

Fixed Point Results with Integral Type Mappings

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Abstract-- In the present paper we proved some fixed point theorems in random fuzzy metric spaces for integral type mappings using the concept of faintly compatible pair of mappings. Our results are motivated by Wadhwa et al. [16-18] and A. Branciari[13]

I. INTRODUCTION & PRELIMINARIES

In 1998, Jungck et al. [11] introduced the notion of weakly compatible mappings and showed that compatible mappings are weakly compatible but not conversely. Al-Thagafi et al. [2] introduced the concept of occasionally weakly compatible (owc) mappings which is more general than the concept of weakly compatible mappings. Aamri et al. [1] generalized the concepts of non-compatibility by defining the notion of (E.A) property in metric space.

Pant et al. [7-9] introduced the concept of conditional compatible maps. Bisht et al. [3] criticize the concept of occasionally weakly compatible (owc) as follows "Under contractive conditions the existence of a common fixed point and occasional weak compatibility are equivalent conditions, and consequently, proving existence of fixed points by assuming occasional weak compatibility is equivalent to proving the existence of fixed points by assuming the existence of fixed points". Therefore use of occasional weak compatibility is a redundancy for fixed point theorems under contractive conditions to remove this redundancy we used faintly compatible mapping in our paper which is weaker than weak compatibility or semi compatibility. Faintly compatible maps introduced by Bisht et al. [3] is an improvement of conditionally compatible maps. Using these concepts Wadhwa et al. [16-18] proved some common fixed point theorems. In this paper we prove some common fixed point for four mappings using the concept of faintly compatible pair of mappings in random fuzzy metric spaces with Integral Type Inequality.

In 2002, A. Branciari [13] analyzed the existence of fixed point for mapping f defined on a complete metric space X , d satisfying a general contractive condition of integral type

Theorem 2.1 (Branciari) Let X, d be a complete metric space, $c \in (0, 1)$ and let $f: X \rightarrow X$ be a mapping such that for each $x, y \in X$,

$$\int_0^{d(fx, fy)} \xi(t) dt \leq c \int_0^{d(fx, fy)} \xi(t) dt \quad 2.1.1$$

Where $\xi: [0, +\infty) \rightarrow [0, +\infty)$ is a Lebesgue integrable mapping which is sum able on each compact subset of $[0, +\infty)$, non negative, and such that for each $\epsilon > 0$, $\int_0^\epsilon \xi(t) dt > 0$, then f has a unique fixed point $a \in X$ such that for each $x \in X$, $\lim_{n \rightarrow \infty} f^n x = a$.

After the paper of Branciari, a lot of a research works have been carried out on generalizing contractive conditions of integral type for different contractive mappings satisfying various known properties. A fine work has been done by Rhoades [15] extending the result of Branciari by replacing the condition [2.1.1] by the following

$$\int_0^{d(fx, fy)} \xi(t) dt \leq \int_0^{\max\{d(x,y), d(x,fx), d(y,fy), \frac{d(x,fy)+d(y,fx)}{2}\}} \xi(t) dt$$

In [9] the author proved the following

Theorem 2.1.3 (14) Let (X, d) be a complete metric space and $f: X \rightarrow X$ such that

$$\int_0^{d(fx, fy)} u(t) dt \leq \alpha \int_0^{d(x,fx)+d(y,fy)} u(t) dt + \beta \int_0^{d(x,y)} u(t) dt + \gamma \int_0^{\max\{d(x,fy)+d(y,fx)\}} u(t) dt$$

For each $x, y \in X$ with non negative reals α, β, γ such that $2\alpha + \beta + 2\gamma < 1$,

where $u: [0, +\infty) \rightarrow [0, +\infty)$ is a Lebesgue integrable mapping which is sum able, non-negative and such that for each $\epsilon > 0$, $\int_0^\epsilon u(t) dt > 0$. Then f has a unique fixed point in X .

There is a gap in the proof of the theorem 2.1.3. In fact; the authors [14] used the inequality

$\int_0^{a+b} u(t)dt \leq \int_0^a u(t)dt + \int_0^b u(t)dt$ for $0 \leq a < b$,
This is not true in general.

