Principles of Fluid Mechanics

1. BASIC PRINCIPLES OF FLUID MECHANICS

Ventilation is the application of the principles of fluid dynamics to the flow of air in underground openings. Airflow is induced from the atmosphere, through the intake opening, underground airways, return opening, and back to the atmosphere again by a differential pressure between intake and return openings of the mine. This pressure differential is usually very small, in the order of 2 to 3% in most cases, when compared to the absolute pressure of the system. Therefore, volume and density changes are neglected without serious loss of accuracy or validity. Airflow in mines is also generally treated as steady state, turbulent, and incompressible. However, in situations where the pressure, temperatures, and humidity changes are large and air conditioning processes are involved, calculations must incorporate compressibility effect of air.

1.1 Definition of A Fluid

A fluid may be defined broadly as a substance which *deforms continuously when subjected to shear stress* . This fluid can be made to flow if it is acted upon by a source of energy. This can be made clear by assuming the fluid being consisted of layers parallel to each other and letting a force act upon one of the layers in a direction parallel to its plane. This force divided by the area of the layer is called shear stress. As long as this shear stress is applied the layer will continue to move relative to its neighboring layers.

If the neighboring layers offer no resistance to the movement of fluid, this fluid is said to be *frictionless fluid* or *ideal fluid*. (Practically speaking, ideal fluids do not exist in nature, but in many practical problems the resistance is either small or is not important, therefore can be ignored.) A fluid is always a continuous medium and there cannot be voids in it. The properties of a fluid, e.g., density, may, however, vary from place to place in the fluid.

In addition to shear force, fluid may also be subjected to *compressive forces*. These compressive forces tend to change the volume of the fluid and in turn its density. If the fluid yields to the effect of the compressive forces and changes its volume, it is *compressible*, otherwise it is *incompressible* .

1.2 Turbulent and Laminar Flows

In a flowing fluid, each particle changes its position with a certain velocity. The magnitudes and directions of the velocities of all particles may vary with position as well as with time. Streamline is used to illustrate this very concept.

Imagine a pipe of circular cross-section containing fluid such as water. For flow to occur without slippage, then the various layers must move at different velocities. The fluid layer adjacent to the pipe wall is virtually stationary, while the layers further out move at increasing higher velocities until a maximum velocity is attained at the center (Fig 4-1).

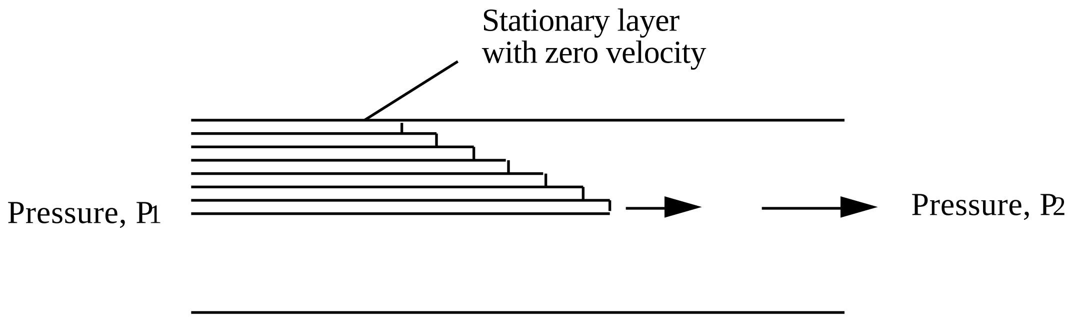

Figure 4-1 Fluid flow through a pipe

A streamline is an imaginary line in a fluid, the tangent to which gives the direction of the flow velocity at that position, as shown in Figure 4-2, where the distance between two streamlines is an inverse measure of the magnitude of the velocity. If the streamlines are smoothly curved and almost parallel to each other, as illustrated in Figure 4-3, the flow is known as streamlined flow or *laminar flow* . On the other hand, if the streamlines are arranged haphazardly as illustrated in Figure 4-4, the flow is known as *turbulent flow* .

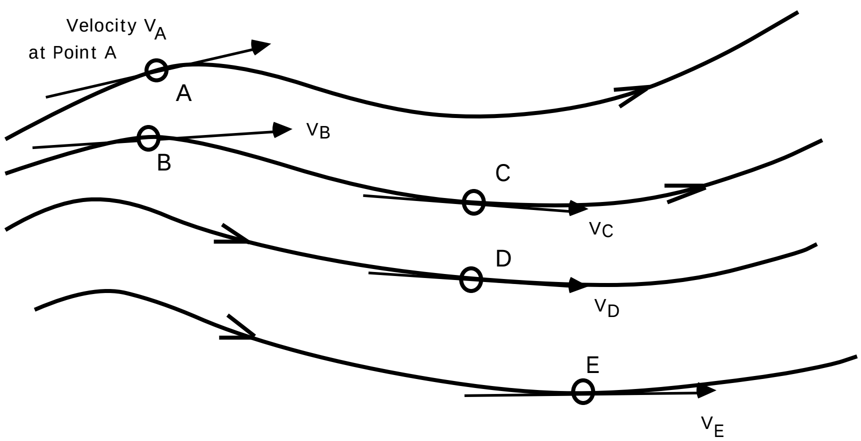

Figure 4-2 Streamlines showing velocity of flow at several points.

