

# Let's call it Nonlocal Quantum Physics

**Manfred Requardt**

email: requardt@theorie.physik.uni-goettingen.de

Institut für Theoretische Physik  
Universität Göttingen  
Bunsenstrasse 9  
37073 Göttingen Germany

## Abstract

In the following we undertake to derive quantum theory as a stochastic low-energy and coarse-grained theory from a more primordial discrete and basically geometric theory living on the Planck scale and which (as we argue) possibly underlies also *string theory*. We isolate the so-called *ideal elements* which represent at the same time the cornerstones of the framework of ordinary quantum theory and show how and why they encode the *non-local* aspects, being ubiquitous in the quantum realm, in a, on the surface, local way. We show that the quantum non-locality emerges in our approach as a natural consequence of the underlying *two-storey* nature of space-time or the physical vacuum, that is, quantum theory turns out to be a residual effect of the geometric depth structure of space-time on the Planck scale. We indicate how the *measurement problem* and the emergence of the *macroscopic sub-regime* can be understood in this framework.

# 1 Introduction

In preceding work we have started to develop a radically discrete mathematical and physical framework aimed at reconstructing, beginning at the Planck scale and working “bottom-up” (so to speak), our present day continuum physics and corresponding mathematics (cf. [1] to [5]). Our main intention is it however to derive both quantum theory and general relativity, i.e. gravitation (and in the last consequence (semi)classical space-time), as emergent and low energy *effective theories* by a coarse graining process from a more primordial discrete substratum.

While papers [1] to [4] deal mostly with the development of the necessary mathematical and physical concepts and tools (typically discrete protoforms of their continuum counterparts), some concrete steps towards a realisation of the more ambitious latter goal were taken in paper [5] as far as the emergence and reconstruction of a protoform of continuum space-time as, what we call, an *order parameter manifold* is concerned. By this we mean an extended coarse-grained *superstructure* displaying a certain collective order on a larger and smoother scale. As in the case of ordinary *order parameters* in, say, condensed matter physics, this emergence of order is usually the result of a *phase transition* and is accompanied by a shrinking of microscopic phase space being occupied by the system. Whereas this programme is far from being completed, its core result (or rather: hypothesis, as not every step in the corresponding analysis is, up to now, rigorously proved) can be summarized as follows.

The physical vacuum or (semiclassical) space-time has to be considered on a certain level of resolution as a two-story structure. It consists of a relatively smooth “surface layer” formed by an intricate web of overlapping *lumps* (the *physical points*) and representing the quasi-continuous medium we experience as ordinary space-time. Beneath this surface there exists a more irregular and wildly fluctuating “underworld” of a distinctly discrete and stochastic nature (stochastic compared to our ordinary level of resolution; at the very bottom the underlying dynamics may well be deterministic!). Its perhaps most characteristic feature is a peculiar *non-local* dynamical behavior observed in [5] and further analyzed below as it plays a decisive role in the understanding of quantum theory. Each of these two stories has its own physical or dynamical *distance function* or *metric* and, typically, there will exist a certain amount of direct interactions or exchange of information in this mentioned underground between regions (or lumps) lying a certain distance apart with respect to the distance concept holding sway in the coarse grained surface structure (i.e. our classical space-time).

Our whole approach is technically based on what we call a *cellular network*, dubbed *QX* for short (“quantum space”), as the most primordial substratum in our framework. This network is assumed to have been evolved in the distant past (“*big bang*”) from a certain chaotic *initial phase* denoted by  $QX_0$  (and which is characterized, among other things, by the complete absence of stable patterns) through a regime of geometric change (called a *geometric phase transition zone*)

into a new phase  $QX/ST$ . This latter phase represents the above described two story *superstructure*, i.e. the underlying primordial network superposed by a coarser network consisting of a web of lumps, that is, certain subgraphs with a particularly dense internal connectivity among the respective nodes (and playing the role of the “*physical points*”). These physical points are considered to be the constituents of the relatively smooth surface structure,  $ST$ . This new phase,  $QX/ST$ , is the epoch our universe is roaming in since the moment when space-time emerged from this mentioned underground as an approximately separate entity.

It is a peculiar feature of this kind of geometric phase transition (described in much more detail in [5]) that it equips the mentioned surface- or superstructure of lumps with a so-called *Nahwirkungsprinzip*, while on a finer level of resolution there remain a lot of additional non-local interactions among distant lumps of, however, a more subtle nature. We will argue in the following that this almost hidden non-local web of exchange of information, which arises quite naturally in our approach, plays a decisive role in the formation of quantum theory as an effective continuum theory incorporating certain non-local gross features of the depth structure of space-time (about such a possibility was already speculated in [6] – and possibly also elsewhere – as a way out of the so-called EPR-paradoxon).

It goes without saying that, given the complexity of the task and the long and entangled history of the subject, such things cannot simply be proved in a rigorous sense of the word as there does not even exist a universally accepted framework from which to start, not even in ordinary (non)orthodox quantum theory. By necessity our approach has to be, at least in this preparatory stage, speculative to some extent and has to be based on a more or less loose (or strong, depending on the point of view) web of arguments mixed with a certain amount of “educated guesswork”.

Furthermore, as the interpretation or epistemology of quantum theory has such a long and involved history of its own we have to refrain from recapitulating too much of this nightmarish and mind boggling subject. That is, lack of space prevents us from giving full credit to many researchers in this field. This would afford a full monograph and would give the paper a perhaps too “philosophical” touch. We therefore concentrate in the rest of this introductory section on mentioning and quoting in loose order those approaches and points of view which appear to be similar (at least to some extent) in spirit to our own working philosophy and make some comments and annotations. In the next section we discuss two approaches in slightly more detail as they are related a little bit closer to our own one in several technical respects.

We begin this brief historical part with two general remarks by von Weizsäcker ([7], similar ideas were also entertained by Wheeler, see e.g. [8]) which strike the key of our paper.

... space-time is not the background but a surface aspect of reality ...

It is extremely improbable that this reality (i.e. quantum reality) will be describable as consisting of events which are localized in space and time.

The translocal phase relations are “surplus information” not lack of information. Quantum theory knows more, not less, than local classical physics.

A well known critic of the orthodox Copenhagen interpretation was Einstein. Here are some illuminating utterances which are in our view very much to the point. More about his original scientific attitude can be found in the beautiful essay of Stachel in [9] or [10]

It is ... to be expected that behind quantum mechanics there lies a lawfulness and a description that refer to the individual system. That it is not attainable within the bounds or concepts taken from classical mechanics is clear.

I do not at all doubt that the contemporary quantum theory (more exactly “quantum mechanics”) is the most complete theory compatible with experience, as long as one bases the description on the concepts of material point and potential energy as fundamental concepts.

(The latter remark is taken from [10]). We want to stress the fundamental importance of the underlying insight being conveyed in these remarks. It is a crucial observation that quantum mechanics (as we know it) happens to be just the description of the “quantum world” if one starts from the core concepts of classical mechanics like e.g. position, momentum etc. Following this line one may get a very specific and biased class of observables while excluding other possible elementary concepts. This contextual and historical dependence of theories, frameworks and whole working philosophies is frequently overlooked or, at least, not sufficiently appreciated. The problems we still have with quantum theory may just result from a too selective choice we have made in the past.

In other words, while quantum theory has made the first steps away from the mechanistic *particle picture*, it has still retained many of its conceptual ingredients and has molded it into the hybrid of the so-called *wave-particle duality*. A similar point of view was held by e.g. Schrödinger (see his beautiful and extensive biography, [12]).

A further point worth of mentioning is Einsteins open attitude towards the *discrete* and the *continuum*. Many quotations can again be found in ([9] p.27ff). Another source (mentioned already in [5]) is his commentary in [11] on the contribution of Menger (*geometry of lumps* and *statistical metrical space*).

Haeretic views were also held by Dirac, a fact which is perhaps not so widely known (cf. e.g. his biography, [13] p.201ff). He in fact tried for many years to revive a modern *aether concept* as a common receptacle for all the physical processes (see also [14]). Similar ideas were uttered by Bell ([15]) or T.D.Lee ([16]), to mention a few.

The arguments for the absence of such an underlying substrate are in our view far from being convincing and are rather typical for what is called a paradigm, i.e. an adopted working philosophy which, when accepted, tends to become very rigid and constellates and frames our whole attitude towards the occurring phenomena and their codification in the form of theoretical concepts. We think that this paradigm is responsible to some extent for the interpretational difficulties and seemingly paradoxical language of orthodox quantum mechanics. We are convinced that quantum mechanics would become considerably less paradoxical if we were prepared to realize the ubiquitous interference phenomena (which are in fact the pivotal point of quantum theory) and the complex structure as part of the quantum information conveyed by extended excitation patterns roaming this largely hidden “underground”.

One of the consequences of taking the possibility of such a hidden and subtly organized substructure not taking into account and regarding, instead of that, space-time as the primordial receptacle is the attitude to consider e.g. *wave functions* and their seeming breakdown as mere subjective artifacts. This becomes particularly apparent in the usual discussion of the *double slit experiment*. In our view it is difficult to deny that there is “something” passing “through” both slits in the undisturbed situation in each individual experiment. But this “something” cannot be so easily detected as long as the intricate substructure of space-time is not realized.

It is no question that the old (mechanical) aether concept is almost empty, but its emptiness resulted from another even older paradigmatic preoccupation of that time, i.e. the fiction of an empty, a priori and independently existing geometric *background space* which is then permeated by some medium called aether. One should rather regard (background) space-time as only a part of an underlying more complex substrate in the way we have indicated above.

This whole bundle of problems and ideas belongs in fact to a much wider topic, which originated already with Leibniz and Mach (see e.g. the lively debate in [17]) and has been lucidly clarified by Einstein. It is the almost universal topic of the role of so-called *ideal elements* in scientific theories. As their role in quantum theory will be a central theme of our paper we give it a closer inspection in the next section and close the introduction with some (as we think) deep remarks and reservations uttered by Scrooge in [18] to which we fully subscribe.

Wave functions are real for the same reason that quarks and symmetries are ... Any system is in a definite state *whether any humans are observing it or not*; the state is not described by a position or a momentum but by a wave function.

It seems to me that none of this forces us to stop thinking of the wave functions as real, it just behaves in ways that we are not used to, including instantaneous changes.

(Weinbergs own utterance on p.143): The positivist concentration on ob-

servables like particle positions and momenta has stood in the way of a realist interpretation of quantum mechanics in which the wave function is the representation of physical reality.