Also we are using effect $\int_0^{2a} f(x)dx = 2 \int_0^a f(x)dx$ which is only true when $f(2a - x) = f(x)$, there is again a gap, to complete our proof that $\int_0^{kf(x)} \phi(t)dt = k \int_0^{f(x)} \phi(t)dt$ which is not true in general.

Throughout this Chapter (Ω, Σ) denotes a measurable space. $\xi: \Omega \rightarrow X$ is a measurable selector. X is any non empty set. \star is continuous t-norm, \mathbf{M} is a fuzzy set in $X^2 \times [0, \infty)$. A binary operation $\star: [0,1] \times [0,1] \rightarrow [0,1]$ is called a continuous t-norm if $([0,1], \star)$ is an abelian Topological monodies with unit 1 such that $a \star b \geq c \star d$ whenever

$a \geq c$ and $b \geq d$, For all $a, b, c, d, \in [0, 1]$

Example of t-norm are $a \star b = a \wedge b$ and $a \star b = \min \{a, b\}$

Definition 2.2. (a): The 3-tuple (X, M, Ω, \star) is called a **Random fuzzy metric space**, if X is an arbitrary set, \star is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions: for all

$\xi x, \xi y, \xi z \in X$ and $s, t > 0$,

$$(RFM-1): M(\xi x, \xi y, 0) = 0$$

$$(RFM-2): M(\xi x, \xi y, t) = 1, \forall t > 0$$

$$(RFM-3): M(\xi x, \xi y, t) = M(\xi y, \xi x, t)$$

$$(RFM-4): M(\xi x, \xi z, t+s) \geq M(\xi x, \xi y, t) \star M(\xi y, \xi z, s)$$

$$(RFM-5): M(\xi x, \xi y, \xi a): [0,1] \rightarrow [0,1] \text{ is left continuous}$$

In what follows, (X, M, Ω, \star) will denote a random fuzzy metric space. Note that $M(\xi x, \xi y, t)$ can be thought of as the degree of nearness between ξx and ξy with respect to t . We identify $\xi x = \xi y$ with $M(\xi x, \xi y, t) = 1$ for all $t > 0$ and

$M(\xi x, \xi y, t) = 0$ with ∞ . In the following example, we know that every metric induces a fuzzy metric.

Example Let (X, d) be a metric space.

Define $a \star b = a \wedge b$, or $ab = \min \{a, b\}$ and for all $x, y, \in X$ and $t > 0$,

$$M(\xi x, \xi y, t) = \frac{t}{t + d(\xi x, \xi y)}$$

Then (X, M, Ω, \star) is a fuzzy metric space. We call this random fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition 2.2. (b): Let (X, M, Ω, \star) is a random fuzzy metric space.

(i) A sequence $\{\xi x_n\}$ in X is said to be convergent to a point $\xi x \in X$,

$$\lim_{n \rightarrow \infty} M(\xi x_n, \xi x, t) = 1$$

(ii) A sequence $\{\xi x_n\}$ in X is called a Cauchy sequence if

$$\lim_{n \rightarrow \infty} M(\xi x_{n+p}, \xi x_n, t) = 1, \forall t > 0$$

(iii) A random fuzzy metric space in which every Cauchy sequence is convergent is said to be Complete.

Let (X, M, \star) is a fuzzy metric space with the following condition.

$$(RFM-6) \quad \lim_{t \rightarrow \infty} M(\xi x, \xi y, t) = 1, \forall \xi x, \xi y \in X$$

Definition 2.2. (c): A function M is continuous in random fuzzy metric space iff whenever

$$\xi x_n \rightarrow \xi x, \xi y_n \rightarrow \xi y \Rightarrow \lim_{n \rightarrow \infty} M(\xi x_n, \xi y_n, t) \rightarrow M(\xi x, \xi y, t)$$

Definition 2.2 (d): Two mappings A and S on random fuzzy metric space X are weakly commuting iff

$$M(AS\xi u, SA\xi u, t) \geq M(A\xi u, S\xi u, t)$$

Definition 2.2 (e): A pair of self-maps (A, S) on a random fuzzy metric space

(X, M, Ω, \star) is said to be

(i) **Non-compatible:** if (A, S) is not compatible, i.e., if there exists a sequence $\{\xi x_n\}$ in X such that $\lim_{n \rightarrow \infty} A\xi x_n = \lim_{n \rightarrow \infty} S\xi x_n = \xi x$, for some $\xi x \in X$, and $\lim_{n \rightarrow \infty} M(AS\xi x_n, SA\xi x_n, t) \neq 1$ or non-existent $\forall t > 0$.

