NIFS 17 (2011), 1, 37-47

Constant intuitionistic fuzzy graphs

M.G.Karunambigai[1] and R. Parvathi[2] and R. Buvaneswari[3]
[1]*Department of Mathematics, Sri Vasavi College,*
Erode - 638 316, Tamilnadu, India.
e-mail: gkaruns@yahoo.com
[2]*Department of Mathematics, Vellalar College for Women,*
Erode - 638 012, Tamilnadu, India.
e-mail: paarvathis@rediffmail.com
[3]*Department of Mathematics, Sankara College of Science and Commerce,*
Coimbatore - 641 035, Tamilnadu, India.
e-mail: rbmksr@gmail.com

In this papar, Constant Intuitionistic Fuzzy Graphs (IFGs), and totally constant IFGs are introduced. Necessary and sufficient conditions under which they are equivalent is studied here. A characterization of constant IFGs on a cycle is given. Some properties of constant IFGs with suitable illustrations are also discussed.

Keywords: μ- degree of a vertex, γ - degree of a vertex, degree of a vertex, constant IFG, totally constant IFG.

1 Introduction

Intuitionistic Fuzzy Graph theory was introduced by Krassimir T Atanassov in [1]. In [4], Karunambigai M G and Parvathi R introduced intuitionistic fuzzy graph as a special case of Atanassov's IFG. In [6], these concepts had been applied to find the shortest path in networks using Dynamic Programming Problem approach. Further in [6], some important operatins on IFGs are defined and their properties are studied.

Regular fuzzy graphs and totally regular fuzzy graphs, degree, size and order of FGs were introduced by A.Nagoor Kani and K.Radha[10]. In this paper, constant IFGs and totally constant IFGs are introduced with suitable illustrations. Necessary and sufficient conditions for their equivalence is studied here.

The paper is organised as follows: Section 2 provides some preliminary concepts which are required for our study. Section 3 gives the definitions of constant

IFG and totally constant IFG. A characterization of constant IFG on a cycle is discussed in Section 4. Some important properties of constant IFG are dealt in Section 5. The paper is concluded in Section 6.

2 Preliminaries

In this section, some basic definitions relating to IFGs are given. [4] A *Minmax Intuitionistic Fuzzy Graph* (IFG) is of the form $G = (V, E)$, where
(i) $V = \{v_1, v_2, \ldots v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degrees of membership and non - membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$, for every $v_i \in V$ $(i = 1, 2, \ldots, n)$.
(ii) $E \subset V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that

$$\mu_2(v_i, v_j) \leq min[\mu_1(v_i), \mu_1(v_j)] \ldots (i)$$
$$\gamma_2(v_i, v_j) \leq max[\gamma_1(v_i), \gamma_1(v_j)] \ldots (ii)$$

and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.
Here the triple $(v_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and degree of non - membership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and degree of non - membership of the edge relation $e_{ij} = (v_i, v_j)$ on V.

Notation: Here after an IFG, G=(V,E) means a Minmax IFG G=(V,E).

Note 1. (i) When $\mu_{2ij} = \gamma_{2ij} = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise there exists an edge between v_i and v_j.
(ii)If one of the inequalities is not satisfied in (i) and (ii), then G is not an IFG.

Definition 2.1. [4] An IFG, $G = \langle V, E \rangle$ is said to be a semi-μ strong IFG if $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$ for every i and j.

Definition 2.2. An IFG, $G = \langle V, E \rangle$ is said to be a semi-γ strong IFG if $\gamma_{2ij} = max(\gamma_{1i}, \gamma_{1j})$ for every i and j.

Definition 2.3. An IFG, $G = \langle V, E \rangle$ is said to be strong IFG if $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = max(\gamma_{1i}, \gamma_{1j})$ for all $(v_i, v_j) \in E$.

Definition 2.4. An IFG, $G = \langle V, E \rangle$ is said to be a complete-μ strong IFG if $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} < max(\gamma_{1i}, \gamma_{1j})$ for all i and j.

Definition 2.5. An IFG, $G = \langle V, E \rangle$ is said to be a complete-γ strong IFG if $\mu_{2ij} < min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = max(\gamma_{1i}, \gamma_{1j})$ for all i and j.

Definition 2.6. An IFG, $G = \langle V, E \rangle$ is said to be a complete IFG if $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = max(\gamma_{1i}, \gamma_{1j})$ for every $v_i, v_j \in V$

Example 2.1. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_1, v_4), (v_3, v_4), (v_1, v_4), (v_4, v_2)\}$.