The following sections, 2 to 4, are of a preparatory nature, that is, they provide the necessary background, motivations and concepts, thus paving the ground for the central sections, 5 and 6, of the paper in which quantum theory is derived as a low-energy effective theory of a more primordial theory, living on the Planck scale. In a short aside we speculate about the possibility that this fundamental theory may also underly *string theory* (near the end of section 5). In section 7 we briefly indicate in what directions our approach has to be further developed in order to address the so-called *measurement problem* and/or the emergence of *macroscopic behavior*.

## 2 The Description of our own Working Philosophy

In this section we want to discuss the pieces of our own working philosophy in more detail and relate it to two other approaches presented in the more recent past, which seem to be developed in a similar spirit. The one of the mentioned approaches is expounded in two longer papers by Smolin ([19],[20]) and stems from the period before he embarked on the *loop quantum gravity program*. The other is the work of 't Hooft about a presumed *cellular automaton* substrate underlying quantum theory (see [21] to [23]). While these two frameworks differ in several respects from each other, each of them shares, on the other side, a bundle of ideas with a certain particular strand in our own approach.

### 2.1 A short Review of Smolin's Ideas, the Role of Ideal Elements and our own Point of View

In this subsection we concentrate mainly on the epistemological and foundational aspects of Smolin's work as they are of particular importance for the understanding of the (sub)structure of quantum theory. The more technical aspects and the concrete implementation are postponed to the following sections. The greater part of the epistemological ideas can be found in [19].

Central in this respect is the notion and role of *ideal elements* in physical model theories. Smolin argues that practically all our theories contain - by necessity - so-called *ideal elements* as they typically deal only with a portion of our universe. He describes them as absolute or background structures which are not themselves determined by solving any dynamical equations or, put differently, elements of the mathematical structure whose interpretation requires the existence of things outside of the dynamical system described by the theory. He argues

that both *Mach's Principle* and the *quantum mechanical measurement problem* are cases in point, both of which are crucial parts of the even greater problem of constructing a sensible quantum cosmology. To give another but related definition, we can say with Einstein that something that acts but is not acted upon serves as an ideal element like e.g. *inertial systems* in special relativity (cf. [24]).

The reason why this theme is so carefully discussed by Smolin is the necessity to formulate a theory of quantum gravity that does not depend on an absolute background space. This is the place where the *Machian Philosophy* comes into the play in form of a sceptical attitude towards the existence of space-time as a metaphysical and a priori substratum. The reader who is interested in the actual content of this (a little bit poetic) principle should consult [17] or e.g. [25]. Another illuminating characterisation, stemming from Westpfahl ([27]), is quoted in [26].

... all tendencies which try to reduce all the phenomena which cannot be described by laws of nature (viz. field equations) to cosmological causes

As our primary concern in this paper is the creation of an underlying more primordial theory which contains ordinary quantum theory as an effective and derived stochastic theory, we will henceforth concentrate more on the role of ideal elements in quantum theory. Perhaps a little bit surprisingly, we will show later that in a certain way such a Machian strategy, i.e. explaining seemingly local features of a theory by a *nonlocal* influence of the (distant) environment, will also work in quantum theory.

Now, what are the ideal elements in quantum theory? Deviating slightly from the analysis of Smolin we concentrate at the moment not so much on the infamous measurement problem but on another (in our view) central structural element of quantum theory, i.e. the *superposition principle* together with the genuine *complex structure* of the theory.

### **Conjecture 2.1 (Ideal Elements in Quantum Theory)**

1. *In a similar sense as Smolin did, we conjecture that the complex Hilbert space structure of ordinary quantum theory (and in particular the superposition principle) are playing the role of ideal elements in quantum theory. They encode in a local way a nonlocal stochastic interaction between the lumps of the surface structure ST.*
2. *Furthermore we claim that a good deal of the observed local quantum fluctuations and randomness has its origin in the fact that ordinary quantum theory is, by necessity, that is, by its very definition, a theory of only a small portion of the universe, with this portion being open to permanent nonlocal interaction with the (distant) regions of the quantum environment.*

3. Put differently, as in the Machian concept of inertia, we assume that the mentioned ideal structural elements of quantum theory encode in  $a$ , on the surface, local way nonlocal effects which originate on a more primordial level and which make the local version of the theory, formulated in macroscopic space-time, a stochastic one. The deeper reason for this is that this local formulation describes (while being unaware of it)  $a$ , in some sense, open system.

The last remark leads to another central theme of a *realistic approach* towards quantum theory, viz. the nature and origin of statistics in the quantum realm. Smolin remarks in [20] by referring to various recent observations in quantum cosmology (e.g. [28]) that there seems to be no local coordinate invariant distinction between *real* statistical fluctuations (in the “ordinary” sense) and *virtual* quantum fluctuations. The lesson we learned from Einstein is then the following.

**Observation 2.2** *If the distinction of two phenomena depends on the system of reference then these, superficially different, phenomena are actually of the same kin.*

Consequently our program can be described as follows.

**Programme 2.3** *Find an underlying more primordial model theory in which virtual quantum fluctuations are ordinary statistical fluctuations.*

This is now the point where the various strategies bifurcate from each other. In [20] Smolin chose to further develop the Nelson programme of stochastic (quantum) mechanics, which is based on the picture of a particle moving in a quasi-brownian environment with, however, a quite peculiar *diffusion behavior* not observed in the classical regime (as to this programme see e.g. [29] or [30]). Smolin then embeds the system in a background gravitational field and argues that this strategy shows that and how the Hilbert space framework has to be transcended. At the end of the paper he discusses certain models dealing with nonlocal hidden variables.

We should say that this was, at least to some extent, a strategy we also pursued in earlier times. In [31] we argued for example that the Nelson approach has to be developed further in the direction of *nonmarkovian mechanics* and made some tentative steps along these lines. Our main argument was that the fluctuating but *passive* background in the Nelson-approach (viz. an ideal element par excellence) has to become dynamical due to the (back)reaction of the randomly moving quantum particle, which, in our view, can no longer be neglected in the quantum regime. If one now averages over this dynamical background one would get an evolution equation for the particle itself which contains now a *memory kernel* (as the averaging process will collect retardation effects), i.e. which becomes *nonmarkovian*. The technical implementation of this programme, however,



turned out to be extraordinarily ambitious and we hesitated at that time to push the work beyond the preprint status.

What all these and related approaches are having in common is that they stick to a particle concept as fundamental building block, i.e. it is assumed that there exists a discrete entity which moves in a fluctuating background. This is, however, exactly the concept we choose to abandon in our more recent approach in which quantum objects are, on the contrary, assumed to be extended excitation patterns roaming our two story network environment but carrying certain discrete particle properties which are observed in suitable measurement set-ups. On the other hand, it may well be that the older ansatz turns out to be a certain useful approximation to this more complex picture.

## 2.2 't Hooft's Framework plus some Comments

The starting point for 't Hooft is a little bit different from the preceding approach. His emphasis lies on the discreteness of physics at the Planck scale and is thus related to the other strand of ideas on which our own framework is based. It is his aim to derive quantum behavior on a large scale (i.e. also by a kind of coarse graining) from suitable deterministic *cellular automaton laws* holding sway on a more primordial scale (see [21] to [23]). There exist earlier related ideas scattered in the literature which were inspired by the concept of cellular automata (e.g. [32] or [33]). In his most recent contribution ([23]) he argues that certain versions of hidden variable theories must be revived in the face of problems in quantum gravity and that space, time and matter all have to be *discrete* at bottom.

Another point he rightly emphasizes is that (obviously) the primordial degrees of freedom are *not* describing electrons or any other particles, but microscopic variables at scales comparable to the Planck scale. This is exactly in line with what we said above and with our own framework. We will readdress this particular aspect in the next subsection under the catchword of *the problem of scales*.

Of particular relevance for our enterprise is the following somewhat related argument against the many critics of such deterministic approaches in the quantum realm. 't Hooft reasons that quantum theory provides a completely adequate framework on its natural scale of resolution and that there is no chance to replace it on this scale by some classical or deterministic model theory. But it may well be that a model theory being deterministic on the Planck scale generates the statistical quantum laws via coarse graining on their natural scale, thus invalidating the consequences of, say, the *Bell inequalities*.

While we are very sympathetic with this programme we would like to comment on some differences as compared to our own framework. The main difference, we think, is that we do not base our analysis on a rigid a priori fixed lattice structure but, instead of that, regard the geometric wiring diagram underlying the cellular network as a full fledged dynamical system of its own which interacts with the node states (which, on the other side, are the only variables in a cellular au-

tomaton). Thus geometry, dimension, metrical properties, near- and far-order all become dynamical *collective quantities* which are assumed to *coevolve*. The underlying philosophy is of course that ultimately both quantum theory *and* gravity emerge as two seemingly different but in fact related large scale aspects of one and the same underlying theory.

Another important aspect of our programme is that we will show in the following that quantum theory encodes in a at first glance local way on its own natural scale hidden *nonlocal long-range* interactions among the really microscopic degrees of freedom, living on a more primordial level.

## 2.3 The Problem of Scales

It is frequently argued that the attempt to relate e.g. quantum physics on its presently accessible middle energy scales to some underlying and largely hidden primordial theory, living on, say, the Planck scale, is virtually impossible due to the huge difference in orders of magnitude between the two regimes. This is called the *problem of scales*. There is certainly more than a grain of truth in this criticism but we think one can turn this seeming difficulty into an advantage by pursuing the following strategy.

As in the physics of the *critical point* (in, say, statistical mechanics or lattice quantum field theory), any continuum theory which is nontrivial, that is, which has correlations and patterns extending over non-zero scales, must necessarily be in a *critical* or at least *near-critical* state on the microscopic scale, as all finite length scales will shrink to zero in the *continuum limit*. That is, it must have very long range correlations on that scale, which is typically only the case near or at the critical point.

By the same token, to a given continuum theory will belong a whole *universality class* of microscopic theories which lead to the same macroscopic consequences. Applied to e.g. general relativity such a point of view is expressed in [34]. A perhaps even more radical opinion is expressed in [35] (there are in fact quite a few other interesting ideas to be found in this book), running under the catchword *random dynamics*. The central hypothesis is that the structure of the theories on the low- or middle-energy side of the energy spectrum is to a large extent independent of the form of the hypothesized fundamental theories on the ultra-high energy side and that the structure of the former ones is rather a consequence of the way how the coarse graining is performed.