(ii) **Conditionally compatible:** if whenever the set of sequences $\{\xi x_n\}$ satisfying $\lim_{n \rightarrow \infty} A\xi x_n = \lim_{n \rightarrow \infty} S\xi x_n$, is non-empty, there exists a sequence $\{\xi z_n\}$ in X such that $\lim_{n \rightarrow \infty} A\xi z_n = \lim_{n \rightarrow \infty} S\xi z_n = \xi t$, for some $\xi t \in X$ and $\lim_{n \rightarrow \infty} M(AS\xi z_n, SA\xi z_n, t) = 1$ for all $t > 0$.

(iii) **Faintly compatible :** if (A, S) is conditionally compatible and A and S commute on a non-empty subset of

the set of coincidence points, whenever the set of coincidence points is nonempty.

(iv) Satisfy the property (E.A.) [1]: if there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} A\xi x_n = \lim_{n \rightarrow \infty} S\xi x_n = \xi x$, for some $\xi x \in X$.

(v) Sub Sequentially continuous: iff there exists a sequence $\{\xi x_n\}$ in X such that $\lim_{n \rightarrow \infty} A\xi x_n = \lim_{n \rightarrow \infty} S\xi x_n = \xi x$, $\xi x \in X$ and satisfy $\lim_{n \rightarrow \infty} AS\xi x_n = A\xi x$, $\lim_{n \rightarrow \infty} SA\xi x_n = S\xi x$.

Note that, compatibility, non- compatibility and faint compatibility are independent concepts. Faintly compatibility is applicable for mappings that satisfy contractive and non contractive conditions.

(vi) Semi-compatible : if $\lim_{n \rightarrow \infty} AS\xi x_n = S\xi x$, whenever is a sequence such that $\lim_{n \rightarrow \infty} A\xi x_n = \lim_{n \rightarrow \infty} S\xi x_n = \xi x \in X$.

Lemma 2.2 (f) [Modified form]: Let $(X, M, *)$ be a random fuzzy metric space and for all $\xi x, \xi y \in X$, $t > 0$ and if there exists a constant $k \in (0, 1)$ such that

$$M(\xi x, \xi y, kt) \geq M(\xi x, \xi y, t) \text{ then } \xi x = \xi y.$$

II. MAIN RESULTS

Theorem 2.1 Let $(X, M, \Omega, *)$ be a random fuzzy metric space with continuous t-norm P, S, Q, T be mappings from X into itself .If there exists $q \in (0,1)$ such that

$$\begin{aligned} & M(P\xi x, Q\xi y, qt) \\ & \int_0^{\infty} \phi(t) dt \\ & \geq \int_0^{\infty} \text{Min}\{M(S\xi x, T\xi y, t), M(S\xi x, P\xi x, t), M(Q\xi y, T\xi y, t), M(P\xi x, T\xi y, t), M(Q\xi y, S\xi x, t)\} \\ & \phi(t) dt \end{aligned} \tag{3.1.1}$$

For all $\xi x, \xi y \in X$ and for all $t > 0$. If pairs (P, S) and (Q, T) satisfies E.A. property with sub sequentially continuous faintly compatible map then P, S, Q, T have a unique common random fixed point in X .

Proof : (P, S) and (Q, T) satisfy E.A property which implies that there exist sequences $\{\xi x_n\}$ and $\{\xi y_n\}$ in X such that $\lim_{n \rightarrow \infty} P\xi x_n = \lim_{n \rightarrow \infty} S\xi x_n = \xi t_1 \in X$ also $\lim_{n \rightarrow \infty} Q\xi x_n = \lim_{n \rightarrow \infty} T\xi x_n = \xi t_2$ for some $\xi t_2 \in X$. Since pairs (P, S) and (Q, T) are faintly compatible therefore conditionally compatibility of (P, S) and (Q, T) implies that there exist sequences $\{\xi z_n\}$ and $\{\xi z'_n\}$ in X satisfying $\lim_{n \rightarrow \infty} P\xi z_n = \lim_{n \rightarrow \infty} S\xi z_n = \xi u$ for some ξu