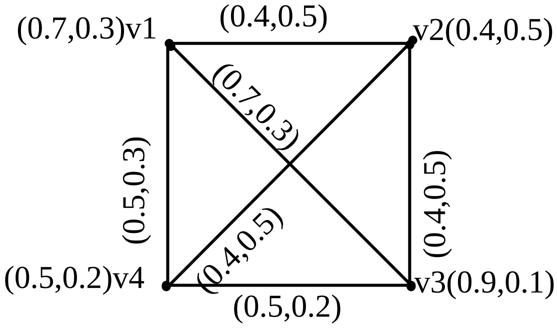

Figure 2.1: Complete Intuitionistic Fuzzy Graph G.

Definition 2.7. [10] Let $G = ((\mu_1, \gamma_1), (\mu_2, \gamma_2))$ be an IFG. The μ- degree of a vertex v_1 is

$$d_\mu(v_i) = \sum_{(v_i,v_j)\in E} \mu_2(v_i, v_j)$$

The γ- degree of a vertex v_1 is

$$d_\gamma(v_i) = \sum_{(v_i,v_j)\in E} \gamma_2(v_i, v_j)$$

The degree of a vertex is

$$d(v_i) = \left[\sum_{v_i,v_j\in E} (\mu_2(v_i, v_j)), \sum_{(v_i,v_j)\in E} (\gamma_2(v_i, v_j)) \right] \text{ and } \mu_2(v_i, v_j) = \gamma_2(v_i, v_j) = 0 \text{ for } v_iv_j \notin E.$$

Example 2.2. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_4, v_3), (v_4, v_1)\}$

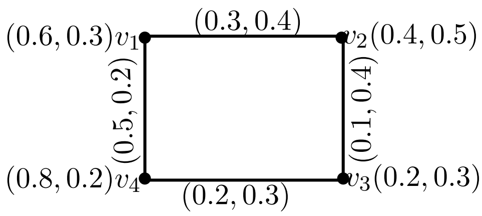

Figure 2.2: Degree of an Intuitionistic Fuzzy Graph G.

In this example, the degree of v_1 is $(0.8, 0.6)$. The degree of v_2 is $(0.4, 0.8)$. The degree of v_3 is $(0.3, 0.7)$. The degree of v_4 is $(0.7, 0.5)$.

Definition 2.8. The minimum μ-degree is $\delta_\mu(G) = \wedge \{d_\mu(v_i)/v_i \in V\}$
The minimum γ-degree is $\delta_\gamma(G) = \wedge \{d_\gamma(v_i)/v_i \in V\}$
The minimum degree of G is $\delta(G) = \wedge \{d_\mu(v_i), d_\gamma(v_i)/v_i \in V\}$
The maximum μ-degree is $\Delta_\mu(G) = \vee \{d_\mu(v_i)/v_i \in V\}$
The maximum γ-degree is $\Delta_\gamma(G) = \vee \{d_\gamma(v_i)/v_i \in V\}$
The maximum degree of G is $\Delta(G) = \vee \{d_\mu(v_i), d_\gamma(v_i)/v_i \in V\}$

Definition 2.9. [11] Let $G =< V, E >$ be an IFG. Then the order of G is defined to be $O(G) = (O_\mu(G), O_\gamma(G))$ where $O_\mu(G) = \sum_{v\in V} \mu_1(v)$ and $O_\gamma(G) = \sum_{v\in V} \gamma_1(v)$.

Definition 2.10. [11] The size of G is defined to be $S(G) = (S_\mu(G), S_\gamma(G))$ where $S_\mu(G) = \sum_{v_i\neq v_j} \mu_2(v_i, v_j)$ and $S_\gamma(G) = \sum_{v_i\neq v_j} \gamma_2(v_i, v_j)$.