This point of view is partly corroborated by the observations we make below when we attempt to derive quantum theory from our network model. On the other side this does not mean (at least in our view) that a particular fundamental theory does not exist or that we shall be unable to discriminate between different model theories in the future (note that a similar standpoint could have been adopted with respect to quantum theory as the underlying theory of classical mechanics). The correct conclusion to draw is that it is not reasonable (in the beginning) to

concentrate too much on certain (possibly wrong or unimportant) microscopic details but better have the gross features right. That is, the real task may rather consist of extracting the possibly few crucial characteristics which the primordial theory must contain and which, in the end, survive the coarse graining limit.

## 3 A brief Résumé of the Properties of the Two-Story Network Substratum

### 3.1 Some General Remarks

As the technical details and underlying working philosophy can to a large part be found in refs. [1] to [5] with special emphasis on [5], we will be very brief. We emulate the underlying substratum of our world, or, more specifically, of our space-time (quantum) vacuum (containing however in addition all the existing quantum and macro objects as extended excitation patterns!) by what we call a cellular network.

This discrete structure consists of elementary *nodes*,  $n_i$ , which interact (or exchange information) with each other via *bonds*,  $b_{ik}$ , playing the role of (in this context) not further reducible (abstract) elements. The possible internal structure of the nodes (modules) or bonds (interaction chanel) is emulated by discrete internal state spaces carried by the nodes/bonds. The node set is assumed to be large but finite or countable. The bond  $b_{ik}$  is assumed to connect the nodes  $n_i, n_k$ . The internal states of the nodes/bonds are denoted by  $s_i, J_{ik}$  respectively. As our philosophy is, to generate complex behavior out of simple models we, typically, make simple choices for them, one being e.g.

$$s_i \in q \cdot \mathbb{Z} \quad , \quad J_{ik} \in \{-1, 0, +1\} \quad (1)$$

with  $q$  an elementary quantum of information.

As in our approach the bond states are dynamical degrees of freedom which, a fortiori, can be switched off or on, the *wiring*, that is the pure *geometry* of the network is also an emergent, dynamical property and is *not* given in advance. Consequently the nodes, bonds are typically not ordered in a more or less regular array, a lattice say, with a fixed near-/far-order. This implies that *geometry* will become to some extent a *relational* (Machian) concept and is no longer an *ideal element* (cf. the discussion in sect. 2).

On the other side, as in cellular automata, the node and bond states are updated (for convenience) in discrete clock time steps,  $t = z \cdot \tau$ ,  $z \in \mathbb{Z}$  and  $\tau$  being an elementary clock time interval. This updating is given by some *local* dynamical law (examples given below). In this context *local* means that the node/bond states are changed at each clock time step according to a prescription with input the overall state of a certain neighborhood (in some topology) of the

node/bond under discussion. We want however to emphasize that  $t$  is *not* to be confounded with some *physical time*, which, for its part, is also considered to be an emergent coarse grained quantity. The well known *problem of time* is, for the time being, not treated in detail in the following, as it is a big problem of its own, needing a careful and separate analysis of its own (see however [36] or [37]). That is, at the moment the above clock time is neither considered to be dynamical nor observer dependent. There is however a brief discussion of the presumed emergence of a new primordial time scale which sets the scale for the regime where quantum fluctuations hold sway (see below).

In [5] we gave examples of local dynamical laws which, we presume, are capable of encoding the kind of geometric unfolding we are expecting. An important ingredient is what we call a *hysteresis dynamics*, that is, the bonds, or more properly the interactions  $J_{ik}$ , are switched off under appropriate conditions of the local network environment. An example of such a local law is the following:

**Definition 3.1 (Example of a Local Law)** *At each clock time step a certain quantum  $q$  is exchanged between, say, the nodes  $n_i, n_k$ , connected by the bond  $b_{ik}$  such that*

$$s_i(t + \tau) - s_i(t) = q \cdot \sum_k J_{ki}(t) \quad (2)$$

(i.e. if  $J_{ki} = +1$  a quantum  $q$  flows from  $n_k$  to  $n_i$  etc.)

The second part of the law describes the back reaction on the bonds (and is, typically, more subtle). This is the place where the so-called hysteresis interval enters the stage. We assume the existence of two critical parameters  $0 \leq \lambda_1 \leq \lambda_2$  with:

$$J_{ik}(t + \tau) = 0 \quad \text{if} \quad |s_i(t) - s_k(t)| =: |s_{ik}(t)| > \lambda_2 \quad (3)$$

$$J_{ik}(t + \tau) = \pm 1 \quad \text{if} \quad 0 < \pm s_{ik}(t) < \lambda_1 \quad (4)$$

with the special proviso that

$$J_{ik}(t + \tau) = J_{ik}(t) \quad \text{if} \quad s_{ik}(t) = 0 \quad (5)$$

On the other side

$$J_{ik}(t + \tau) = \begin{cases} \pm 1 & J_{ik}(t) \neq 0 \\ 0 & J_{ik}(t) = 0 \end{cases} \quad \text{if} \quad \lambda_1 \leq \pm s_{ik}(t) \leq \lambda_2 \quad (6)$$

In other words, bonds are switched off if local spatial charge fluctuations are too large, switched on again if they are too small, their orientation following the sign of local charge differences, or remain inactive.

Remark: Another interesting law arises if one exchanges the role of  $\lambda_1$  and  $\lambda_2$  in the above law, that is, bonds are switched off if the local node fluctuations are too small and are switched on again if they exceed  $\lambda_2$ . We emulated all these laws on a computer and studied a lot of network properties. The latter law has the peculiar feature that it turned out to have very short *transients* in the simulations, i.e. it reaches an *attractor* in a very short clock time. Furthermore this attractor turned out to be very regular, that is, it had a very short period of typically six, the whole network returned in a previous state after only six clock time steps, which is quite remarkable, given the seeming complexity of the evolution and the huge phase space ([38]).

Some characteristic features of these class of laws are the following.

**Observation 3.2 (Gauge Invariance)**

1. *The dynamics depends only on the local charge differences,  $s_i - s_k$  and nowhere on the absolute values  $s_i$  itself, i.e. it is to some extent relational.*
2. *The total charge,  $Q := \sum_{n_i} s_i$ , is conserved under the evolution. One could e.g. choose a boundary condition like  $Q = 0$ , which may be considered as a kind of gauge fixing.*

The following point we consider to be of central importance, irrespective of the concrete network law under discussion.

**Observation 3.3** *We expect that really interesting fundamental laws display the following generic patterns. They typically consist of more or less two parts, encoding the interaction of two primordial substructures, described a little bit sloppily by the catchwords geometry and matter.*

1. *geometry acting on matter*
2. *matter acting on geometry*

*Usually the first part of the dynamical law seems to be relatively simple and transparent, while the second part is typically much more involved.*

Remark: Note that these criteria are fulfilled by our above example, where the first part is more or less a *conservation law*. The geometric structure is the wiring of the network, i.e. the global bond state. A classical case in point is *general relativity*, where the first part consists of the *geodesic motion* of matter, the latter part of the *Einstein equations*. We will later show that even quantum theory is already of this type if understood or looked upon in a certain (new?) way.

In [5] we chose to concentrate on the geometric structure of the network, thus neglecting most of the details of the microscopic network state and the

dynamics. In the corresponding reduced graphical representation a bond,  $b_{ik}$  was drawn between the nodes  $n_i, n_k$  in the time-dependent graph,  $G(V, E(t))$ , iff the bond-state,  $J_{ik}(t)$ , was different from zero ( $V, E$  the set of nodes, edges (bonds) respectively). In a next step these graphs were considered as members of a certain probability space,  $\mathcal{G}(n, p)$ , of *random graphs* over  $n$  nodes and with the *edge probability*  $0 \leq p \leq 1$ .

**Remark 3.4** *One can as well choose a slightly higher resolution by keeping trace of the sign of the bond-interaction, that is  $J_{ik} = +/ - 1$ , and identify  $J_{ik} = +1$  with a directed bond,  $d_{ik}$ , pointing from  $n_i$  to  $n_k$  or vice versa for  $J_{ik} = -1$ . This would lead to a so-called directed graph.*

In [5] we were particularly interested in certain subgraphs of a typical random graph taken from  $\mathcal{G}(n, p)$ , their size, number, degree of overlap and entanglement. These particular subgraphs are called *cliques* in graph theory and are (in a technical sense) maximal complete subgraphs or subsimplices, that is, all the pairs of nodes belonging to a clique are connected by a bond and the cliques are the *maximal* elements in the respective chains of subsimplices (ordered by inclusion). In more physical terms we also called them *lumps* or *physical points*.

For later purposes we note that a graph carries a *natural metric*:

**Definition 3.5** *The (natural) distance,  $d(n_i, n_k)$ , between two nodes,  $n_i, n_k$ , is the length of a minimal path (a geodesic) connecting them, its length given by the number of bonds of the path. This distance defines a metric on  $G$  ( $d := \infty$  if the nodes are lying in different components).*

Remark: There are other interesting notions of distance one can envisage on a graph. One is studied in [3] and is related to similar concepts in *non-commutative geometry*. Another is discussed at the end of [19].

## 3.2 The Web of Lumps

We argued in [5] that what we experience as (quasi)classical space-time and (on a higher resolution) as *quantum vacuum*, consists roughly of two or rather three regimes. At the very bottom we have the level of the primordial network with its corresponding primordial length- and (clock) time scales, correlation lengths/times etc. On the next level we have the web of lumps or physical points, i.e. the web of overlapping cliques. this level defines a new group of corresponding (length) scales, as it is usually the case if a new phase emerges. We conjecture that these emerging scales are the infamous *Planck-units*, e.g.  $l_P, t_P$  etc. On the macroscopic surface level, which is the regime directly accessible to us, the internal structure of the physical points is no longer visible, we observe a (quasi) continuum as *background space* which, on a slightly finer scale, is roamed by *quantum fluctuations*, representing the *residual* low-energy effects of what is happening on the deeper levels.

In [5] we made a relatively detailed analysis of this web of lumps within the framework of *random graphs*. We calculated the typical size of these lumps, their number, mutual overlap, expected size of the infinitesimal neighborhood of a typical lump etc. On the other side, some problems remained open for which we have, at the moment, only partial answers (which is however no wonder, given the enormous complexity of the behavior of the underlying network). Note that in the random graph approach we concentrated solely on the wiring diagram of the network and studied its properties in a purely statistical way. It became apparent that in order to follow its dynamical evolution in more detail, something like a non-equilibrium statistical mechanics for such systems is called for. Furthermore, the pure random graph picture is possibly (or rather: probably) not sufficient to explain every aspect of the *unfolding process* of the network towards the expected new phase,  $QX/ST$ . This, however, has to be expected since we have learned that the unfolding towards a level both of higher order and diversity may need some *fine-tuning* and is not the expected to be the ordinary situation (a catchword being “complexity at the edge of chaos”; see [39] to [41]).

