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Fixed Point Results Related To Compatible Mappings In Random Fuzzy Metric Spaces

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Abstract-- In the present paper we find some common fixed point theorems in random fuzzy metric spaces related to compatible mappings. Our results are generalized form of many known results.

Keywords: **Random Fuzzy metric spaces, Common fixed point, compatible mappings**

AMS subject Classification: 47H10

I. Introduction And Preliminaries

Probabilistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing probabilistic models in the applied sciences. The study of fixed points of random operators forms a central topic in this area. The Prague school of probabilistic initiated its study in the 1950. However, the research in this area flourished after the publication of the survey article of Bharucha-Reid [17]. Since then many interesting random fixed point results and several applications have appeared in the literature; for example the work of Beg and Shahazad [14-16], Lin [23], O'Regan [24].

In recent years, the study of random fixed points have attracted much attention. In particular ,random iteration schemes leading to random fixed point of random operators have been discussed in [18-20].

Weak compatibility is one of the weaker forms of the commuting mappings. Many researchers use this concept to prove the existence of unique common fixed point in fuzzy metric space. Al-Thagafi and Shahzad [2] introduced the concept of occasionally weakly compatible (owc) and weaken the concept of nontrivial weakly compatible maps.

Recently, R.K. Bist and R. P. Pnat [3] criticize the concept of owc as follows "Under contractive conditions the existence of a common fixed point and occasional weak compatibility are equivalent conditions, and consequently, proving existence of fixed points by assuming owc is equivalent to proving the existence of fixed points by assuming the existence of fixed points".

Therefore use of owc is a redundancy for fixed pint theorems under contractive conditions.

This redundancy can be also seen in recent result of A. Jain et.al. [5]. To remove this we used faintly compatible mapping in our paper which is weaker than weak compatibility or semi compatibility. Faintly compatible maps introduced by Bisht and Shahzad [4] as an improvement of conditionally compatible maps, Pant and Bisht [8], introduced the concept of conditional compatible maps. This gives the existence of a common fixed point or multiple fixed point or coincidence points under contractive and non-contractive conditions.

The aim of this chapter is remove redundancy of results of A. Jain et.al. [5], and prove the existence of common fixed point using faintly compatible maps in random fuzzy metric space motivated by Wadhwa et.al.[11-13]

In this section, we recall some definitions and useful results which are already in the literature .Throughout this Chapter (Ω, Σ) denotes a measurable space. $\xi : \Omega \to X$ is a measurable selector. X is any non empty set. $\star$ is continuous t-norm, **M** is a fuzzy set in $X^2 \times [0, \infty)$ .A binary operation $*$:[0,1]x[0,1]→[0,1] is called a continuous t-norm if ([0,1],$*$) is an abelian Topological monodies with unit 1 such that $a * b \geq c * d$ whenever

$a \geq c$ and $b \geq d$, For all a, b, c, d, $\in [0, 1]$

Example of t-norm are $a * b = a\,b$ and $a * b = \min\{a, b\}$

Definition2.1. (a): The 3-tuple (X, M, Ω $*$) is called a **Random fuzzy metric**

space, if X is an arbitrary set,* is a continuous t-norm and M is a fuzzy set in X^2 x $[0,\infty)$ satisfying the following conditions: for all

ξx, ξy, $\xi z \in X$ and s, t > 0,

$$(RFM-1): M\left(\xi x, \xi y, 0\right) = 0$$

$$(RFM-2): M\left(\xi x, \xi y, t\right) = 1, \forall t \succ \quad : y$$

$$(RFM-3): M\left(\xi x, \xi y, t\right) = M(\xi y, \xi x, t)$$

$$(RFM-4): M\left(\xi x, \xi z, t+s\right) \geq M(\xi x, \xi y, t) * M(\xi z, \xi y, s)$$

$$(RFM-5): M(\xi x, \xi y, \xi a): [0,1) \rightarrow [0,1] \, is \, left \, continuous$$

In what follows, (X, M, Ω ,*) will denote a random fuzzy metric space.

Note that M (ξ x, ξ y, t) can be thought of as the degree of nearness between ξ x and ξ y with respect to t. We identify ξ x = ξ y with M (ξ x, ξ y, t) = 1 for all t > 0 and M (ξ x, ξ y, t) = 0 with ∞. In the following example, we know that every metric induces a fuzzy metric.

Example Let (X, d) be a metric space.

Define a ∗b = a b, or ab =min {a, b}) and for all x, y, ∈ X and t > 0,

$$M\left(\xi x, \xi y, t\right) = \frac{t}{t + d\left(\xi x, \xi y\right)}$$

Then (X, M, Ω , *) is a fuzzy metric space. We call this random fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition2.1. (b): Let (X, M, Ω , *) is a random fuzzy metric space.