Definition 2.11. [4] If $v_i, v_j \in V \subseteq G$, the $\mu-$ strength of connectedness between v_i and v_j is $\mu_2^\infty(v_i, v_j) = \sup\{\mu_2^k(v_i, v_j) \mid k = 1, 2, \ldots, n\}$ and $\gamma-$strength of connectedness between v_i and v_j is $\gamma_2^\infty(v_i, v_j) = \inf\{\gamma_2^k(v_i, v_j) \mid k = 1, 2, \ldots, n\}$.
If u, v are connected by means of paths of length k then $\mu_2^k(u, v)$ is defined as $\sup\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2)\wedge, \mu_2(v_2, v_3) \ldots \wedge \mu_2(v_{k-1}, v) \mid (u, v_1, v_2 \ldots v_{k-1}, v \in V)\}$ and $\gamma_2^k(u, v)$ is defined as $\inf\{\gamma_2(u, v_1) \vee \gamma_2(v_1, v_2)\vee, \gamma_2(v_2, v_3) \ldots \vee \gamma_2(v_{k-1}, v) \mid (u, v_1, v_2 \ldots v_{k-1}, v \in V)\}$.

Definition 2.12. [4](v_i, v_j) is said to be a bridge in G, if either $\mu_{2xy}^{'\infty} < \mu_{2xy}^{\infty}$ and $\gamma_{2xy}^{'\infty} \geq \gamma_{2xy}^{\infty}$ or $\mu_{2xy}^{'\infty} \leq \mu_{2xy}^{\infty}$ and $\gamma_{2xy}^{'\infty} > \gamma_{2xy}^{\infty}$, for some $v_x, v_y \in V$.
In other words, deleting an edge (v_i, v_j) reduces the strength of connectedness between some pair of vertices (or) (v_i, v_j) is a bridge if there exist vertices v_x, v_y such that (v_i, v_j) is an edge of every strongest path from v_x to v_y.

Definition 2.13. [4] A vertex v_i is said to be a *cut-vertex* in G if deleting a vertex v_i reduces the strength of connectedness between some pair of vertices or v_i is a cut vertex if and only if there exists v_x, v_y such that v_i is a vertex of every strongest parth from v_x to v_y. In other words, $\mu_{2xy}^{'\infty} \leq \mu_{2xy}$ and $\gamma_{2xy}^{'\infty} < \gamma_{2xy}$ or $\mu_{2xy}^{'\infty} < \mu_{2xy}$ and $\gamma_{2xy}^{'\infty} \leq \gamma_{2xy}$, for some $v_x, v_y \in V$.

3 Constant IFG

Definition 3.1. Let Let $G : [(\mu_{1i}, \gamma_{1i}), (\mu_{2ij}, \gamma_{2ij})]$ be an IFG on $G^* : (V, E)$. If $d_\mu(v_i) = k_i$ and $d_\gamma(v_j) = k_j$ for all $v_i, v_j \in V$ i.e the graph is called as (k_i, k_j)-IFG (or) constant IFG of degree (k_i, k_j)

Example 3.1. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4\}$,

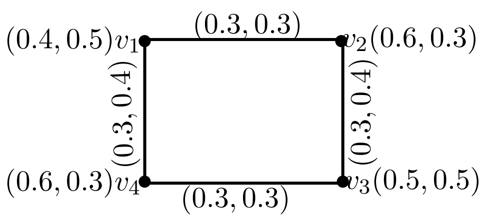

Figure 3.1: Constant IFG G of degree (k_i, k_j).

In this example, the degree of v_1,v_2,v_3,v_4 is $(0.6, 0.7)$.

Remark 3.1. G is a (k_i, k_j)-constant IFG iff $\delta = \Delta = k$, where $k = k_i + k_j$.

Example 3.2. The following example shows that a complete IFG need not be a constant IFG. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4\}$, . Refer figure 3.2.

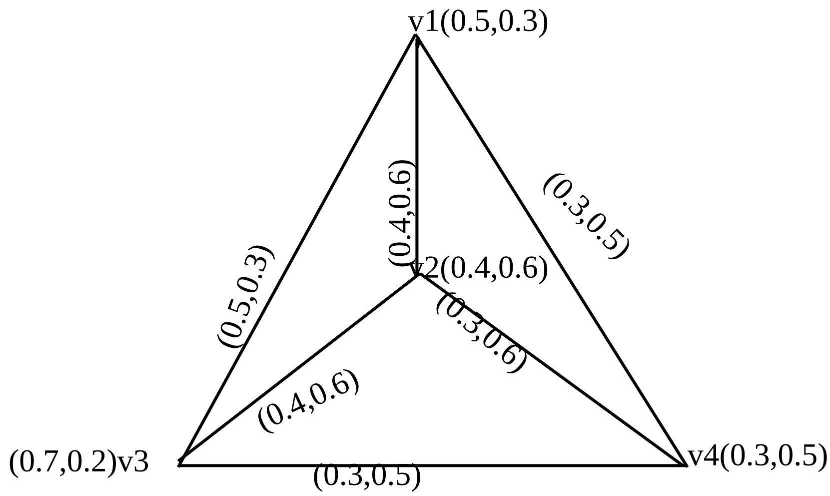

Figure 3.2: G is complete, but not constant IFG.