As far as the derivation of quantum theory as an *effective theory* is concerned we therefore will assume that our network has made a transition into this new phase,  $QX/ST$ , consisting, on a limit scale of magnification, of a web of lumps, fluctuating around some stable positions and/or average shape with their degree of overlap and their mutual (macroscopic) distance also fluctuating around some average value. Some aspects of this picture are then very reminiscent of model systems studied in the past (investigations initiated by Menger et al; see e.g. [42] and further references given there). We sum up what we have said so far in the following brief résumé. (Note that in the following, in contrast to [5], we denote lumps or physical points by  $P_i$  for notational convenience).

### Observation 3.6 (The Two-Story Concept of $QX/ST$ )

1. *Given a network or graph,  $G$ , of the above kind, we can construct its associated clique graph  $\mathcal{C}_G$  (vertices being the lumps or cliques, the bonds given by overlap of cliques, see [5]), and thus establish the two story concept, mentioned already in the introduction. We hence have two kinds of distances and metric (causal) relations in the network, the one defined by the original node distance in  $G$ , the other by the distance between lumps (defined by overlap) in  $\mathcal{C}_G$ .*
2. *It is important that two lumps,  $P_1, P_2$ , which are some distance apart in  $\mathcal{C}_G$ , may nevertheless be connected by a certain (possibly substantial) number of interbonds or short paths, extending from nodes in  $P_1$  to nodes in  $P_2$  (see the construction of the cliques described in the preceding sections).*
3. *That is, there may exist two types of information transport or correlation being exchanged in the network. A relatively coherent (and presumably*

quasi-classical) one, exchanged among the lumps, obeying a so-called Nah-wirkungsprinzip and a more stochastic and less organized one (of quantum nature) between individual groups of nodes lying in lumps, which may be a certain distance apart, and which, nevertheless, can be almost instantaneous.

As we are in the following mainly concerned with the information flow between the various lumps,  $P_i$ , making up the orderparameter manifold,  $ST$ , we develop below a couple of useful concepts and tools which are adapted this new emergent level of description. Particularly important for the *near-/far-order* in  $ST$  (which is related to its *causal structure*) are the various degrees of connectedness among the physical points,  $P_i$ . The following abbreviation is useful. For  $n_i, n_k$  (not being) connected by a bond we write

$$n_i \sim n_k \quad (n_i \not\sim n_k) \quad (7)$$

We then have

**Observation 3.7** *From the definition of the cliques it follows*

1.  $n_i \not\sim n_k$  implies that they are lying in different  $P_\nu$ 's.
2.  $P_\nu, P_\mu$  are disjoint, i.e.  $P_\nu \cap P_\mu = \emptyset$  iff

$$\forall n_\nu \in P_\nu \exists n_\mu \in P_\mu \text{ with } n_\nu \not\sim n_\mu \text{ or vice versa} \quad (8)$$

This shows that it may well be that  $P_\nu \cap P_\mu = \emptyset$  while the two lumps have still a lot of *interbonds*, i.e. bonds connecting the one with the other. The guiding idea is however that the respective vertex sets  $V_{P_\nu}$  and  $V_{P_\mu}$ , as a whole, will typically be weaker entangled with each other than the nodes within  $P_\nu$  or  $P_\mu$  when the unfolding process is fully developed.

**Observation/Definition 3.8** *With respect to the above clique graph or web of lumps we can speak of an*

1. interior bond of a given  $P_\nu$ , i.e:

$$b_{ik} \text{ with } n_i, n_k \in P_\nu \quad (9)$$

2. exterior bond with respect to a given  $P_\nu$ , i.e:

$$b_{ik} \text{ with } n_i, n_k \notin P_\nu \quad (10)$$

3. an interbond, i.e:

$$b_{ik} \text{ with } n_i \in P_\nu, n_k \in P_\mu, \nu \neq \mu \quad (11)$$



4. a common bond of  $P_\nu, P_\mu$  if  $b_{ik}$  is an interior bond both of  $P_\nu$  and  $P_\mu$ .

5. a true interbond  $b_{ik}$  if for  $\nu \neq \mu$ :

$$n_i \in P_\nu, n_k \in P_\mu, n_k \notin P_\nu \quad (12)$$

6. We then have the relation for given  $P_\nu, P_\mu$ :

$$\{\text{interbonds}\} - \{\text{common bonds}\} = \{\text{true interbonds}\} \quad (13)$$

We noted above that we now have two (metric) structures on the network or graph, the original one with its neighborhood structure and distance function,  $d(n_i, n_j)$ , and the superstructure given by the clique graph and its coarse grained neighborhood structure of physical points and coarse grained distance function,  $d_{cl}(S_i, S_j)$ , which we regard as a protoform of our ordinary macroscopic distance. Note that there may still exist a substantial number of interbonds on the lower level between supernodes  $P_i, P_j$  with  $d_{cl}(S_i, S_j) \gg 1$ .

In the physics of many degrees of freedom what really matters, or gives “distance” a physical content, is not so much some abstract notion of distance but the strength of interaction or correlation between the various constituents. Given two node sets  $A, B$  or the respective subgraphs we can count the number of bonds connecting them and regard this as a measure of their direct mutual dynamical coupling.

**Definition 3.9 (Connectivity of Subgraphs)** *With  $A, B$  being two node sets in a given graph, we denote by  $|A \sim B|$  the actual number of bonds connecting the nodes of  $A$  with the nodes of  $B$  and by  $|A \sim B|_m$  the maximal possible number. Then we call*

$$0 \leq c_{AB} := |A \sim B| / |A \sim B|_m \leq 1 \quad (14)$$

*the connectivity of the pair  $A, B$ . It represents the probability that a randomly chosen pair of nodes  $n_A \in A, n_B \in B$  is connected by a bond.  $|A \sim B|_m$  depends however on their relative position in  $G$ .*

**Observation 3.10** *We have the following relations*

$$A \cap B = \emptyset \rightarrow |A \sim B|_m = |A| \cdot |B| \quad (15)$$

*( $|A|, |B|$  the respective number of nodes), hence*

$$c_{AB} = |A \sim B| / |A| \cdot |B| \quad (16)$$

$$A = B \rightarrow |A \sim B|_m = \binom{|A|}{2} \quad (17)$$

For their intersection we have in general

$$A \cap B \neq \emptyset \rightarrow |A \sim B|_m = |(A - B) \sim (B - A)|_m \\ + |(A \Delta B) \sim (A \cap B)|_m + |(A \cap B) \sim (A \cap B)|_m \quad (18)$$

i.e.:

$$|A \sim B|_m = |A - B| \cdot |B - A| + |A \cap B| \cdot (|A - B| + |B - A|) + \binom{|A \cap B|}{2} \quad (19)$$

with  $A \Delta B$  being the symmetric difference of  $A$  and  $B$ .

In our papers, cited above, we argued that it is reasonable to treat certain (bulk) aspects of the network properties and its evolution in a statistical way. This is particularly necessary if we want to extract some coarse-grained information from it which depends on the collective behavior of many nodes and bonds. That is, we have to take averages over certain portions of the network and/or (possibly appreciable) clock-time intervals (which may, nevertheless, correspond to infinitesimal intervals on a more macroscopic scale). In the following we introduce and describe only those collective variables which may become relevant in the further analysis.

We will deal with our network mainly on the level of the web of lumps or the *clique graph*, abbreviated by  $ST$ . At each clock-time step or whole clock-time interval,  $QX/ST$  consists of a certain overlapping web of lumps or cliques,  $P_i$ , having some average size (in graph theory usually called *order*), that is, number of nodes

$$\langle r(P) \rangle := \langle \text{order of clique} \rangle \quad (20)$$

where the statistical average is taken over the network and/or an appropriate clock-time interval. We assume of course that the phase,  $QX/ST$ , the network or clique graph is occupying, is sufficiently stable or slowly varying, so that the actual clique size,  $\langle r(P_i)(t) \rangle$  is assumed to fluctuate not too much around this average value,  $\langle r(P) \rangle$ . In other words, the lumps are assumed to be *fuzzy*.

In the same sense the average vertex degree can be defined

$$\langle v(P) \rangle := \langle \text{vertex degree} \rangle \quad (21)$$

the average number of active bonds per clique

$$\langle N(P) \rangle := \langle \text{number of active bonds per clique} \rangle \quad (22)$$

and the respective averages over the bonds pointing *inward* or *outward*, that is, connecting two nodes, the one lying inside, the other outside the lump under discussion.

$$\langle N_{in,out}(P) \rangle := \langle \text{number of in- , out - bonds} \rangle \quad (23)$$

Note that the bond is counted as *in* if the *orientation* is  $J_{ik} = +1$  with  $n_i$  lying outside the lump.

We consider these latter variables (averaged or non-averaged) as being particularly relevant as they tell us something about the *charge fluctuations* inside the lumps and through the (fuzzy) boundary. At each clock-time step we have an internal charge,  $Q(t; P)$ , of the respective lump,  $P$ , and due to our dynamical network law it holds

$$\begin{aligned} Q(t + \tau; P) - Q(t; P) &=: \Delta Q(t; P) = \sum_{n_i \in P} s_i(t + \tau) - \sum_{n_i \in P} s_i(t) \\ &= q(N_{in}(t; P) - N_{out}(t; P)) = q\left(\sum_{in} J_{ik}(t; P) - \sum_{out} J_{ik}(t; P)\right) \quad (24) \end{aligned}$$

and the corresponding equations when taken with the respective averages.

## 4 An Alternative Look upon Quantum Theory

### 4.1 Isolating the Pure “Quantum Phenomenon”

After this series of preparatory steps we now enter the central part of the paper. We have our model system  $QX/ST$  and want to derive quantum theory from it as an effective theory living near the continuous “surface” of this structure. In doing this we have first to clarify two points. What do we actually mean by quantum theory (understood as a general conceptual framework) and, second, what are the large-scale phenomena we expect to emerge or survive in this low-energy limit (compared to the primordial Planck scale).

As to the first question, typical models of, say, quantum field theory are usually inspired by their classical counterparts which are then “quantized” (following a certain, one may venture to say heuristic, scheme). Furthermore, *covariance*, *spectrum condition etc.* are usually imposed. In a first step we think it is easier to concentrate on, what we regard as the essential and model-independent quantum phenomena, leaving, for the moment, aside all the additional complications. Take e.g. *special relativity*. To derive it as a macroscopic phenomenon from our underlying  $QX/ST$ -network we need a more detailed understanding of the emergence of *macroscopic time*, which is a veritable problem of its own and will not be dealt with here (this does not mean that it cannot be done; it means rather it has to be done in a separate investigation due to natural limits of space), for reviews see e.g. [36] and [37].