If the streamline pattern of a flow remains constant with time, the flow is *steady* . If it does not, the flow is *unsteady* and in this case a streamline picture is an instantaneous one, valid only for a particular instant of time. A greatly enlarged view of any small region of a turbulent flow shows that the flow is a randomly unsteady laminar flow. If the mean flow values are unchanged over a period of time, it is called a steady turbulent flow.

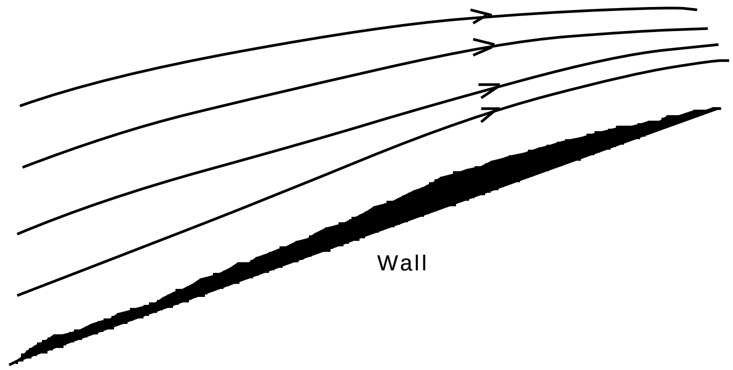

Figure 4-3 Streamlines in a laminar flow

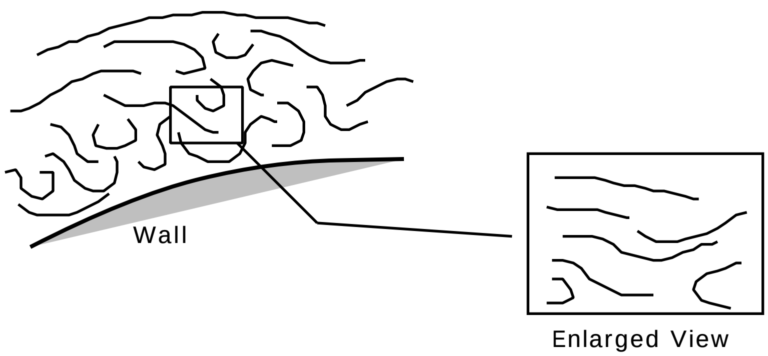

Figure 4-4 Instantaneous picture of streamlines in turbulent flow

It was recognized earlier on that the type of flow depended upon the velocity and viscosity of the fluid. It was not until the 1880s this relationship was mathematically expressed in terms of the ratio of inertial and viscous forces by Professor Osborne Reynolds through many experiments and dimension analysis,

$$N_{Re} = \frac{\rho \; D \; V}{\mu}$$

ρ is the fluid density; $\nu = \mu/\rho$, is kinematic viscosity;
μ is absolute viscosity; D is diameter of conduit;
V is velocity

Reynolds number (N_{Re}) is a dimensionless number. If this number is less than 2000, viscous forces prevail and the flow will be laminar. The condition of the flow is less well defined for this number beyond 2000 for which turbulence starts to occur. It has been common practice to regard the flow as turbulent when Reynolds number is larger than 4000,

Laminar flow - for $N_{Re} < 2,000$
Turbulent flow - for $N_{Re} > 4,000$

Example 4-1: A ventilation shaft of diameter 5 m passes an airflow of 200 m^3/sec at a mean density of 1.2 kg/m^3 and a mean temperature of 18°C (64.4°F). Determine the Reynolds number for the shaft.

Solution : For air at 18°C, $\mu_{air} = (17.0 + 0.045 \times t) \times 10^{-6}$ where t is temperature in °C

$\mu_{air} = (17.0 + 0.045 \times 18) \times 10^{-6} = 17.81 \times 10^{-6}$ Nsec/m^2

$$\text{Air velocity} = V = \frac{Q}{A} = \frac{200}{\pi(5)^2/4} = 10.186 \text{ m/sec}$$

$$N_{Re} = \frac{\rho\ D\ V}{\mu} = \frac{1.2 \times 5 \times 10.186}{17.81 \times 10^{-6}} = 3.432 \times 10^6$$

Reynolds number indicates that the flow will be turbulent in the shaft.

1.3 Bernoulli's Equation

Mine ventilation is normally an example of a steady flow process, in which none of the variables of flow changes with time. Transitions and losses in energy are involved in such process. Energy changes and their mathematical expressions are basic to the calculation of mine air quantity and pressure are developed as follows.