Definition 3.2. Let G be an IFG. The total degree of a vertex $v \in V$ is defined as

$$td(v) = \left[\sum_{v_1 v_2 \in E} d_{\mu_2}(v) + \mu_1(v), \sum_{v_1 v_2 \in E} d_{\gamma_2}(v) + \gamma_1(v) \right]$$

If each vertex of G has the same total degree (r_1, r_2), then G is said to be an IFG of total degree (r_1, r_2) or a (r_1, r_2)-totally constant IFG.

Example 3.3. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$. Refer figure 3.3.

Theorem 3.1. Let G be an IFG. Then (μ_1, γ_1) is a constant function iff the following are equivalent.

(i) G is a constant IFG.

(ii) G is a totally constant IFG.

Proof.
Suppose that (μ_1, γ_1) is a constant fuction. Let $\mu_1(v_i) = c_1$ and $\gamma_1(v_i) = c_2$ for all $v_i \in V$ where c_1 and c_2 are constants. Assume that G is a (k_1, k_2)-constant IFG. Then, $d_\mu(v_i) = k_1$ $d_\gamma(v_i) = k_2$ for all $v \in V$.So, $td_\mu(v_i) = d_\mu(v_i) + \mu_1(v_i)$, $td_\gamma(v_i) = d_\gamma(v_i) + \gamma_1(v_i)$ for all $v_i \in V$, $td_\mu(v_i) = k_1 + c_1$, $td_\gamma(v_i) = k_2 + c_2$ for all $v_i \in V$. Hence G is a totally constant IFG. Thus (i) ⇒(ii) is proved.
Now, suppose that G is a (r_1, r_2)-totally constant IFG. Then, $td_\mu(v_i) = r_1$, $td_\gamma(v_i) = r_2$ for all $v \in V$. $d_\mu(v_i) + \mu_1(v_i) = r_1$, $d_\mu(v_i) + c_1 = r_1$, $d_\mu(v_i) = r_1 - c_1$. Similarly,$d_\gamma(v_i) + \gamma_1(v_i) = r_2$,$d_\gamma(v_i) + c_2 = r_2$,$d_\gamma(v_i) = r_2 - c_2$. So, G is a constant IFG. Thus (ii) ⇒(i) is proved. Hence (i) and (ii) are equivalent.
Conversely, assume that (i) and (ii) are equivalent ie G is a constant IFG iff G is a totally constant IFG.
Suppose (μ_1, γ_1) is not a constant function.Then, $\mu_1(v_1) \neq \mu_1(v_2)$, $\gamma_1(v_1) \neq \gamma_1(v_2)$ for atleast one pair of vertices $v_1, v_2 \in V$.Let G be a (k_1, k_2)-constant IFG. Then, $d_\mu(v_1) = d_\mu(v_2) = k_1$,

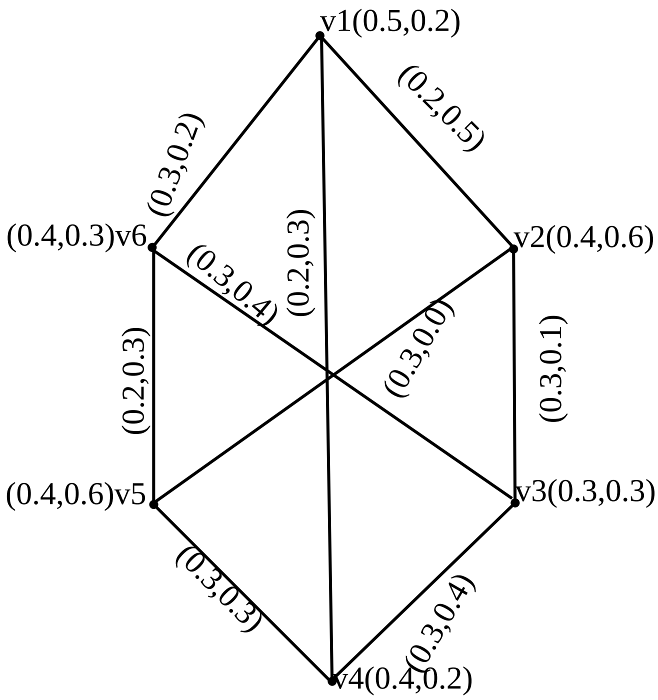

Figure 3.3: Totally constant IFG G with total degree (r_1, r_2).