**Conjecture 4.1** *We conjecture that the model-independent content of “the” quantum phenomenon is mainly encoded in the generically complex structure together with the superposition principle, both making up the complex Hilbert-space structure of ordinary quantum theory and leading to the seemingly paradoxical entanglement phenomena.*

Remark: The soundness of this conjecture will be further illuminated below and is perhaps underpinned by the following citations taken from [12] p.216, 218 (see also [43]):

(Schrödinger) ... the complex structure as carrier of the unobservable phase information ...  
 (Dirac) ... the phase quantity was very well hidden in nature ...

These model-independent ingredients can, in a first step, best be studied in the non-relativistic regime by investigating the (hidden) structure of e.g. the Schrödinger equation.

In this context it is sometimes argued that some of the peculiar quantum phenomena displayed by the Schrödinger equation, e.g. its instantaneous spreading, is an artefact of its lack of *relativistic covariance*. We think, this is beside the point to some extent. We carefully analyzed this issue in [44] and [45] and showed that exactly the same processes are at work in both the relativistic and the non-relativistic regime, pointing to a kind of underlying *entanglement* or *non-locality* of quantum phenomena, being ubiquitous in the whole field. Put sloppily one may say:

**Conjecture 4.2** *Only observables behave locally or causally while states do, typically, not. This is also the case in the relativistic regime. On the other side, the Schrödinger equation describes the evolution of a state!, not of a quantum field.*

The underlying reason for this is the following. The *energy-momentum content* or *transfer* of an *observable* is typically *two-sided*, that is, the Fourier-support or spectrum (in fact an operator-valued distribution as long as the operator is not smeared with a testfunction)

$$\hat{A}(p) := (2\pi)^{-2} \int e^{ipx} A(x) d^4x \quad (25)$$

$p = (p_0, \mathbf{p}), x = (x_0, \mathbf{x})$  four-vectors,  $px$  the Minkowski-scalar-product

$$A(x) = e^{iPx} \cdot A \cdot e^{-iPx} \quad P = (H, \mathbf{P}) \quad (26)$$

has the following property:

**Lemma 4.3** *With  $p$  belonging to the energy-momentum support of  $A = A^*$ , put sloppily  $\hat{A}(p) \neq 0$  at  $p$  in a distributional sense,  $-p$  belongs also to the support of  $A$  or  $\hat{A}$ .*

Proof:

$$(\hat{A})(p)^* = \left( \int e^{ipx} A(x) d^4x \right)^* = \int e^{-ipx} A(x) d^4x = \hat{A}(-p) \quad (27)$$

that is,  $\hat{A}(p) \neq 0$  at  $p$  as operator-valued distribution implies the same for its adjoint at  $-p$   $\square$

Remark: In the theory of operator algebras the above spectrum is usually called the *Arveson-spectrum* (see e.g. [46] or [47]).

States, on the other side, are prepared by applying such local observables (or field operators) to the ground state or *vacuum*,  $\Omega$ . As this is the state with lowest energy (usually set to zero), the negative energy support of  $A$ , when applied to  $\Omega$ , is by definition cut off, that is,

$$\text{supp}(A \cdot \Omega) \subset V^+ \quad (28)$$

( $V^+$  the closed *forward cone*). This inevitable asymmetry in the support of states compared to observables leads to their different (causal) behavior as has been analysed in our above mentioned papers and has nothing to do with Lorentz-covariance. It is rather a pure *quantum phenomenon*.

## 4.2 A Different Look at Schrödingers Equation

In standard quantum theory the Schrödinger equation is considered to be only one of a couple of possible representations of quantum dynamics, i.e. the *configuration-space* version. We want to argue in this subsection that one should perhaps change ones point of view in at least two respects. For one, as we learned from general relativity, space and time seem to have a very peculiar significance of their own and do not seem to be a mere mode of representation of physics among many other ones being possible (in contrast to the point of view, suggested by e.g. ordinary quantum theory). For another, Schrödingers equation is at first glance linear (which was severely criticized by Einstein), but it is only linear with respect to its *complex structure* which makes a big difference as we will show.

In our (perhaps heretic) view, shared however by quite a few others, it rather represents an intricate *dynamical entanglement* of two underlying and in the ordinary approach largely hidden quantities, which have survived the coarse-graining process if one goes “bottom-up”, starting from the Planck-scale. If one disentangles this single complex-linear equation it becomes the non-linear coupled evolution of two real equations. This fact has long been known, we think however that our interpretation of this phenomenon is a different one.

The two perhaps most widely known fields where this has also been done are the *stochastic mechanics* developed by Nelson and several precursors, and the so-called *Bohmian mechanics* of Bohm et al. To keep the length of our paper reasonable we will mention only very few sources, where the interested reader can look up more references, and make up his own mind concerning the pros and cons of the various contributions. We mentioned already [20] and [29] to [31] which

deal primarily with stochastic mechanics. From the many papers about Bohmian mechanics we cite the following, as we think, quite readable accounts [48] to [50] with reference [49] being perhaps particularly interesting as in it Nelson's approach has been compared with Bohm's own approach.

In more recent times so-called *stochastic collapse models* have also become fashionable (a small selection being [51] to [53]). As far as the randomly fluctuating environment, employed in some of these models, can be viewed as a coarse-grained epiphenomenon deriving from a more fundamental layer of microphysics, they may be considered to be phenomenological or effective theories, describing a more complex underlying dynamics. Note however that in our approach the fluctuating environment is a dynamical agent of its own which acts but is also acted upon by "matter". Furthermore we do not employ a *particle picture* in the form of, say, small objects immersed in a random medium. It may however have its value as a certain approximation.

In disentangling the Schrödinger equation we follow a traditional line of non-orthodox quantum theory but with a to some extent different working philosophy in mind, formulated in observation 3.3. That is, we conjecture that a really fundamental law consists always of two parts, i) "geometry" acting on "matter" and ii) "matter" acting on "geometry", with the first equation being typically significantly simpler than the latter one. With

$$\psi = \rho^{1/2} \cdot e^{iS/\hbar} \quad (29)$$

Schrödinger's equation

$$i\hbar\partial_t\psi = -\hbar^2/2m \cdot \Delta\psi + V\psi \quad (30)$$

decomposes into the conservation equation

$$\partial_t\rho = -\nabla \cdot (\rho \cdot \mathbf{v}) \text{ with } \mathbf{v} = 1/m \cdot \nabla S \quad (31)$$

and the dynamical equation

$$-\partial_t S = 1/2m \cdot (\nabla S)^2 + V - \hbar^2/2m \cdot \Delta\sqrt{\rho}/\sqrt{\rho} \quad (32)$$

which may be considered as a *quantum deformation* of the *Hamilton-Jacobi equation*, the deformation being the so-called *quantum potential* (Bohm)

$$V_q := -\hbar^2/2m \cdot \Delta\sqrt{\rho}/\sqrt{\rho} \quad (33)$$

**Observation 4.4** *The quantum potential,  $V_q$ , is the only place where the "quantum" openly enters. Thus, any attempt to explain quantum mechanics as arising from a more primordial level has to give an explanation for the emergence of this term.*

We will show that, in fact, this term encodes in our framework the non-local entanglement between the various lumps making up  $ST$  in our network  $QX/ST$ .

**Remark 4.5** *Note that in the conservation equation the quantity  $\rho$  enters, while in the second equation it is  $\sqrt{\rho}$ , the peculiar statistical nature of which will play a considerable role in the following sections.*

There exist corresponding equations for several particles. For two particles we have e.g.

$$\partial_t \rho = -(\nabla_1 \cdot (\rho \nabla_1 S / m_1) + \nabla_2 \cdot (\rho \nabla_2 S / m_2)) \quad (34)$$

$$\begin{aligned} -\partial_t S = & (2m_1)^{-1}(\nabla_1 S)^2 + (2m_2)^{-1}(\nabla_2 S)^2 + V(x_1, x_2) \\ & - (\hbar^2/2m_1)\Delta_1(\sqrt{\rho})/\sqrt{\rho} - (\hbar^2/2m_2)\Delta_2(\sqrt{\rho})/\sqrt{\rho} \end{aligned} \quad (35)$$

In the following conjecture we indicate how this disentangled Schrödinger equation fits in our general picture.

**Conjecture 4.6** *The conservation equation encodes the action of “geometry” on “matter”, the deformed Hamilton-Jacobi equation the action of “matter” on “geometry”, where  $\rho$  is supposed to relate to “matter”, the phase-function  $S$  to “geometry”. These structural elements survive the huge gap between the Planck-scale and the middle-energy regime of, say, quantum theory in the form of large scale excitation patterns.*

The role of the above quantities in ordinary quantum theory is the following. We have

$$\int \bar{\psi} i \partial_t \psi d^3x = - \int \rho \partial_t S d^3x + i \int \rho^{1/2} \partial_t \rho^{1/2} d^3x \quad (36)$$

The latter term on the rhs equals  $(i/2)\partial_t \int \rho d^3x = 0$ . Hence  $-\rho \cdot \partial_t S$  may be interpreted as an *energy density*. Correspondingly we have for the momentum:

$$\int \bar{\psi} i^{-1} \partial_j \psi d^3x = \int \rho \partial_j S d^3x + (2i)^{-1} \int \partial_j \rho d^3x \quad (37)$$

The last term on the rhs is a surface term and hence vanishes; thus  $\rho \cdot \partial_x S$  can be regarded as a *momentum density*.

Before we go on we want to address again the longstanding question of the reality of, say, the wave function  $\psi$  or of its constituents  $\rho$  and  $S$ . This issue was already discussed in subsection 2.2 and we now want to add a few more facets. One of the most prominent scientists in favor of an undulatory ontological nature of  $\psi$  was Schrödinger (see e.g. his contribution in the de Broglie volume, which

contains quite a few remarkable observations supporting his point of view). One argument against this interpretation is the so-called problem of *polydimensions*, i.e. the structure of  $\psi$  in the case of several particles, which is in that context defined over the cartesian product of, say,  $\mathbb{R}^3$  and hence can, at first glance, no longer be interpreted as an extended excitation living in  $\mathbb{R}^3$ . This point was already raised by Heisenberg at the 1927-Solvay conference (see [12] p.240f).