$\in X$, such that $M(PS\xi z_n, SP\xi z_n, t) = 1$, also $\lim_{n \rightarrow \infty} Q\xi z'_n = \lim_{n \rightarrow \infty} T\xi z'_n = \xi v$ for some $\xi v \in X$, such that $M(QT\xi z'_n, TQ\xi z'_n, t) = 1$. As the pairs (P, S) and (Q, T) are sub sequentially continuous, we get $\lim_{n \rightarrow \infty} PS\xi z_n = P\xi u$, $\lim_{n \rightarrow \infty} SP\xi z_n = S\xi u$ and so $P\xi u = S\xi u$, also $\lim_{n \rightarrow \infty} QT\xi z'_n = Q\xi v$, $\lim_{n \rightarrow \infty} TQ\xi z'_n = T\xi v$ and so $Q\xi v = T\xi v$. Since pairs (P, S) and (Q, T) are faintly compatible, we get $PS\xi u = SP\xi u$ & So $PP\xi u = PS\xi u = SP\xi u = SS\xi u$ also $QT\xi v = TQ\xi v$ & So $QQ\xi v = QT\xi v = TQ\xi v = TT\xi v$.

Now we show that $P\xi u = Q\xi v$.

Let $\xi x = \xi u$ and $\xi y = \xi v$ in equation (1) we have

$$\begin{aligned} & M(P\xi u, Q\xi v, qt) \\ & \int_0^{\infty} \phi(t) dt \\ & \geq \int_0^{\infty} \text{Min}\{M(S\xi u, T\xi v, t), M(S\xi u, P\xi u, t), M(Q\xi v, T\xi v, t), M(P\xi u, T\xi v, t), M(Q\xi v, S\xi u, t)\} \\ & \phi(t) dt \\ & \geq \int_0^{\infty} \text{Min}\{M(S\xi u, T\xi v, t), 1, M(Q\xi v, T\xi v, t), M(P\xi u, T\xi v, t), M(Q\xi v, S\xi u, t)\} \\ & \phi(t) dt \end{aligned}$$

$$\geq \int_0^{\text{Min}\{M(P\xi u, T\xi v, t), 1, 1, M(P\xi u, T\xi v, t), M(T\xi v, P\xi u, t)\}} \phi(t) dt$$

Hence from the lemma it is clear that $P\xi u = Q\xi v$

Now we have to show that $PP\xi u = P\xi u$

Let $\xi x = P\xi u$ and $\xi y = \xi v$ in equation (1)

We get

$$\begin{aligned} & \int_0^{M(PP\xi u, P\xi u, qt)} \phi(t) dt \\ & \geq \int_0^{\text{Min}\{M(SP\xi u, T\xi v, t), M(SP\xi u, PP\xi u, t), M(Q\xi v, T\xi v, t), M(PP\xi u, T\xi v, t), M(Q\xi v, SP\xi u, t)\}} \phi(t) dt \\ & \geq \int_0^{\text{Min}\{M(PP\xi u, Q\xi v, t), 1, 1, M(PP\xi u, Q\xi v, t), M(\{P\xi u, PP\xi u, t\})\}} \phi(t) dt \end{aligned}$$

Now by the lemma it is clear that $PP\xi u = P\xi u$

Now we have to show that $P\xi u = QQ\xi v$

Let $\xi x = \xi u$ and $\xi y = Q\xi v$ in equation (1)

We get

$$\begin{aligned} & \int_0^{M(P\xi u, QQ\xi v, qt)} \phi(t) dt \\ & \geq \int_0^{\text{Min}\{M(S\xi u, TQ\xi v, t), M(S\xi u, P\xi u, t), M(QQ\xi v, TQ\xi v, t), M(P\xi u, TQ\xi v, t), M(QQ\xi v, S\xi u, t)\}} \phi(t) dt \\ & \geq \int_0^{\text{Min}\{M(S\xi u, TQ\xi v, t), M(S\xi u, P\xi u, t), M(QQ\xi v, TQ\xi v, t), M(P\xi u, TQ\xi v, t), M(QQ\xi v, S\xi u, t)\}} \phi(t) dt \\ & \geq \int_0^{\text{Min}\{M(P\xi u, QQ\xi v, t), 1, 1, M(P\xi u, QQ\xi v, t), M(QQ\xi v, P\xi u, t)\}} \phi(t) dt \end{aligned}$$