$d_\gamma(v_1) = d_\gamma(v_2) = k_2$. So, $td_\mu(v_1) = d_\mu(v_1) + \mu_1(v_1) = k_1 + \mu_1(v_1)$ and $td_\mu(v_2) = k_1 + \mu_1(v_2)$. Similarly,$td_\gamma(v_1) = k_2 + \gamma_1(v_1)$,$td_\gamma(v_2) = k_2 + \gamma_1(v_2)$. Since,$\mu_1(v_1) \neq \mu_1(v_2)$,$\gamma_1(v_1) \neq \gamma_1(v_2)$. We have,$td_\mu(v_1) \neq td_\mu(v_2)$, $td_\gamma(v_1) \neq td_\gamma(v_2)$.So, G is not totally constant IFG which is a contradiction to our assumption.
Now, let G be a totally constant IFG.Then, $td_\mu(v_1) = td_\mu(v_2)$, $d_\mu(v_1) + \mu(v_1) = d_\mu(v_2) + \mu(v_2)$,$d_\mu(v_1) - d_\mu(v_2) = \mu(v_2) - \mu(v_1)(ie \neq 0)$, $d_\mu(v_1) \neq d_\mu(v_2)$. Similarly,$d_\gamma(v_1) \neq d_\gamma(v_2)$. So, G is not constant which is a contradiction to our assumption. Hence (μ_1, γ_1) is a constant function.

Example 3.4. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_4, v_3), (v_4, v_1)\}$

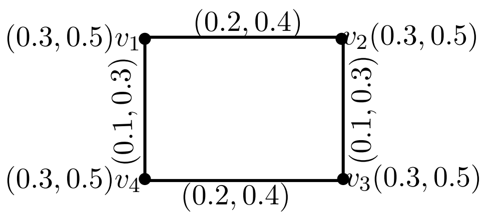

Figure 3.4: (μ_1, γ_1) is a constant fuction, then G is constant and totally constant.

Theorem 3.2. If an IFG is both constant and totally constant, then (μ_1, γ_1) is a constant function.
Proof.
Let G be a (k_1, k_2)-constant and (r_1, r_2)-totally constant IFG. So, $d_\mu(v_1) = k_1$, $d_\gamma(v_1) = k_2$,

for $v_1 \in V$ and $td_\mu(v_1) = r_1$, $td_\gamma(v_1) = r_2$, for all $v \in V$. $td_\mu(v_1) = r_1$, for $v \in V$, $d_\mu(v_1) + \mu_1(v_1) = r_1$, for all $v \in V$. $k_1 + \mu_1(v) = r_1$, for all $v \in V$. $\mu_1(v) = (r_1 - k_1)$, for all $v \in V$. Hence $\mu_1(v_1)$ is a constant function. Similarly, $\gamma_1(v_1) = (r_2 - k_2)$, for all $v \in V$.

Remark 3.2. Converse of Theorem 3.2 need not be true.

Example 3.5. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_4, v_3), (v_4, v_1)\}$

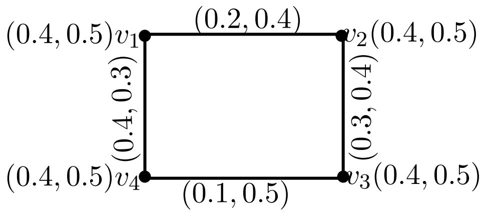

Figure 3.5: (μ_1, γ_1) is a constant fuction, but G is neither constant IFG nor totally constant IFG.

4 A Characterization of constant IFG on a cycle

Theorem 4.1. Let G be an IFG where crisp graph G is an odd cycle. Then G is constant IFG iff (μ_2, γ_2) is a constant function.

Proof.

If $(\mu_2, \gamma 2)$ is a constant function say $\mu_2 = c_1$ and $\gamma_2 = c_2$ for all $(v_i v_j) \in E$, then,$d_\mu(v_i) = 2c_1$ and $d_\gamma(v_i) = 2c_2$ for every $v_i \in V$, So G is a constant IFG.

