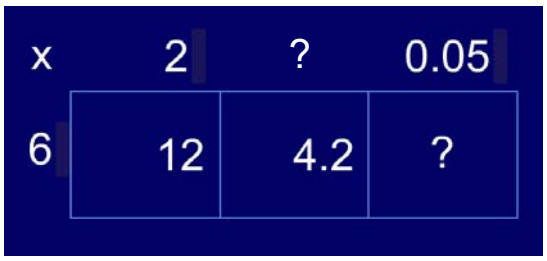
- Give children regular practice in using known multiplication facts to derive linked decimal facts. For example, children should be able to use 6 × 8 = 48 to derive:
  0.6 × 8 = 4.8 0.06 × 8 = 0.48 0.006 × 8 = 0.048 0.6 × 0.8 = 0.48, and so on.

- Make sure that children can explain each step of their whole-number written method for multiplication before extending this into working with decimals. They should use the value of the digit they are working with as part of their explanation, for example, by saying two tens or 20 rather than just two.

- Compact methods for multiplication are efficient but often do not make the value of each digit explicit. When introducing multiplication of decimals, it is sensible to take children back to an expanded form such as the grid method where the value of each digit is clear, to ensure that children understand the process.

- Insist that children make an estimate for the answer to every written calculation before carrying it out. This can then be used to check that the answer they get is reasonable.

- Give children experience of calculations involving gaps. Ask children to work out the missing number or digit:



- Build on children's understanding of written methods for whole numbers. Demonstrate multiplication of a decimal number alongside its whole number equivalent: For example:

326	3.26
× 8	× 8
2400	24.00
160	1.60
48	0.48
2608	26.08