(i) A sequence { ξ x_n} in X is said to be convergent to a point ξ x ∈ X,

$$\lim_{n\to\infty} M(\xi x_n, \xi x, t) = 1$$

(ii) A sequence { ξ x_n} in X is called a Cauchy sequence if

$$\lim_{n\to\infty} M(\xi x_{n+p}, \xi x_n, t) = 1, \forall t \succ \quad \succ$$

(iii) A random fuzzy metric space in which every Cauchy sequence is convergent is said to be Complete.

Let (X.M,*) is a fuzzy metric space with the following condition.

(RFM-6) $\lim_{t\to\infty} M(\xi x, \xi y, t) = 1, \forall \xi x, \xi y \ \varepsilon \ X$

Definition2.1. (c): A function M is continuous in random fuzzy metric space iff whenever

$$\xi x_n \rightarrow \xi x, \xi y_n \rightarrow \xi y \Rightarrow \lim_{n\to\infty} M(\xi x_n, \xi y_n, t) \rightarrow M(\xi x, \xi y, t)$$

Definition 2.1. (d): Two mappings A and S on random fuzzy metric space X are weakly commuting iff

M (AS ξ u, SA ξ u, t) ≥ M (A ξ u, S ξ u, t)

Definition 2.1. (e): A pair of self-maps (A, S) on a random fuzzy metric space

(X, M, Ω ∗)is said to be

(i) Non-compatible: if (A, S) is not compatible, i.e., if there exists a sequence {ξx_n} in X such that $\lim_{n\to\infty} A\xi x_n = \lim_{n\to\infty} S\xi x_n = \xi x$, for some $\xi x \in X$, and $\lim_{n\to\infty} M(AS\xi x_n, SA\xi x_n, t) \neq 1$ or non-existent ∀t > 0.

(ii) Conditionally compatible [8]: if whenever the set of sequences {ξx_n} satisfying $\lim_{n\to\infty} A\xi x_n = \lim_{n\to\infty} S\xi x_n$, is non-empty, there exists a sequence {ξz_n} in X such that $\lim_{n\to\infty} A\xi z_n = \lim_{n\to\infty} S\xi z_n = \xi t$, for some $\xi t \in X$ and

$\lim_{n\to\infty} M(AS\xi x_n, SA\xi x_n, t) = 1$ for all t > 0.

(iii) Faintly compatible [4]: if (A, S) is conditionally compatible and A and S commute on a non-empty subset of the set of coincidence points, whenever the set of coincidence points is nonempty.

(iv) Satisfy the property (E.A.) [1]: if there exists a sequence {x_n} in X such that $\lim_{n\to\infty} A\xi x_n = \lim_{n\to\infty} S\xi x_n = \xi x$, for some $\xi x \in X$.

(v) Sub Sequentially continuous [11]: iff there exists a sequence {ξx_n} in X such that $\lim_{n\to\infty} A\xi x_n = \lim_{n\to\infty} S\xi x_n = \xi x$, $\xi x \in X$ and satisfy $\lim_{n\to\infty} AS\xi x_n = A\xi x$, $\lim_{n\to\infty} SA\xi x_n = S\xi x$.

Note that, compatibility, non- compatibility and faint compatibility are independent concepts. Faintly compatibility is applicable for mappings that satisfy contractive and non contractive conditions.

(vi) Semi-compatible [5]: if $\lim_{n\to\infty} AS\xi x_n = S\xi x$, whenever is a sequence such that $\lim_{n\to\infty} A\xi x_n = \lim_{n\to\infty} S\xi x_n = \xi x \in X$.

Lemma 2.1(f)[Modified form of 6]: Let (X, M, *) be a random fuzzy metric space and for all $\xi x, \xi y \in X$, t > 0 and if there exists a constant k ∈(0, 1) such that

M(ξx, ξy, kt) ≥ M(ξx, ξy, t) then ξx = ξy.

Now we write a modified basic result for random fuzzy metric spaces motivated by

A. Jain et.al. [5], as follows

Theorem2.2[5]: Let A, B, S and T be self mappings of a complete random fuzzy metric space (X, M, Ω , *). Suppose that they satisfy the following conditions:

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(2.2.1) $A(X)\subset T(X)$, $B(X)\subset S(X)$;

(2.2.2) the pair (A, S) is semi-compatible and (B, T) is occasionally weakly compatible;

(2.2.3) there exists $k\in(0, 1)$ such that $\forall \xi x, \xi y\in X$ and $t>0$,

$M(A\xi x, B\xi y, kt) \geq \min\{M(B\xi y, T\xi y, t), M(S\xi x, T\xi y, t), M(A\xi x, S\xi x, t)\}$.

Then A, B, S and T have a unique fixed point in X.

Now we prove some common fixed point theorems for pair of faintly compatible mappings in random fuzzy metric spaces.