Conversely, suppose that G is a (k_1, k_2)-regular IFG. Let $e_1, e_2, ...e_{2n+1}$ be the edges of G in that order. Let $\mu_2(e_1) = c_1$, $\mu_2(e_2) = k_1 - c_1$, $\mu_2(e_3) = k_1 - (k_1 - c_1) = c_1$, $\mu_2(e_4) = k_1 - c_1$ and so on. Similarly, $\gamma_2(e_1) = c_2$, $\gamma_2(e_2) = k_2 - c_2$, $\gamma_2(e_3) = k_2 - (k_2 - c_2) = c_2$, $\gamma_2(e_4) = k_2 - c_2$ and so on.

Therefore

$$\mu_2(e_i) = \begin{cases} c_1, \text{if } i \text{ is odd} \\ k_1 - c_1, \text{if } i \text{ is even} \end{cases} .$$

Hence $\mu_2(e_1) = \mu(e_{2n+1}) = c_1$. So, if e_1 and e_{2n+1} incident at a vertex v_1, then $d_\mu(v_1) = k_1$, $d(e_1) + d(e_{n+1}) = k_1$,$c_1 + c_1 = k_1$, $2c_1 = k_1$, $c_1 = \frac{k_1}{2}$.

Remark 4.1. The above theorem does not hold for totally constant IFG.

Example 4.1. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3\}$,and $E = \{(v_1, v_2), (v_2, v_3), (v_1, v_3)\}$.

Theorem 4.2. Let G be an IFG where crisp graph G is an even cycle. Then G is constant IFG iff either (μ_2, γ_2) is a constant function or alternate edges have same membershp values and non-membership values.

Proof.

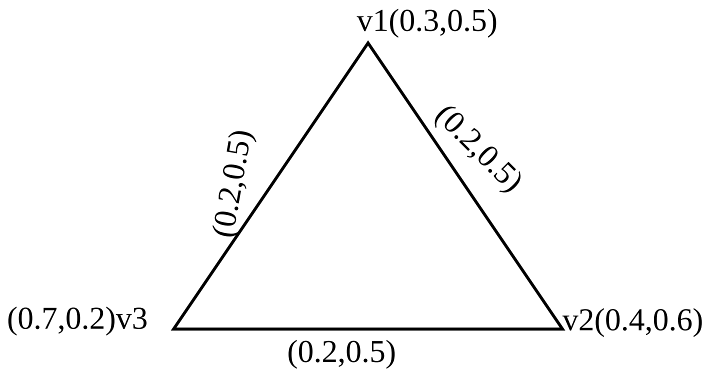

Figure 4.1: (μ_2, γ_2) is a constant fuction, but not totally constant IFG.

If either (μ_2, γ_2) is a constant function or alternate edges have same membershp values and non-membership values, then G is a constant IFG. Conversely, suppose G is a (k_1, k_2) constant IFG. Let $e_1, e_2, ...e_{2n}$ be the edges of even cycle G^* in that order. Proceeding as in the above theorem,

$$\mu_2(e_i) = \begin{cases} c_1, \text{if } i \text{ is odd} \\ k_1 - c_1, \text{if } i \text{ is even} \end{cases}.$$

Similarly,

$$\gamma_2(e_i) = \begin{cases} c_2, \text{if } i \text{ is odd} \\ k_2 - c_2, \text{if } i \text{ is even.} \end{cases}.$$

If $c_1 = k_1 - c_1$, the (μ_2, γ_2) is a constant function. If $c_1 \neq k_1 - c_1$, then alternate edges have same membershp values and non-membership values.

Remark 4.2. The above theorem does not hold for totally constant IFG.

Example 4.2. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_4, v_3), (v_4, v_1)\}$

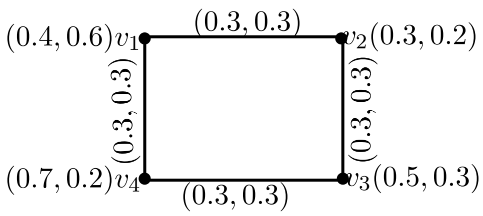

Figure 4.2: (μ_2, γ_2) is a constant fuction, then G is a constant IFG. But it is not totally constant IFG.