... I see nothing in the calculations of Mr. Schrödinger that justifies his hope that it will be possible to explain or understand in three dimensions the results from polydimensions

While Schrödinger did not seem to have a coherent underlying theory supporting his own point of view, he evidently envisioned excitations in  $\mathbb{R}^3$  interpenetrating each other (perhaps in a soliton-like manner). We will sketch our own ideas concerning this important question in a preliminary form in the following conjecture.

**Conjecture 4.7** [*N-Particle Wave Functions*]

1. *We think the system-theoretic task, solved by “Nature” with the “invention” of quantum theory, consists of storing effectively and stably a certain amount of information in a noisy background, represented by QX/ST.*
2. *As the individual grains,  $P_i$ , i.e. the physical points, comprise in our picture still a lot of internal degrees of freedom (the nodes and bonds belonging to  $P_i$ ), there should be ample internal space to store the local pieces of different excitation patterns, living and interpenetrating each other in one and the same environment (by the way, a task also solved by the human brain).*
3. *One possible method consists of letting only a small fraction of internal degrees of freedom contribute to each extended wave pattern. This appears to be reasonable anyhow, as quantum theory as we understand it is actually a weak, low-energy excitation of QX/ST as compared to, say, the Planck energy.*
4. *An interesting situation is expected to emerge when the number of particles,  $N$ , becomes appreciable or macroscopic. There should exist a critical local occupation rate above which this weak-field-approximation breaks down. By the same token, the picture of interpenetrating (and to a certain extent individual) particle excitations will become problematical. This impossibility to store a too large amount of information in a finite space and the respective threshold are in our view the interface region where quantum mechanical many-body systems start to behave macroscopically. This picture has to be worked out in much more detail and will be treated elsewhere as it draws on a huge corpus of material of its own which has accumulated in the past (for a cursory discussion see however section 7).*



## 5 The Collective Dynamics of the Web of Lumps

In the preceding sections we described how a certain extended structure of lumps may emerge within the network as the consequence of a geometric phase transition or geometric reorganisation of the underlying network. In this section we want to argue that this new phase, dubbed  $QX/ST$ , is accompanied by the emergence of a new class of characteristic *collective variables* and their respective *cooperative behavior* which does not yet exist on the more primordial level and which is the reason why this new geometric phase may be rightly called an *orderparameter manifold*.

Remark: It is not accidental that such a point of view and/or language is a little bit in the spirit of the physics of *self-organisation* or *synergetics* (see e.g. [54] or [55]) as it is in our view pretty much to the point. After all, our underlying medium is a complex *dynamical system* consisting of a huge number of elementary constituents. It is hence reasonable to employ the corresponding arsenal of tools and concepts. We will, due to limits of space, however only introduce a limited amount of the technical machinery below.

The above described superstructure,  $ST$ , overlying the primordial network,  $QX$ , is the deepest of, presumably, a whole hierarchy of increasingly coarse-grained and smooth levels, each of which typically supporting and generating its own emergent set of natural variables and laws. In a sense  $ST$  functions as a *shell* which *decouples* and *shields* the upper stories of the hierarchy from the most primordial one. The crucial question to answer is the following.

**Programme 5.1** *Find the modes in which the system operates on a given scale of resolution!*

This is the characteristic question, emerging also in the physics of self-organisation, the Landau-picture of elementary excitations in, say, quantum fluids, or the theory of renormalisation in high-energy physics, to mention a few fields.

**Conjecture 5.2** *As to the cooperative behavior of our web of lumps we assume the following: The orderparameter manifold,  $ST$ , overlying the primordial network,  $QX$ , enslaves (a notion taken also from synergetics, see [54]) the more primordial degrees of freedom, that means in our context the node- and bond-variables and forces them into a specific cooperative undulatory behavior, put differently, the geometric phase transition manifest itself, among other things, by means of a new collective excitation mode. In brief, order parameters are collective variables which enslave subsystems.*

*This new collective mode is a spatio-temporal undulation pattern of the  $Q(P)$ - and the  $N_{in,out}(P)$ -field (see the end of section 3.2) being entangled with it via the underlying dynamical law. The emergent characteristic parameters of this*

*excitation mode* are an oscillation- or correlation time,  $t_{pl}$ , and a characteristic (correlation- or oscillation-) length,  $l_{pl}$ , supposed to characterize the Planck-regime.

While we cannot prove this conjecture at the moment, as we are presently unable to solve the very complicated dynamical behavior of our network in greater detail and follow it through its presumed phase transition regime into the new phase,  $QX/ST$ , we will at least try to transform it into an *educated guess* by providing a row of more or less coherent arguments supporting this hypothesis.

1. We mentioned in section 3 in the remark after the definition of a, as we think, typical model of a dynamical network law (definition 3.1) that a slight variation of the law yields a new model having very short *transients* and reaches *periodic state cycles* or (*attractors*) having only period six, i.e. the whole network state returns into exactly the same state after only four time steps. For sufficiently small networks (only a few nodes) one can do the calculations by hand and follow the evolution and oscillation in detail. For networks up to several thousand nodes the evolution has been implemented on a computer (see [38]). Note that such a behavior is quite remarkable, given the huge accessible phase space and the relatively complicated evolution law.

Remark: Such puzzling and still quite mysterious phenomena were also mentioned by S.Kauffman in his study of so-called *switching nets* (cf. e.g. [39] p.112 ff or [40]). Such *oscillating media* are also observed in synergetics (cf. e.g. section 4 in [56]).

This shows that such things may already happen on the level of the primordial network. In other respects however, such a peculiar law is too regular as it does not allow for a diversified pattern creation on the higher levels. The interesting evolution laws are sitting, according to the working philosophy expounded e.g. in the above cited literature (see also [41]) and which we are largely sharing, at the edge between *chaos* and *order*.

2. On the other side, the evolution law given in definition (3.1) itself, i.e. with the other rule of switching-on and -off of interaction,  $J_{ik}$ , does not show these short transients and periodic state cycles already on the level of its primordial nodes and bonds and therefore seems to behave more erratically at least on the most fundamental level (as we learned from a numerical investigation of its characteristics which were also studied in quite some detail in [38]). It evidently behaves more stochastically and may be nearer to this mentioned edge between order and chaos (sitting supposedly on the other side). Unfortunately our computer capacities were not large enough to study it on the higher level of the web of lumps., which would have implied, among other things, a permanent calculation of these cliques and

their dynamics (as to such analytical techniques cf. [5]). It may, however, be possible that it supports such oscillating modes in its fluctuation spectrum on this more advanced level of lumps.

3. One can however try to get some qualitative glimpses how such networks may behave by concentrating on a fixed given lump,  $P$ , say. Let us assume that at a certain clock-time,  $t_0$ , its charge,  $Q(t_0; P)$ , happens to be somewhat below the average of the charges of the surrounding lumps directly interacting with it (i.e. more or less the so-called *local group*, see [5]). While we cannot make an exact prediction about the corresponding states of the bonds in the immediate environment of  $P$ , the laws which we introduced above suggest however that a deficiency of charge in  $P$  will induce after some clock-time cycles a reorientation of bonds in favor of more bonds pointing inward (with, and this is important, a certain tendency of *overcompensating*!). In other words, after a certain lapse of time the charge in  $P$  will be above average with the excess charge now being presumably greater than the deficiency of charge in the cycle before. Again this surplus charge induces an even more pronounced reorientation of neighboring bonds, leading to an even greater deficiency of charge within  $P$ , and so on. This process will not stop until it has reached a *characteristic excitation level* being typical for both the network (law) under discussion and the specific phase,  $QX/ST$  it is occupying.

The above qualitative analysis shows that there may well be such a spatio-temporal *undulation pattern* within the excitation spectrum of the network, extending over the whole web of lumps and being perhaps similar to an array of coupled self-oscillating subunits. The characteristic *oscillation period* of these subunits (which are assumed to comprise more or less the individual lumps and their immediate neighborhoods) is the *Planck-time*,  $t_P$ , the characteristic *wave- or coherence length* is the *Planck-length*,  $l_P$ , being, on the other side, a measure of the typical diameter of a lump or its local neighborhood. These emergent and autonomously generated quantities figure then as the elementary building blocks of the corresponding continuum concepts, length, time, energy etc. on the smoother, that is, more coarse-grained scales and show how a complex system is capable of generating its own intrinsic scales by a process of *self-organisation*. That is, these elementary units need not be put in by hand! That is, our subclass of networks seem to belong to the class of *oscillating media* described in e.g. [56].

It seems now worthwhile to introduce a limited amount of machinery, being employed in the theory of self-organization or dynamical systems (see e.g. [54] to [58]), in order to make the following analysis more concise. Our networks are, among other things, complex dynamical systems. On the most fundamental level their dynamics is assumed to be *deterministic* (whereas this is not necessarily a crucial point due to the *shielding phenomenon* which decouples the various levels from each other to some extent). In contrast to most of the dynamical

systems, discussed in the literature, the number of their constituents is, on the one side, very large. On the other side, both the evolution and the phase space is *discrete*. This prevents the immediate application of the usual tools of continuum mathematics in the analysis of the geometry of, say, *attracting sets* and the like. On the other side, after a certain coarse-graining, when dealing e.g. with the web of lumps, the dynamics and the medium may be considered, in a good approximation, to be quasi-continuous. On the other side, on the higher levels the dynamics is no longer deterministic due to the “integrating out” of degrees of freedom and corresponding loss of information. Instead of that we may get certain phenomenological dynamical field equations superposed by a stochastic component implementing the additional noise in the network. This then is the typical scenario of synergetics. It is our aim to show that low-energy quantum theory is exactly of this sort, that is, certain emergent dynamical field laws plus a *non-local* stochastic component.

To exhibit the interplay of statistical averaging and the underlying microscopic evolution, we introduce the following concepts. We take a particular initial state,  $x_0$ , say, which lies in the *basin of attraction* of an attracting set of states,  $X$ , say, or already in  $X$  itself. We assume a discrete evolution law, i.e. an *iterated map*,  $M$ . In general,  $M$  is not *invertible* but only an *endomorphism*. For technical reasons it is usually assumed that it is *onto*. Then we can follow the path the system takes with starting point  $x_0$ , i.e:

$$x_n := M^n x_0 \quad , \quad M^n := M \cdots M \text{ (n-times)} \quad (38)$$

For  $n \rightarrow \infty$  the states,  $x_n$ , wander through the attractor,  $X$ . With  $f$  an observable, defined on the microstate  $x$ , we can define its *time average* (provided it exists):

$$\bar{f} := \lim_{T \rightarrow \infty} 1/T \sum_{n=0}^T f(M^n x_0) \quad (39)$$

Under certain conditions there exists an *invariant measure*,  $\mu$ , on  $X$  so that time averages become ensemble averages with respect to  $\mu$ , the averages being independent of the starting point,  $x_0$ . Systems with this property are called *ergodic* (cf. [59]). How this may come about can be seen as follows. Instead of  $x_0$  we take a full initial probability distribution,  $\rho_0$ . Under the map  $M$  it goes over in a new distribution,  $\rho_1$ :

$$M : \rho_0(x) \rightarrow \rho_1(x) \quad (40)$$

and in general

$$\rho_{n+1} = \int \rho_n(y) \cdot \delta(x - My) dy \quad (41)$$

(called for whatever reason the Frobenius-Perron-equation).