2.3 Main Results:

Theorem 2.3.1: Let (X, M, Ω,*) be a random fuzzy metric space and let A, B, S, T, P and Q be self mappings of X such that

(2.3.1.1) the pairs (A, SP) and (B, TQ) are non compatible, sub sequentially continuous faintly compatible;

(2.3.1.2) Pair (A, P), (S, P), (B, Q), (T, Q) are commuting;

(2.3.1.3) there exists $k\in(0,1)$ such that $\forall$ $\xi x, \xi y\in X$ and $t >$

$$0, M(A\xi x, B\xi y, kt)\geq \min\left\{\frac{aM(SP\xi x,A\xi x,t)+bM(B\xi y,TQ\xi y,t)}{a+b}, \frac{cM(SP\xi x,B\xi y,t)+dM(SP\xi x,TQ\xi y,t)}{cM(B\xi y,TQ\xi y,t)+d}, \frac{eM(A\xi x,TQ\xi y,t)+fM(SP\xi x,A\xi x,t)}{eM(SP\xi x,TQ\xi y,t)+f}\right\}$$

where $a,b,c,d,e,f\geq 0$ with a&b, c&d and e&f cannot be simultaneously 0.

Then A, B, S, T, P and Q have a unique random common fixed point in X.

Proof: Non compatibility of (A, SP) and (B, TQ) implies that there exist sequences $\{\xi x_n\}$ and $\{\xi y_n\}$ in X such that

$\lim_{n\to\infty}A\xi x_n=\lim_{n\to\infty}(SP)\,\xi x_n=t_1$ for some $t_1\in X$, and $M(A(SP)\,\xi x_n,(SP)A\xi x_n,t)\neq 1$

or nonexistent $\forall t > 0$; Also

$\lim_{n\to\infty}B\xi x_n=\lim_{n\to\infty}(TQ)\,\xi x_n=t_2$ for some $t_2\in X$, and $M(B(TQ)\,\xi x_n,(TQ)B\xi x_n,t)\neq 1$ or nonexistent $\forall$ $t>0$.

Since pairs (A, SP) and (B, TQ) are faintly compatible therefore conditionally compatibility of (A, SP) and (B, TQ) implies that there exist sequences $\{\xi z_n\}$ and $\{\xi z_n'\}$ in X satisfying

$\lim_{n\to\infty}A\xi z_n = \lim_{n\to\infty}(SP\xi)z_n = \xi u$ for some $\xi u\in X$, such that

$M(A(SP)\,\xi z_n, (SP)A\xi z_n, t)=1$;

Also $\lim_{n\to\infty}B\xi z_n' = \lim_{n\to\infty}(TQ)\,\xi z_n' = \xi v$ for some $\xi v\in X$, such that $M(B(TQ)\,\xi z_n',(TQ)B\xi z_n', t)=1$.

As the pairs (A, SP) and (B, TQ) are sub sequentially continuous, we get

$\lim_{n\to\infty}A(SP)\,\xi z_n = A\xi u$, $\lim_{n\to\infty}(SP)A\xi z_n = (SP)\,\xi u$

and so $A\xi u = (SP)\,\xi u$ i.e. (u is coincidence point of A and (SP));

Also $\lim_{n\to\infty}B(TQ)\,\xi z_n' = B\xi v$, $\lim_{n\to\infty}(TQ)B\xi z_n' = (TQ)\,\xi v$

and so $B\xi v = (TQ)\,\xi v$ i.e. (ξv is coincidence point of B and (TQ)).

Since pairs (A, SP) and (B, TQ) are faintly compatible, we get

$A(SP)\,\xi u=(SP)A\xi u$ & so
$AA\xi u=A(SP)\,\xi u=(SP)A\xi u=(SP)(SP)\,\xi u$;

and Also $B(TQ)\,\xi v=(TQ)B\xi v$ & so
$BB\xi v=B(TQ)\,\xi v=(TQ)B\xi v=(TQ)(TQ)\,\xi v$.

Now we show that $A\xi u=B\xi v$, $AA\xi u= A\xi u$, $BB\xi v=B\xi v$, $PA\xi u=A\xi u$ and $QA\xi u=A\xi u$.

By taking $\xi x=\xi u$ and $\xi y=\xi v$ in (5.3.1.3),

$$M(A\xi u, B\xi v, kt)\geq \min\left\{\frac{aM(SP\xi u,A\xi u,t)+bM(B\xi v,TQ\xi v,t)}{a+b}, \frac{cM(SP\xi u,B\xi v,t)+dM(SP\xi u,TQ\xi v,t)}{cM(B\xi v,TQ\xi v,t)+d}, \frac{eM(A\xi u,TQ\xi v,t)+fM(SP\xi u,A\xi u,t)}{eM(SP\xi u,TQ\xi v,t)+f}\right\};$$

$$M(A\xi u, B\xi v, kt)\geq \min\left\{\frac{aM(A\xi u,A\xi u,t)+bM(B\xi v,B\xi v,t)}{a+b}, \frac{cM(A\xi u,B\xi v,t)+dM(A\xi u,B\xi v,t)}{cM(B\xi v,B\xi v,t)+d}, \frac{eM(A\xi u,B\xi v,t)+fM(A\xi u,A\xi u,t)}{eM(A\xi u,B\xi v,t)+f}\right\};$$

$M(A\xi u, B\xi v, kt)\geq \min\{1, M(A\xi u, B\xi v, t), 1\}$;

$M(A\xi u, B\xi v, kt) \geq M(A\xi u, B\xi v, t)$, $\Rightarrow A\xi u=B\xi v$.