5 Properties of constant IFG

Theorem 5.1. The size of a (k_1, k_2) constant IFG is $\left(\frac{pk_1}{2}, \frac{pk_2}{2}\right)$ where $p = |V|$.
Proof

The size of G is $S(G) = \left[\sum_{v_1v_2 \in E} \mu_2(v_1v_2), \sum_{v_1v_2 \in E} \gamma_2(v_1v_2)\right]$, since G is (k_1, k_2) regular IFG, $d_\mu(v) = k_1, d_\gamma(v) = k_2$ for all $v \in V$. We have, $\sum_{v \in E} d_G(v) = 2\left[\sum \mu_2(v_1v_2), \sum \gamma_2(v_1v_2)\right]$= 2 S(G). So, $2S(G) = \sum_{v \in E} d_G(v)$, $\left[\sum_{v \in V} k_1, \sum_{v \in V} k2\right]$, $[pk_1, pk_2]$, $S(G) = \left(\frac{pk_1}{2}, \frac{pk_2}{2}\right)$.

Theorem 5.2. If G is a $(k, k^{'})$-totally constant IFG, then 2S(G) + O(G) = $(pk, pk^{'})$ where $p = |V|$.

Proof

Since G is a $(k, k^{'})$ totally constnat IFG, $k = td_\mu(v) = d_\mu(v) + \mu_1(v)$ and $k^{'} = td_\gamma(v) = d_\gamma(v) + \gamma_1(v)$, for all $v \in V$. Therefore, $\sum_{v \in V} k = \sum_{v \in V} d_G(v) + \sum_{v_1 \in V} \mu_1(v_1)$, $\sum_{v \in V} k^{'} = \sum_{v \in V} d_G(v) + \sum_{v_1 \in V} \gamma_1(v_1)$.

$pk^{'} = 2S_\mu(G) + O_\mu(G)$. $pk = 2S_\gamma(G) + O_\gamma(G)$ and thus $pk + pk^{'} = 2\left[S_\mu(G) + S_\gamma(G)\right] + O_\mu(G) + O_\gamma(G)$.

Corollary 5.3. *If G is a (k_1, k_2) constant and a (r_1, r_2)-totally constant IFG, then $O_\mu(G) = p(r_1 - k_1)$, $O_\gamma(G) = p(r_2 - k_2)$.*

From the above theorem, $2S(G) = \left[\frac{pk_1}{2}\right]$ or $2S(G) = pk_1$, $2S(G) = \left[\frac{pk_2}{2}\right]$ or $2S(G) = pk_2$. $2S_\mu(G) + O_\mu(G) = pr_1$, $2S_\gamma(G) + O_\gamma(G) = pr_2$. So, $O_\mu(G) = pr_1 - 2S_\mu(G)$,$pr_1 - pk_1$, $p(r_1 - k_1)$. Similarly, $O_\gamma(G) = p(r_2 - k_2)$, $O(G) = O_\mu(G) + O_\gamma(G)$,$\Rightarrow = pr_1 - pk_1 + pr_2 - pk_2$, $\Rightarrow p\left[r_1 - k_1 + r_2 - k_2\right]$.

Theorem 5.4. A constant IFG on an odd cycle does not have a IF bridge. Hence it does not have an IF cut vertex.

Proof

Assume that G is a constant IFG on an odd cycle of crisp graph G. Then (μ_2, γ_2) is a constant function. So removal of any edge does not reduce the strength of connectedness between any pair of vertices. Hence G has no IF bridge. Hence by definition 2.12, G does not have a IF cut vertex.

Remark 5.1. The above theorem does not hold for totally constant IFG.

Example 5.1. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3\}$,and $E = \{(v_1, v_2), (v_2, v_3), (v_1, v_3)\}$. Refer figure 5.1.

Theorem 5.5. Let G be a constant IFG on an even cycle of a crisp graph G. Then either G does not have an IF bridge or it has q/2 IF bridges where q =$|E|$. Also G does not have an IF cutvertex.

Proof

Assume that G is a constant IFG on an even cycle of crisp graph G. Then by Theorem 4.2, either (μ_2, γ_2) is a constant function or alternate edges have same membership values and non-membership values.