We arrive at an *invariant density* if we have a *fixed point*, that is:

$$\rho_{n+1}(x) = \rho_n(x) = \rho(x) \quad (42)$$

In general it is reasonable to switch to a slightly more general point of view and consider *invariant measures* instead of densities (or to allow for distributional densities), that is  $\rho(x) \rightarrow \mu(A)$  with  $A$  some measurable set. The notion of invariance is now expressed as

$$\mu(A) = \mu(M^{-1}A) \quad \text{for all measurable sets} \quad (43)$$

We then have

$$\langle f \rangle = \bar{f} := \lim 1/T \sum_{n=0}^T f(M^n x_0) = \int_X f(x) d\mu \quad \text{independent of } x_0 \quad (44)$$

While  $M$  is not necessarily invertible, invariance of  $\mu$  implies that it induces an *isometric* map on the function space (Hilbert space)  $L^2(\mu)$ , that is, it holds

$$\int |U_M f|^2 d\mu = \int |f|^2 d\mu \quad \text{with} \quad (U_M f)(x) := f(Mx) \quad (45)$$

For this to make sense the above mentioned technical property that  $M$  is onto is needed.

**Observation 5.3** *Note that  $\mu(A)$  measures in effect the average time the system occupies states belonging to  $A$ .*

Another useful tool in the analysis of such complex dynamical systems is the method of the *correlation functions* and their *spectral representation*. Suppose again that  $f(x)$  is an observable defined on our state space. Its *time-autocorrelation function* is defined by

$$\langle f(t_1) \cdot f(t_2) \rangle^C := \langle f(t_1) \cdot f(t_2) \rangle - \langle f \rangle^2 = \lim 1/T \sum_{t=0}^T (f(t+t_1) - \bar{f}) \cdot (f(t+t_2) - \bar{f}) \quad (46)$$

(provided that such limits do exist). Fourier transformation leads to

$$\langle f(t_1) \cdot f(t_2) \rangle^C = (2\pi)^{-1/2} \cdot \int e^{-i(t_1-t_2)} \cdot c(\omega) d\omega \quad (47)$$

with  $c(\omega)d\omega$  a positive measure, the so-called *power spectrum* of the respective observable. If we have an invariant measure these correlations can alternatively

be calculated in the ensemble approach (note that, as the evolution is usually *dissipative*, that is, one-sided or not-invertible,  $t_1, t_2$  have to be chosen positive).

The power spectrum can be used to characterize the type of evolution or/and attractor. If there are e.g. sharp peaks in the spectrum they signal the existence of extended oscillating modes, buried in the (possibly continuous) background noise. This is exactly the situation we are speculating about in the case of our networks and the particular phase,  $QX/ST$ , which is, in the language of dynamical systems, an attractor. What we will say in the following about the qualitative dynamics of our network or the web of lumps overlying it, should be considered within this wider context which we only briefly sketched above. One should however note that our networks are far more complex than the dynamical systems usually considered in the corresponding literature. For the time being we simply have to assume that our phase  $QX/ST$  corresponds to an attracting set, that long-time averages are practically independent of the initial configuration and that an invariant measure exists on the attractor, corresponding to  $QX/ST$ , so that time averages can be expressed by averages with respect to this measure. This lays the basis for a *statistical treatment* of the problems to be discussed in the following.

Our qualitative discussion of the propensity for an oscillating behavior of our medium (the web of lumps) suggests that we will find a sharp peak (actually two as the spectrum is symmetric) in the Fourier spectrum of the (clock-)time autocorrelation function of the charge,  $Q(P, t)$ , of a given fixed lump,  $P$ . That is, we conjecture

$$\langle Q(P, t) \cdot Q(P, t + t_1) \rangle^C = mode(\omega_{pl}) + \int remainder \quad (48)$$

The spatio-temporal excitation pattern, resulting from the cooperation of these individual resonating lumps, will presumably be much more complex. It could be tested via the correlation among different lumps, i.e:

$$\langle Q(P', t') \cdot Q(P, t) \rangle \quad (49)$$

As we want to concentrate in this investigation on the derivation of low-energy quantum theory, we will postpone a more detailed discussion of the leading collective modes being prevalent in the vacuum on the Planck-scale. We will only briefly indicate what kind of excitation patterns we are having in mind, as it shows that our web of lumps may already contain the so-called *string-bits*, i.e the prerequisites to allow for string-like excitations as cooperative patterns made of local clusters of lumps ([60]).

One possible excitation pattern may have the structure of densely entangled *chain mail*, built from elementary loops (the chain-links) consisting on their side of lumps which resonate in a cooperative manner so that a certain amount of charge is pulsating around the respective loop. It is therefore perhaps not too

far-fetched to tentatively associate the lumps with the notorious *D0-branes* and their entangled interaction being modelled by a *matrix-model* (see e.g. [61],[62]).

Remark: We denote in the following these presumed local clusters of cooperating lumps by  $C_i$ .

What is important for our further discussion is that the *wave-number*,  $k_{pl}$ , the pendant of  $\omega_{pl}$ , should not be associated with some plane-wave excitations. The relevant *normal modes* are rather such entangled collective excitations as, say, the above mentioned chain mail. The characteristic parameter,  $k_{pl}$ , is then just the dual of  $l_{pl}$ , which, on its side, characterizes the diameter of the local resonating lumps or the elementary patterns being built from them, that is the, the local clusters  $C_i$ .

The characteristic parameters of our web of lumps are related to each other as follows. We assume that the characteristic frequency is the *Planck-frequency*, the characteristic wave-number,  $k_{pl}$ , the *Planck-wave-number* etc. These variables are related with each other via:

$$E_{pl} = \hbar \cdot \omega_{pl} \quad p_{pl} = \hbar \cdot k_{pl} \quad (50)$$

$$E_{pl} \cdot t_{pl} = \hbar \quad p_{pl} \cdot l_{pl} = \hbar \quad (51)$$

$$t_{pl} = l_{pl}/c \quad l_{pl} = (G\hbar/c^3)^{1/2} \quad (52)$$

with  $E_{pl}, p_{pl}, t_{pl}, l_{pl}$  Planck-energy, -momentum, -time, -length respectively. The remaining contributions in the spectrum we assume to be a certain amount of patternless noise plus longer wave-length modulations of this *ground oscillation* with typical wave-length  $l_{pl}$  and oscillation-time  $t_{pl}$  (see the next section).

To simplify the following discussion and to exhibit the red thread, we neglect, in a first step, all the stochastic fluctuations and possible modulations of this ground wave and concentrate on the leading mode contribution. That is, we write for a fixed but arbitrary lump (suppressing for the moment an additional phase factor)

$$Q(P, t) \approx Q_{av} + Q_0 \cdot \cos(\omega_{pl} \cdot t) \quad (53)$$

$Q_{av}$  is the average charge of the lump under discussion, which we assume to be the same over the web of lumps,  $Q_0$  is the amplitude of the oscillation. In the following section we have to deal with our network and/or the overlying web of lumps on several clearly separated scales. The same holds for the respective natural observables emerging on the various scales. The scale of our web of lumps we assume to be associated with the Planck-scale, abbreviated by  $[L_{pl}]$ . We further introduce the scale of ordinary quantum theory, denoted by  $[L_{qm}]$ , with the property

$$[L_{qm}] \gg [L_{pl}] \quad (54)$$

On the scale  $[L_{pl}]$  we can incorporate the additional spatio-temporal fine structure, induced by the resonating local clusters,  $C_i$ , of the above described ground mode by introducing a space-time dependent phase-factor,  $\varphi(x, t)$ , which varies over the local clusters of lumps forming the elementary building blocks of, say, the above mentioned chain mail. As it varies spatially on these very short scales, that is,  $l_{pl}$ , it is almost invariant on average on larger scales as e.g.  $[L_{qm}]$  and can practically be ignored on this larger scale. Furthermore, it should not change the imposed frequency,  $\omega_{pl}$  in an appreciable way. We hence expect:

$$\partial_t \varphi \ll \omega_{pl} \quad , \quad |\partial_x \varphi| = \mathcal{O}(k_{pl}) \quad (55)$$

#### Assumption 5.4 (Fine Structure of the Physical Vacuum)

$$Q(x, t) = Q_{av} + Q_0 \cdot \cos(\omega_{pl}t + \varphi(x, t)) + \text{noise} \quad (56)$$

## 6 Quantum Theory as a Low-Energy Limit of the Dynamics of the Web of Lumps

### 6.1 The Building Blocks

We now embark on the derivation of the building blocks of *low-energy quantum theory* as coarse-grained quantities from our web of lumps. Most important are the two quantities  $\rho$  and  $S$ . From the *quantum-Hamilton-Jacobi equation* of section 4 we surmise that  $S$  will play a particularly significant role as a unifying concept, mediating between the Planck-, the quantum- and the classical regime.

We begin with some heuristic considerations concerning the supposed role of  $S$  as mediator between these very different scales. Adopting so-called *natural* units with  $c$  and  $\hbar$  chosen dimensionless, the phase function  $S$  becomes a dimensionless quantity which should be considered as a sort of *generalized action*. On the one side our working philosophy is that physics is discrete on the fundamental level. On the other side we want to interpret quantities like  $S$ , occurring in ordinary quantum theory, as something which really exists. This leads to the following conjecture.