Now by the lemma it is clear that $P\xi u = QQ\xi v$

Now we have

$$PP\xi u = SP\xi u = P\xi u$$

$$P\xi u = QQ\xi v = QP\xi u$$

And

$$P\xi u = QQ\xi v = TQ\xi v = TP\xi u$$

Since $Q\xi v = P\xi u$,

Hence we have $P(P\xi u) = S(P\xi u) = Q(P\xi u) = T(P\xi u)$

Let $P\xi u = \xi w$

$P(\xi w) = S(\xi w) = Q(\xi w) = T(\xi w)$

Where ξw is a common fixed point of P, S, Q, and T

Hence the uniqueness of the fixed point holds from equation (1).

Hence Proved.

Theorem 2.2 Let $(X, M, \Omega, \square, *)$ be a complete random fuzzy metric space and let P, S, Q, T are mappings from X into itself such that

$$\begin{aligned}
 M(P\xi x, Q\xi y, qt) & \int_0^1 \phi(t) dt \\
 & \geq \int_0^1 \phi\{M(S\xi x, T\xi y, t), M(S\xi x, P\xi x, t), M(Q\xi y, T\xi y, t), M(P\xi x, T\xi y, t), M(Q\xi y, S\xi x, t)\} \phi(t) dt \\
 & \quad (3.2.2)
 \end{aligned}$$

For all $\xi x, \xi y \in X$ and $\phi: [0,1]^5 \rightarrow [0,1]$ such that $\phi(t, 1, 1, t, t) > t$

for all $0 < t < 1$.

If pairs (P, S) and (Q, T) satisfies E.A. property with sub sequentially continuous faintly compatible map then P, S, Q, T have a unique common fixed point in X.

Proof : (P, S) and (Q, T) satisfy E.A property which implies that there exist sequences $\{\xi x_n\}$ and $\{\xi y_n\}$ in X such that $\lim_{n \rightarrow \infty} P\xi x_n = \lim_{n \rightarrow \infty} S\xi x_n = \xi t_1$ for some $\xi t_1 \in X$ also $\lim_{n \rightarrow \infty} Q\xi x_n = \lim_{n \rightarrow \infty} T\xi x_n = \xi t_2$ for some $\xi t_2 \in X$. Since pairs (P, S) and (Q, T) are faintly compatible therefore conditionally compatibility of (P, S) and (Q, T) implies that there exist sequences $\{\xi z_n\}$ and $\{\xi z'_n\}$ in X satisfying $\lim_{n \rightarrow \infty} P\xi z_n = \lim_{n \rightarrow \infty} S\xi z_n = \xi u$ for some $\xi u \in X$, such that $M(PS\xi z_n, SP\xi z_n, t) = 1$, also

$\lim_{n \rightarrow \infty} Q\xi z'_n = \lim_{n \rightarrow \infty} T\xi z'_n = \xi v$ for some $\xi v \in X$, such that

$M(QT\xi z'_n, TQ\xi z'_n, t) = 1$. As the pairs (P, S) and (Q, T) are sub sequentially continuous, we get $\lim_{n \rightarrow \infty} PS\xi z_n = P\xi u$, $\lim_{n \rightarrow \infty} SP\xi z_n = S\xi u$ and so $P\xi u = S\xi u$. Also

$\lim_{n \rightarrow \infty} QT\xi z'_n = Q\xi v$, $\lim_{n \rightarrow \infty} TQ\xi z'_n = T\xi v$ and so $Q\xi v = T\xi v$.

Since pairs (P, S) and (Q, T) are faintly compatible, we get $PS\xi u = SP\xi u$ & So

$PP\xi u = PS\xi u = SP\xi u = SS\xi u$ also $QT\xi v = TQ\xi v$

& So $QQ\xi v = QT\xi v = TQ\xi v = TT\xi v$.