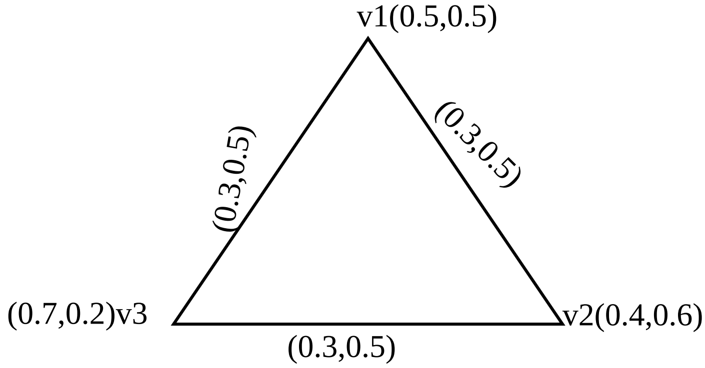

Figure 5.1: (μ_2, γ_2) is a constant fuction, but neither an IF bridge nor an IF cutvertex.

Case 1: (μ_2, γ_2) is a constant function. Then the removal of any edge does not reduce the strength of connectedness between any pair of vertices. So G does ot have a IF bridge and hence does not have an IF cut vertex.

Case 2: Alternate edges have same membership values and non-membership values. Then by Theorem 2.11, edges with greater membership values and smaller non-membership values are the IF bridges of G. There are q/2 such edges where q = $|E|$. Hence G has q/2 IF bridges. But then no vertex is a common vertex of two IF bridges. So G does not have an IF cut vertex.

Remark 5.2. The above theorem does not hold for totally constant IFG.

Example 5.2. Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2),$ $(v_2, v_3), (v_4, v_3), (v_4, v_1)\}$

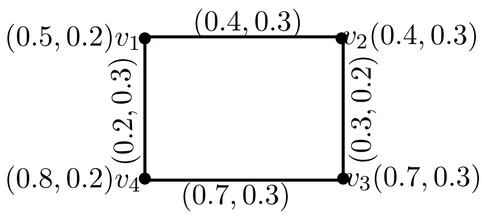

Figure 5.2:

6 Conclusion

The concept of constant IFG in graphs is very rich both in theoretical developments and applications. In this paper, we introduced constant IFG and totally constant IFG and some interesting properties of these new concepts are proved.

7 Acknowledgement

The authors would like to thank the University Grants Commission, New Delhi, India for its financial support to UGC Research Award No. F. 30 - 1/ 2009 (SA - II) dated 2^{nd} July 2009 and to Minor Research Project F. MRP - 3365/ 10 (MRP/UGC - SERO) dated 23^{rd} July 2010 and the Bilateral Scientific Cooperation Research Programme INT/Bulgaria/B-2/08 and BIn - 02 - 09 supported by Department of Science and Technology, NewDelhi, India, Ministry of Education and Science, Sofia, Bulgaria.

References

[1] K. Atanassov, *Intuitionistic Fuzzy Sets: Theory and Applications*, Physica-Verlag, New York (1999).

[2] J. A. Bondy and U. S. R. Murthy, *Graph Theory with Applications*, American Elesevier Publishing Co., New York (1976).

[3] E. J. Cockayne and S. T. Hedetnieme,Towards a Theory of Domination in Graphs, *Networks* **7** (1977), 247-261.

[4] M. G. Karunambigai and R. Parvathi, *Intuitionistic Fuzzy Graphs*, Proceedings of 9th Fuzzy Days International Conference on Computational Intelligence, *Advances in soft computing: Computational Intelligence,Theory and Applications*, Springer-Verlag, **20** (2006), 139-150.

[5] Mordeson N. John and Nair S. Premchand, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica-Verlag, New York (2000).

[6] R. Parvathi, M. G. Karunambigai and K. Atanassov, *Operations on Intuitionistic Fuzzy Graphs*, Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), August 2009, 1396-1401.

[7] A. Somasundram and S. Somasundaram, Domination in Fuzzy Graphs-I, *Pattern Recognition Letters*, **19** (1998), 787-791.

[8] Yan Luo and Changrui Yu, A Fuzzy Optimization Method for Multi-Criteria Decision Making Problem Based on the Inclusion Degrees of Intuitionistic Fuzzy Sets, *Journal of Information and Computing Science*, **3**(2) (2008), 146-152.

[9] L. A. Zadeh, Fuzzy Sets, *Information Sciences*, **8** (1965), 338-353.

[10] A.Nagoor Gani and K.Radha, On Regular Fuzzy Graphs, *Journal of Physical Sciences*, **(**12) (2008), 33-140.

[11] A.Nagoor Gani and S.Shajitha Begum, Degree,Order and Size in Intuitionistic Fuzzy Graphs, *International Journal of Algorithms, Computing and Mathematics*, **(**3)3 (2010).