By taking $\xi x=A\xi u$ and $\xi y=\xi v$ in (5.3.1.3),

$$M(AA\xi u, B\xi v, kt)\geq \min\left\{\frac{aM(SPA\xi u,AA\xi u,t)+bM(B\xi v,TQ\xi v,t)}{a+b}, \frac{cM(SPA\xi u,B\xi v,t)+dM(SPA\xi u,TQ\xi v,t)}{cM(B\xi v,TQ\xi v,t)+d}, \frac{eM(AA\xi u,TQ\xi v,t)+fM(SPA\xi u,AA\xi u,t)}{eM(SPA\xi u,TQ\xi v,t)+f}\right\};$$

$$M(AA\xi u, B\xi v, kt) \geq \min\left\{\begin{array}{l}\frac{aM(AA\xi u,AA\xi u,t)+bM(B\xi v,B\xi v,t)}{a+b},\\ \frac{cM(AA\xi u,B\xi v,t)+dM(AA\xi u,B\xi v,t)}{cM(B\xi v,B\xi v,t)+d},\\ \frac{eM(AA\xi u,B\xi v,t)+fM(AA\xi u,AA\xi u,t)}{eM(AA\xi u,B\xi v,t)+f}\end{array}\right\};$$

$M(AA\xi u, B\xi v, kt) \geq \min\{1, M(AA\xi u, B\xi v, t), 1\};$

$M(AA\xi u, B\xi v, kt) \geq M(AA\xi u, B\xi v, t),$

$\Rightarrow AA\xi u = B\xi v = A\xi u.$

By taking $\xi x = \xi u$ and $\xi y = B\xi v$ in (2.3.1.3),

$$M(A\xi u, BB\xi v, kt) \geq \min\left\{\begin{array}{l}\frac{aM(SP\xi u,A\xi u,t)+bM(BB\xi v,TQB\xi v,t)}{a+b},\\ \frac{cM(SP\xi u,BB\xi v,t)+dM(SP\xi u,TQB\xi v,t)}{cM(BB\xi v,TQB\xi v,t)+d},\\ \frac{eM(A\xi u,TQB\xi v,t)+fM(SP\xi u,A\xi u,t)}{eM(SP\xi u,TQB\xi v,t)+f}\end{array}\right\};$$

$$M(A\xi u, BB\xi v, kt) \geq \min\left\{\begin{array}{l}\frac{aM(A\xi u,A\xi u,t)+bM(BB\xi v,BB\xi v,t)}{a+b},\\ \frac{cM(A\xi u,BB\xi v,t)+dM(A\xi u,BB\xi v,t)}{cM(BB\xi v,BB\xi v,t)+d},\\ \frac{eM(A\xi u,BB\xi v,t)+fM(A\xi u,A\xi u,t)}{eM(A\xi u,BB\xi v,t)+f}\end{array}\right\};$$

$M(A\xi u, BB\xi v, kt) \geq \min\{1, M(A\xi u, BB\xi v, t), 1\};$

$M(A\xi u, BB\xi v, kt) \geq M(A\xi u, BB\xi v, t),$

$\Rightarrow A\xi u = BB\xi v \Rightarrow BB\xi v = A\xi u = B\xi v.$

Now we have $AA\xi u = (SP)A\xi u = A\xi u$, $A\xi u = BB\xi v = BA\xi u$ and

$A\xi u = BB\xi v = (TQ)B\xi v = (TQ)A\xi u$ since $B\xi v = A\xi u$.

Hence $AA\xi u = (SP)A\xi u = BA\xi u = (TQ)A\xi u = A\xi u$

i.e. $A\xi u$ is a common coincidence point of A, B, SP and TQ.