Now we show that $P\xi u = Q\xi v$.

Let $\xi x = \xi u$ and $\xi y = \xi v$ in equation (2) we have

$$\begin{aligned}
 M(P\xi u, Q\xi v, qt) & \int_0^1 \phi(t) dt \geq \int_0^1 \phi\{M(S\xi u, T\xi v, t), M(S\xi u, P\xi u, t), M(Q\xi v, T\xi v, t), M(P\xi u, T\xi v, t), M(Q\xi v, S\xi u, t)\} \phi(t) dt \\
 & \geq \int_0^1 \phi\{M(S\xi u, T\xi v, t), 1, M(Q\xi v, T\xi v, t), M(P\xi u, T\xi v, t), M(Q\xi v, S\xi u, t)\} \phi(t) dt
 \end{aligned}$$

$$\begin{aligned} &\geq \int_0^{\phi\{M(P\xi u, T\xi v, t), 1, 1, M(P\xi u, T\xi v, t), M(P\xi u, T\xi v, t)\}} \emptyset(t) dt \\ &\geq \int_0^{M(P\xi u, Q\xi v, t)} \emptyset(t) dt \end{aligned}$$

Hence from the lemma it is clear that $P\xi u = Q\xi v$

Now we show that $PP\xi u = P\xi u$

Let $\xi x = P\xi u$ and $\xi y = \xi v$

We get

$$\begin{aligned} &\int_0^{M(PP\xi u, P\xi u, qt)} \emptyset(t) dt \geq \int_0^{\phi\{M(SP\xi u, T\xi v, t), M(SP\xi u, PP\xi u, t), M(Q\xi v, T\xi v, t), M(PP\xi u, T\xi v, t), M(Q\xi v, SP\xi u, t)\}} \emptyset(t) dt \\ &\geq \int_0^{\phi\{M(PP\xi u, Q\xi v, t), 1, 1, M(PP\xi u, Q\xi v, t), M\{P\xi u, PP\xi u, t\}\}} \emptyset(t) dt \\ &\geq \int_0^{M(PP\xi u, P\xi u, t)} \emptyset(t) dt \end{aligned}$$

It is clear that $PP\xi u = P\xi u$

Now we have to show that $P\xi u = QQ\xi v$

Let $\xi x = \xi u$ and $\xi y = Q\xi v$ in equation (2)

We get

$$\begin{aligned} &\int_0^{M(P\xi u, QQ\xi v, qt)} \emptyset(t) dt \\ &\geq \int_0^{\phi\{M(S\xi u, TQ\xi v, t), M(S\xi u, P\xi u, t), M(QQ\xi v, TQ\xi v, t), M(P\xi u, TQ\xi v, t), M(QQ\xi v, S\xi u, t)\}} \emptyset(t) dt \\ &\geq \int_0^{\phi\{M(S\xi u, TQ\xi v, t), M(S\xi u, P\xi u, t), M(QQ\xi v, TQ\xi v, t), M(P\xi u, TQ\xi v, t), M(QQ\xi v, S\xi u, t)\}} \emptyset(t) dt \\ &\geq \int_0^{\phi\{M(P\xi u, QQ\xi v, t), 1, 1, M(P\xi u, QQ\xi v, t), M(QQ\xi v, P\xi u, t)\}} \emptyset(t) dt \\ &\geq \int_0^{M(P\xi u, QQ\xi v, t)} \emptyset(t) dt \end{aligned}$$

Now by the lemma it is clear that $P\xi u = QQ\xi v$

Now we have

$$PP\xi u = SP\xi u = P\xi u$$

$$P\xi u = QQ\xi v = QP\xi u$$

and

$$P\xi u = QQ\xi v = TQ\xi v = TP\xi u$$

$$\text{Since } Q\xi v = P\xi u,$$

$$\text{Hence we have } P(P\xi u) = S(P\xi u) = Q(P\xi u) = T(P\xi u)$$

$$\text{Let } P\xi u = \xi w$$

$$P(\xi w) = S(\xi w) = Q(\xi w) = T(\xi w)$$

Where ξw is a common fixed point of P, S, Q, and T

Hence the uniqueness of the fixed point holds.

Hence Proved.

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