According to Newton's Second Law, the force F used to accelerate a body must be equal to the product of the mass *m* of the body and its acceleration *a*. If this law is applied to an ideal (frictionless), incompressible (constant–density) fluid –*Euler's Law* of conservation of momentum is obtained.

Assuming a fluid moving steadily through an imaginary fixed volume element xyz, in the x–direction (across the yz–plane), as shown in Figure 4-5. If pressure p_1 and p_2 act upon the two opposite faces. The resultant force on the fluid in the volume element considered is:

$$F = P_1yz - P_2yz + Bw\ xyz$$

where:

B = body or mass forces per unit mass
w = constant fluid density

if x is very small distance, the acceleration along it is constant,

$$a = (V_2 - V_1)/t$$

where *t* is the time for the fluid through distance x.

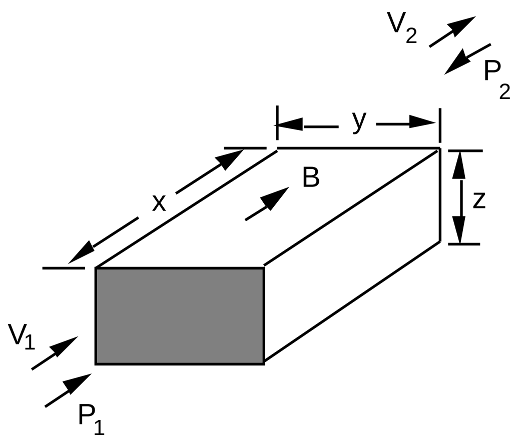

Figure 4-5 Illustrations of Euler's Law

$$a = (V_2 - V_1)/t$$

where t is the time for the fluid through distance x.

Since the mass of the fluid element is m = w xyz, with Newton's Law $F = m\,a$:

$$P_1\,yz - P_2\,yz + Bw\,xyz = w\,xyz\,\frac{(V_2 - V_1)}{t}$$

or, $$P_1 - P_2 + Bw\,x = w\,x\,\frac{(V_2 - V_1)}{t} \qquad (4-2)$$

The mean velocity over the distance x is $(V_2 + V_1)/2 = x/t$

Substituting:

$$P_1 - P_2 + Bw\,x = w\,\frac{(V_2^2 - V_1^2)}{2}$$

This is *Euler's Equation* for the x-direction, also known as the momentum equation.

If the x–direction is taken as vertically upwards as shown in Figure 4-6, such that P_1 & V_1 are at height Z_1 and P_2 & V_2 at height Z_2, the distance is $x = Z_2 – Z_1$.

The force B per unit mass would now be due to gravitation, and B = – g

$$P_1 - P_2 - gw\,(Z_2 - Z_1) = w\,(V_2^2 - V_1^2)/2$$

or $$P_1 + w(V_1^2/2) + gw\, Z_1 = P_2 + w\, (V_2^2/2) + gwZ_2 \quad \text{. . . . .} \quad (4\text{–}3)$$

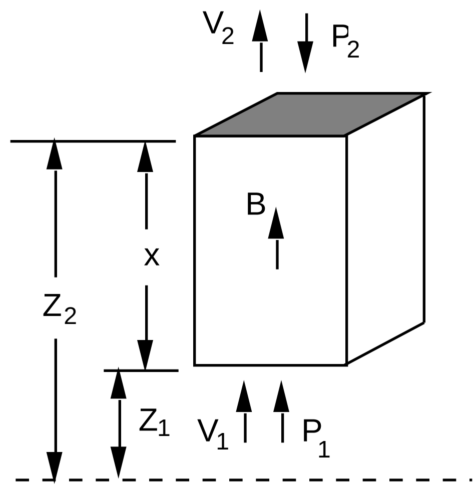

Figure 4-6 Illustrations of Euler's Law for vertical control body

This is *Bernoulli's Equation* for the ideal incompressible fluid in terms of pressure (Conservation of Momentum). The *Second Law of Thermodynamics* limits the directions in which energy can be transformed or transferred. It is well known that friction causes a degradation of useful "pressure" energy. So, the heat will flow from a hot body to cold body; in terms of the Second Law it is impossible for the reverse to occur.

or $$P_1/wg + V_1^2/2g + Z_1 = P_2/wg + V_2^2/2g + Z_2 \quad \text{. . . . .} \quad (4\text{–}4)$$

This is Bernoulli's Equation (in SI unit) for the ideal incompressible fluid in terms of pressure head where P is in Pa (Nm^{-2}). The pressures P_1 and P_2 in equations 2 and 3 are referred to as *static* pressures; they act in all directions regardless of the direction of the flow. The terms $V_1^2/2$ and $V_2^2/2$ in Equation 4–3 are referred as *velocity* or *dynamic* pressures. The summation of the two is referred as the *total* pressure of the oncoming fluid stream, also called the *facing* pressure. Each pressure term is expressed in *length* (m).