By taking $\xi x = PA\xi u$ and $\xi y = A\xi u$ in (2.3.1.3),

$$M(A\xi x, B\xi y, kt) \geq \min\left\{\begin{array}{l}\frac{aM(SP\xi x,A\xi x,t)+bM(B\xi y,TQ\xi y,t)}{a+b},\\ \frac{cM(SP\xi x,B\xi y,t)+dM(SP\xi x,TQ\xi y,t)}{cM(B\xi y,TQ\xi y,t)+d},\\ \frac{eM(A\xi x,TQ\xi y,t)+fM(SP\xi x,A\xi x,t)}{eM(SP\xi x,TQ\xi y,t)+f}\end{array}\right\};$$

$$M(APA\xi u, BA\xi u, kt) \geq \min\left\{\begin{array}{l}\frac{aM(SPPA\xi u,APA\xi u,t)+bM(BA\xi u,TQA\xi u,t)}{a+b},\\ \frac{cM(SPPA\xi u,BA\xi u,t)+dM(SPPA\xi u,TQA\xi u,t)}{cM(BA\xi u,TQA\xi u,t)+d},\\ \frac{eM(APA\xi u,TQA\xi u,t)+fM(SPPA\xi u,APA\xi u,t)}{eM(SPPA\xi u,TQA\xi u,t)+f}\end{array}\right\}$$

;

Since (A, P) and (S, P) are commuting, therefore

$$M(PA\xi u, BA\xi u, kt) \geq \min\left\{\begin{array}{l}\frac{aM(PA\xi u,PA\xi u,t)+bM(BA\xi u,A\xi u,t)}{a+b},\\ \frac{cM(PA\xi u,BA\xi u,t)+dM(PA\xi u,A\xi u,t)}{cM(BA\xi u,A\xi u,t)+d},\\ \frac{eM(PA\xi u,A\xi u,t)+fM(PA\xi u,PA\xi u,t)}{eM(PA\xi u,A\xi u,t)+f}\end{array}\right\};$$

$$M(PA\xi u, A\xi u, kt) \geq \min\left\{\begin{array}{l}\frac{aM(PA\xi u,PA\xi u,t)+bM(A\xi u,A\xi u,t)}{a+b},\\ \frac{cM(PA\xi u,A\xi u,t)+dM(PA\xi u,A\xi u,t)}{cM(A\xi u,A\xi u,t)+d},\\ \frac{eM(PA\xi u,A\xi u,t)+fM(PA\xi u,PA\xi u,t)}{eM(PA\xi u,A\xi u,t)+f}\end{array}\right\};$$

$M(PA\xi u, A\xi u, kt) \geq \min\{1, M(PA\xi u, A\xi u, t), 1\};$

$M(PA\xi u, A\xi u, kt) \geq M(PA\xi u, A\xi u, t),$

$\Rightarrow PA\xi u = A\xi u.$

By taking $\xi x = A\xi u$ and $\xi y = QA\xi u$ in ((2.3.1.3),

$$M(AA\xi u, BQA\xi u, kt) \geq \min\left\{\begin{array}{l}\frac{aM(SPA\xi u,AA\xi u,t)+bM(BQA\xi u,TQQA\xi u,t)}{a+b},\\ \frac{cM(SPA\xi u,BQA\xi u,t)+dM(SPA\xi u,TQQA\xi u,t)}{cM(BQA\xi u,TQQA\xi u,t)+d},\\ \frac{eM(AA\xi u,TQQA\xi u,t)+fM(SPA\xi u,AA\xi u,t)}{eM(SPA\xi u,TQQA\xi u,t)+f}\end{array}\right\}$$

;

Since (B, Q) and (T, Q) are commuting, therefore

$$M(A\xi u, QA\xi u, kt) \geq \min\left\{\begin{array}{l}\frac{aM(A\xi u,A\xi u,t)+bM(QA\xi u,QA\xi u,t)}{a+b},\\ \frac{cM(A\xi u,QA\xi u,t)+dM(A\xi u,QA\xi u,t)}{cM(QA\xi u,QA\xi u,t)+d},\\ \frac{eM(A\xi u,QA\xi u,t)+fM(A\xi u,A\xi u,t)}{eM(A\xi u,QA\xi u,t)+f}\end{array}\right\};$$

$M(A\xi u, QA\xi u, kt) \geq \min\{1, M(A\xi u, QA\xi u, t), 1\};$

$M(A\xi u, QA\xi u, kt) \geq M(A\xi u, QA\xi u, t),$

$\Rightarrow A\xi u = QA\xi u.$

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Therefore AAξu=(SP)Aξu=BAξu=(TQ)Aξu=Aξu

$\Rightarrow$ AAξu=SPAξu=SAξu and BAξu=TQAξu=TAξu.

Hence AAξu=BAξu=SAξu=TAξu=PAξu=QAξu=Aξu, i.e. Au is a common fixed point of A, B, S, T, P and Q in X.

The uniqueness follows from (2.3.1.3), This completes the proof of the theorem.

If we take P=Q=I (the identity map on X) in theorem 5.3.1 then condition (5.3.1.3), trivially satisfied and we get the following corollary:

Corollary 2.3.2: Let (X, M, Ω ,*) be a random fuzzy metric space and let A, B, S, T, P and Q be self mappings of X such that

(2.2.2.1) the pairs (A, S) and (B, T) are non compatible, sub sequentially continuous faintly compatible;

(2.3.2.2) there exists k$\in$(0,1) such that $\forall$ ξx, ξy$\in$X and t > 0,

$$M(A\xi x, B\xi y, kt) \geq \min\left\{\frac{aM(S\xi x,A\xi x,t)+bM(B\xi y,T\xi y,t)}{a+b}, \frac{cM(S\xi x,B\xi y,t)+dM(S\xi x,T\xi y,t)}{cM(B\xi y,T\xi y,t)+d}, \frac{eM(A\xi x,T\xi y,t)+fM(S\xi x,A\xi x,t)}{eM(S\xi x,T\xi y,t)+f}\right\};$$

where a,b,c,d,e,f$\geq$0 with a&b, c&d and e&f cannot be simultaneously 0;

Then A, B, S and T have a unique common fixed point in X.

Proof: The proof is similar to the proof of theorem 2.3.1 without required condition (2.3.1.3),

Remark : If we take a=c=e=0 and P= Q=I in theorem2.3.1 then we get the result of A. Jain et.al. [5], for faintly compatibility and sequentially continuous map for random fuzzy metric spaces.

Theorem 2.3.3: Let (X, M, Ω ,*) be a random fuzzy metric space and let A, B, S, T, P and Q be self mappings of X such that

(2.3.3.1) the pairs (A, SP) and (B, TQ) are non compatible, sub sequentially continuous faintly compatible;

(2.3.3.2) Pair (A, P), (S, P), (B, Q), (T, Q) are commuting;

(2.3.3.3) there exists k$\in$(0,1) such that $\forall$ ξx, ξy$\in$X and t > 0,

$$M(A\xi x, B\xi y, kt) \geq \phi\left(\min\left\{\frac{aM(SP\xi x,A\xi x,t)+bM(B\xi y,TQ\xi y,t)}{a+b}, \frac{cM(SP\xi x,B\xi y,t)+dM(SP\xi x,TQ\xi y,t)}{cM(B\xi y,TQ\xi y,t)+d}, \frac{eM(A\xi x,TQ\xi y,t)+fM(SP\xi x,A\xi x,t)}{eM(SP\xi x,TQ\xi y,t)+f}\right\}\right)$$

where a,b,c,d,e,f$\geq$0 with a&b, c&d and e&f cannot be simultaneously 0 and ϕ :[0,1]$\rightarrow$[0,1] such that $\phi(t) > t$ $\forall$ $0 < t < 1$;

Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: The prove follows from theorem 2.3.1.

Now we are giving more improved form of theorem 3.1 as follows:

Theorem 2.3.4: Let (X, M, Ω ,*) be a random fuzzy metric space and let A, B, S, T, P and Q be self mappings of X such that

(2.3.4.1) the pairs (A, SP) and (B, TQ) are non compatible, sub sequentially continuous faintly compatible;

(2.3.4.2) Pair (A, P), (S, P), (B, Q), (T, Q) are commuting;

(2.3.4.3) there exists k$\in$(0,1) such that $\forall$ ξx, ξy$\in$X and t > 0,

$$M(A\xi x, B\xi y, kt) \geq \phi\left\{\frac{aM(SP\xi x,A\xi x,t)+bM(B\xi y,TQ\xi y,t)}{a+b}, \frac{cM(SP\xi x,B\xi y,t)+dM(SP\xi x,TQ\xi y,t)}{cM(B\xi y,TQ\xi y,t)+d}, \frac{eM(A\xi x,TQ\xi y,t)+fM(SP\xi x,A\xi x,t)}{eM(SP\xi x,TQ\xi y,t)+f}\right\};$$

where a, b, c, d, e, f $\geq$ 0 with a & b, c & d and e & f cannot be simultaneously 0 and

$\phi : [0, 1]^3 \rightarrow [0, 1]$ such that $\phi(1, t, 1) > t$ $\forall$ $0 < t < 1$;

Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Non compatibility of (A, SP) and (B, TQ) implies that there exist sequences $\{\xi x_n\}$ and $\{\xi y_n\}$ in X such that

$\lim_{n\to\infty} A\xi x_n = \lim_{n\to\infty}(SP)\,\xi x_n = \xi t_1$ for some $\xi t_1 \in X$, and $M(A(SP)\,\xi x_n,(SP)A\xi x_n, t) \neq 1$ or nonexistent $\forall t > 0$; Also

$\lim_{n\to\infty} B\xi x_n = \lim_{n\to\infty}(TQ)\,\xi x_n = \xi t_2$ for some $\xi t_2 \in X$, and

$M(B(TQ)\,\xi x_n,(TQ)B\xi x_n, t) \neq 1$ or nonexistent t $\forall t>0$.

Since pairs (A, SP) and (B, TQ) are faintly compatible therefore conditionally compatibility of (A, SP) and (B, TQ) implies that there exist sequences $\{\xi z_n\}$ and $\{\xi z_n'\}$ in X satisfying

$\lim_{n\to\infty} A\xi z_n = \lim_{n\to\infty}(SP)\,\xi z_n = \xi u$ for some $\xi u \in X$,

such that $M(A(SP)\,\xi z_n, (SP)A\xi z_n, t)=1$;

Also $\lim_{n\to\infty} B\xi z_n' = \lim_{n\to\infty}(TQ)\,\xi z_n' = \xi v$ for some $\xi v \in X$, such that $M(B(TQ)\,\xi z_n',(TQ)B\xi z_n', t)=1$.

As the pairs (A, SP) and (B, TQ) are sub sequentially continuous, we get

$\lim_{n\to\infty} A(SP)\,\xi z_n = A\xi u$, $\lim_{n\to\infty}(SP)A\xi z_n = (SP)\,\xi u$

and so $A\xi u$ = (SP) ξu i.e. (u is coincidence point of A and (SP));

Also $\lim_{n\to\infty} B(TQ)\,\xi z_n' = B\xi v$, $\lim_{n\to\infty}(TQ)B\xi z_n' = (TQ)\,\xi v$

and so Bv = (TQ)v i.e. (v is coincidence point of B and (TQ)).

Since pairs (A, SP) and (B, TQ) are faintly compatible, we get

$A(SP)\,\xi u=(SP)A\xi u$ & so
$AA\xi u=A(SP)\,\xi u=(SP)A\xi u=(SP)(SP)\,\xi u$;

and Also $B(TQ)\,\xi v=(TQ)B\xi v$ & so
$BB\xi v=B(TQ)\,\xi v=(TQ)B\xi v=(TQ)(TQ)\,\xi v$.

Now we show that $A\xi u=B\xi v$, $AA\xi u= A\xi u$., $PA\xi u=A\xi u$ and

$QA\xi u=A\xi u$.

By taking $\xi x=\xi$ u and $\xi y=\xi$ v in (2.3.4.3),

$$M(A\xi u, B\xi v, kt) \geq \phi\left\{\begin{array}{c}\frac{aM(SP\xi u,A\xi u,t)+bM(B\xi v,TQ\xi v,t)}{a+b},\\ \frac{cM(SP\xi u,B\xi v,t)+dM(SP\xi u,TQ\xi v,t)}{cM(B\xi v,TQ\xi v,t)+d},\\ \frac{eM(A\xi u,TQ\xi v,t)+fM(SP\xi u,A\xi u,t)}{eM(SP\xi u,TQ\xi v,t)+f}\end{array}\right\};$$

$$M(A\xi u, B\xi v, kt) \geq \phi\left\{\begin{array}{c}\frac{aM(A\xi u,A\xi u,t)+bM(B\xi v,B\xi v,t)}{a+b},\\ \frac{cM(A\xi u,B\xi v,t)+dM(A\xi u,B\xi v,t)}{cM(B\xi v,B\xi v,t)+d},\\ \frac{eM(A\xi u,B\xi v,t)+fM(A\xi u,\xi Au,t)}{eM(A\xi u,B\xi v,t)+f}\end{array}\right\};$$

$M(A\xi u, B\xi v, kt) \geq \phi\{1, M(A\xi u, B\xi v, t), 1\}$;

$M(A\xi u, B\xi v, kt) \geq M(A\xi u, B\xi v, t)$,

$\Rightarrow A\xi u=B\xi v$.

By taking $\xi x=A\xi u$ and $\xi y= \xi v$ in (2.3.4.3),

$$M(AA\xi u, B\xi v, kt) \geq \phi\left\{\begin{array}{c}\frac{aM(SPA\xi u,AA\xi u,t)+bM(B\xi v,TQ\xi v,t)}{a+b},\\ \frac{cM(SPA\xi u,B\xi v,t)+dM(SPA\xi u,TQ\xi v,t)}{cM(B\xi v,TQ\xi v,t)+d},\\ \frac{eM(AA\xi u,TQ\xi v,t)+fM(SPA\xi u,AA\xi u,t)}{eM(SPA\xi u,TQ\xi v,t)+f}\end{array}\right\};$$

$$M(AA\xi u, B\xi v, kt) \geq \phi\left\{\begin{array}{c}\frac{aM(AA\xi u,AA\xi u,t)+bM(B\xi v,B\xi v,t)}{a+b},\\ \frac{cM(AA\xi u,B\xi v,t)+dM(AA\xi u,B\xi v,t)}{cM(B\xi v,B\xi v,t)+d},\\ \frac{eM(AA\xi u,B\xi v,t)+fM(AA\xi u,AA\xi u,t)}{eM(AA\xi u,B\xi v,t)+f}\end{array}\right\};$$

$M(AA\xi u, B\xi v, kt) \geq \phi\{1, M(AA\xi u, B\xi v, t), 1\}$;

$M(AA\xi u, B\xi v, kt) \geq M(AA\xi u, B\xi v, t)$,

$\Rightarrow AA\xi u=B\xi v=A\xi u$.

Similarly we can show $BB\xi v=B\xi v$ By taking $\xi x= \xi u$ and $\xi y=B\xi v$ in (2.3.4.3).

Now we have $AA\xi u=(SP)A\xi u=A\xi u$,

$A\xi u= BB\xi v=BA\xi u$ and $A\xi u= BB\xi v=(TQ)B\xi v=(TQ)A\xi u$ since $B\xi v=A\xi u$.

Hence $AA\xi u=(SP)A\xi u=BA\xi u=(TQ)A\xi u=A\xi u$

i.e. $A\xi u$ is a common coincidence point of A, B, SP and TQ.

By taking $\xi x=PA\xi u$ and $\xi y=A\xi u$ in (2.3.4.3),

$$M(A\xi x, B\xi y, kt) \geq \phi\left\{\begin{array}{c}\frac{aM(SP\xi x,A\xi x,t)+bM(B\xi y,TQ\xi y,t)}{a+b},\\ \frac{cM(SP\xi x,B\xi y,t)+dM(SP\xi x,TQ\xi y,t)}{cM(B\xi y,TQ\xi y,t)+d},\\ \frac{eM(A\xi x,TQ\xi y,t)+fM(SP\xi x,A\xi x,t)}{eM(SP\xi x,TQ\xi y,t)+f}\end{array}\right\};$$

$$M(APA\xi u, BA\xi u, kt) \geq \phi\left\{\begin{array}{c}\frac{aM(SPPA\xi u,APA\xi u,t)+bM(BA\xi u,TQA\xi u,t)}{a+b},\\ \frac{cM(SPPA\xi u,BA\xi u,t)+dM(SPPA\xi u,TQA\xi u,t)}{cM(BA\xi u,TQA\xi u,t)+d},\\ \frac{eM(APA\xi u,TQA\xi u,t)+fM(SPPA\xi u,APA\xi u,t)}{eM(SPPA\xi u,TQA\xi u,t)+f}\end{array}\right\}$$

;

Since (A, P) and (S, P) are commuting, therefore

$$M(PA\xi u, BA\xi u, kt) \geq \phi\left\{\begin{array}{c}\frac{aM(PA\xi u,PA\xi u,t)+bM(BA\xi u,A\xi u,t)}{a+b},\\ \frac{cM(PA\xi u,BA\xi u,t)+dM(PA\xi u,A\xi u,t)}{cM(BA\xi u,A\xi u,t)+d},\\ \frac{eM(PA\xi u,A\xi u,t)+fM(PA\xi u,PA\xi u,t)}{eM(PA\xi u,A\xi u,t)+f}\end{array}\right\};$$

$$M(PA\xi u, A\xi u, kt) \geq \phi\left\{\begin{array}{c}\frac{aM(PA\xi u,PA\xi u,t)+bM(A\xi u,A\xi u,t)}{a+b},\\ \frac{cM(PA\xi u,A\xi u,t)+dM(PA\xi u,A\xi u,t)}{cM(A\xi u,A\xi u,t)+d},\\ \frac{eM(PA\xi u,A\xi u,t)+fM(PA\xi u,PA\xi u,t)}{eM(PA\xi u,A\xi u,t)+f}\end{array}\right\};$$

$M(PA\xi u, A\xi u, kt) \geq \phi\{1, M(PA\xi u, A\xi u, t), 1\}$;

$M(PA\xi u, A\xi u, kt) \geq M(PA\xi u, A\xi u, t)$,

$\Rightarrow PA\xi u= A\xi u$.

Similarly we can show $A\xi u=QA\xi u$, by taking $\xi x=A\xi u$ and $\xi y=QA\xi u$ in (5.3.4.3).

Therefore $AA\xi u=(SP)A\xi u=BA\xi u=(TQ)A\xi u=A\xi u$

$\Rightarrow AA\xi u=SPA\xi u=SA\xi u$ and $BA\xi u=TQA\xi u=TA\xi u$.

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Hence $AA\xi u=BA\xi u=SA\xi u=TA\xi u=PA\xi u=QA\xi u=A\xi u$, i.e. $A\xi u$ is a common fixed point of A, B, S, T, P and Q in X.

The uniqueness follows from (2.3.4.3).This completes the proof of the theorem.

II. CONCLUSION

Our theorem 2.3.1 is an improvement and generalization of theorem 3.1 of A. Jain et.al. [5], in the following way:

(i) Requirement of the semi-compatibility replaced by weaker form faintly compatibility for random fuzzy metric spaces
(ii) Completeness of the space has been removed completely for random fuzzy metric spaces
(iii) Our results never require the containment of the ranges for random fuzzy metric spaces
(iv) In the light of [3], owc mappings have been replaced by faintly compatible mappings for random fuzzy metric spaces.
(v) Our results are special form for random fixed point theory of Wadhwa et.al.[11-13]

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