**Conjecture 6.1** *The phase function  $S$  describes the coarse-grained effect of an undulation phenomenon on the level of the web of lumps. More precisely, it counts the (dimensionless) number of oscillations or switches of or within the underlying fundamental medium with respect to a fixed but arbitrary reference point.  $-\partial_t S$  and  $\nabla S$  have the dimensions of energy and momentum in natural units, that is, inverse time and inverse length or frequency and wave number characterizing the undulations in this presumed substratum.*



**Observation 6.2** *Note that  $\rho$  is intrinsically positive in ordinary quantum theory while the phase function  $S$  can be positive or negative. This makes no problems in the ordinary interpretation where  $\rho$  is a probability density and  $S$  has a relatively fictitious meaning. If we want to attribute some ontological meaning to them the situation is different (see below). It turns out that this “problem of signs” may be used as a guiding principle in isolating the relevant quantities which survive the coarse-graining process (cf. the related “problem of scales”, discussed in section 2.3)*

The collective excitation pattern, we have described so far, will now serve as the *carrier wave* of smooth long-wave-length undulations, which *modulate* the ground excitation pattern and which vary on their natural scale,  $[L_{qm}]$ . In other words, we will relate the objects of, say, *low-energy quantum theory* to certain *low-frequency/long-wave-length modulations* of this underlying very-high-frequency oscillation mode.

**Assumption 6.3 (Quantum Theory)** *Staying within our approximative picture, we expect the following modulation of the dominant mode in the carrier wave if there are low-energy quantum objects around. These quantum objects are assumed to be implemented by modulations of small amplitude and frequency (compared to the Planck-characteristics,  $Q_0$  and  $\omega_{pl}$ ). That is*

$$Q(P, t) \approx Q_{av} + (Q_0 + a(P, t)) \cdot \cos(\omega_{pl} \cdot t + \varphi(P, t) + \varepsilon(P, t)) \quad (57)$$

with

$$a(x, t), \varepsilon(x, t) \text{ varying on scale } [L_{qm}] \gg [L_{pl}] \quad (58)$$

and

$$|a(P, t)| \ll Q_0, \partial_t \varepsilon(P, t) \ll \omega_{pl} \quad (59)$$

In other words, a quantum object is assumed to consist of some extended information pattern, being impressed on the high-frequency carrier wave, representing on its side the *physical vacuum*. This impressed information consists of both a component implemented as *amplitude modulation* (i.e.,  $a(P, t)$ ) and a component being realized as *phase-modulation* (that is,  $\varepsilon(P, t)$ ).

We interpret this modulation pattern in the following way on the level of lumps. A space-time dependent tiny fraction,  $a(P, t)$ , of the nodes within the given lump,  $P$ , joins the number of nodes (is slaved!),  $Q_0$ , oscillating collectively (in the case  $a > 0$ ) or leaves this set, i.e. falls out of phase (for  $a < 0$ ). At the same time the *momentary frequency* is also changed by a tiny amount,  $\partial_t \varepsilon$ .

**Observation/Definition 6.4** *We now relate  $|a(x, t)|$  with  $\rho(x, t)$  and  $\varepsilon(x, t)$  with  $-S(x, t)$ . ( $\omega_{pl} \cdot t$ ) counts, as kind of a generalized action, the (dimensionless) number of oscillations or switches with respect to some arbitrary reference*

time (or -point).  $\varepsilon(x, t)$  or  $-S(x, t)$  is the local deviation from this global quantity (measured on scale  $[L_{qm}]$ ), induced by the presence of quantum objects. It is satisfying that  $a(x, t)$  can be both positive or negative in contrast to  $\rho(x, t)$ . We assume however that it is either (in this low-energy approximation) positive or! negative, as we should expect both particle- and hole-excitations in our medium. These may correspond (in the old but perhaps not outdated Dirac-picture) to particles and anti-particles. The detection of the presence of an excitation (or particle) in low-energy quantum theory should, on the other side, not depend on the sign of  $a(x, t)$ .

With the help of the above formula (57) we can now give  $-\partial_t S$  and  $\nabla S$  a precise microscopic meaning.

**Observation 6.5**  $-\partial_t S$  describes the local deviation of the momentary frequency of the undulation pattern from the vacuum value,  $\omega_{pl}$ , on scale  $[L_{qm}]$ .  $-\nabla S \cdot dx$  measures on the one side the differential change of the number of periods of oscillation with respect to the reference point,  $\omega_{pl} \cdot t$ . On the other side, in order to be a physical observable, it must also have a meaning which can be locally measured in the medium.  $\varphi(x, t)$  varies spatially on scale  $l_{pl}$ , i.e. it yields a wave number of order  $k_{pl}$ .  $-\nabla S \cdot (dx/|dx|)$  is the local deviation of the wave number from  $k_{pl}$  in direction  $dx$ , measured on scale  $[L_{qm}]$ .  $\nabla S$  itself points in the direction of the maximal decrease.

Proof of the latter statement: Take a change of phase of  $2\pi$  on scale  $L_{qm}$  in direction  $dx$ . We have

$$2\pi = \nabla S \cdot dx = \nabla S \cdot (dx/|dx|)|dx| \quad (60)$$

Hence

$$\lambda = |dx| = 2\pi(\nabla S \cdot dx/|dx|)^{-1} \Rightarrow k = 2\pi/\lambda = \nabla S \cdot dx/|dx| \quad (61)$$

## 6.2 The Quantum Mechanical Continuity Equation

We now come to the derivation of the two defining equations making up quantum theory in e.g. the Schroedinger picture. For one particle we have

$$\partial_t \rho = -\nabla(\rho \cdot \nabla S/m) \quad (62)$$

For a particle excitation we assume  $\rho(x, t) = a(x, t) \geq 0$ . Gauss-law yields

$$\partial_t \int_V a(x, t) d^3x = - \int_{\partial V} (a(x, t) \cdot \nabla S/m) d\mathbf{o} \quad (63)$$

The amount of surplus charge,  $\int a(x, t) d^3x$ , activated by the presence of the quantum excitation in the vacuum and participating in the collective motion, is

conserved in time.  $\rho \cdot \nabla S$  is a momentum density,  $\rho \cdot \nabla S/m$  a “velocity density”. We want to understand the above first law microscopically.

The above equation, interpreted on the level of the web of lumps, tells us that the surplus charge in  $V$  has the tendency to move in the direction of decreasing wave number,  $\nabla S$ , or longer wavelength, with a prefactor,  $m^{-1}$ , which may be considered as a measure of the stiffness of the excitation against change. This seems to be a reasonable behavior.

**Remark 6.6**  *$m$  was already detected in classical mechanics as a measure of resistance of particle motion in the background medium called vacuum. The above microscopic interpretation seems to be consistent with this observation.*

Increase of wave number means shorter wave length, i.e. smaller extension of the local clusters,  $C_i$ , in the direction of  $\nabla S$ . This may imply a more intense coupling among the lumps cooperating in the respective  $C_i$ ’s, which may have the effect that more elementary charges,  $q$ , are participating in the cooperative movement, hence, an increase of  $\int_V a(x, t) d^3x$  in the volume  $V$ . That is, the first equation of low-energy quantum theory may be a reasonable formula also on the microscopic scale.

We want to add at this place a comment about the *normalisation condition* of ordinary quantum mechanics as a probabilistic theory. In our realistic approach, in which  $\rho$  is not considered as some probability density but as kind of an (abstract) amount of charge or information per lump, participating in a collective undulating motion, a conservation law like  $\int \rho d^3x = \text{const}$  may be reasonable. On the other hand, a normalisation to, say,  $\text{const} = 1$  does not make physical sense in this more general non-probabilistic framework. What may, however, be reasonable is a (*projective*) *ray-interpretation* as it is sometimes employed anyhow in quantum theory (as to this more geometric aspect cf. the interesting paper by Ashtekar and Schilling; [63]).

In the same sense as mentioned before (cf. section 3.1), physics on this primordial scale may be independent of the absolute values of node-charges in this *weak-field regime*. What may rather matter is the relational information content being stored in the shape of the excitation pattern. This idea does however not contradict the strong probabilistic flavor of ordinary quantum theory. In this latter framework probabilities are closely linked with observations and measurements. Their outcomes, on the other side, are of course related to the information content of the excitation patterns under discussion but this relation may be a subtle one (see section 7).

### 6.3 The Quantum-Hamilton-Jacobi Equation, the Local Contribution

More demanding is the interpretation of the second equation of low-energy quantum theory. This applies in particular to a microscopic understanding of the

so-called *quantum potential*, which contains the core of quantum behavior. We begin with the first (drift-)term on the rhs of equation (32). Our working philosophy is that the quantum potential contains the non-local stochastic effects while the other terms encode the local and more coherent contributions.

Our analysis in the preceding sections shows that a local change (on scale  $[L_{qm}]$ ) of the *action*,  $S(x, t)$  relative to

$$S_0(x, t) := \omega_{pl}t \text{ or } (\omega_{pl}t + \varphi(x, t)) \quad (64)$$

is accompanied by a change of local wave-number,  $k(x, t)$ . As we have a coherent web of elementary oscillating circuits, a variation of  $k(x, t)$ , that is,  $\nabla S(x, t)$ , will, by the same token, induce a local change of frequency,  $-\partial_t S(x, t)$ . Hence there must exist a *dispersion law*

$$\partial_t S = F(\nabla S) \quad (65)$$

(we neglect, for the moment, the other contributions in (32)).

In the non-relativistic regime we can relate  $d\omega/dk$  to a velocity. The role of velocity in our context is played by  $\nabla S/m$ . Hence, relating  $-\text{partial}_t S$  with  $\omega$  and assuming a power-law behavior, we can infer

#### Observation 6.7

$$-\partial_t S = (2m)^{-1}(\nabla S)^2 \quad (66)$$

On the other side, the potential term,  $V(x)$ , encodes some external, effective force and is model dependent. The really crucial contribution is however the quantum potential,  $V_q$ .

## 6.4 The Quantum Potential

We now come to the most mysterious term in the (re)interpretation of the equations of low-energy quantum mechanics, that is, the quantum potential

$$V_q = -(\hbar^2/2m) \cdot \Delta(\rho^{1/2}) \cdot \rho^{-1/2} \quad (67)$$

We will argue that  $V_q$ , as non-relativistic quantum theory is still relatively near to the classical regime, being perhaps only a small deformation (compared to the Planck scale), is the only component comprising truly stochastic elements in the above representation. By the same token, it is the term being responsible for the seemingly non-local phenomena, being almost ubiquitous in quantum theory. We will phrase it that way:

**Conjecture 6.8** *The quantum potential,  $V_q$ , encodes the non-local aspects of quantum theory in a, superficially, local way.*

**Remark 6.9** *This non-locality is more hidden in the one-particle situation or the self-interaction among the terms within the excitation pattern belonging to a specific quantum object. It becomes more apparent when several quantum objects are involved. On the other hand, the interpretation of the meaning of the physical quantities becomes much more subtle in the latter case (polydimensions); cf. section 7.*

If we ignore this term, we have sort of a classical field theory (as is of course well-known). In the free case ( $V = 0$ ) a solution of

$$\partial_t \rho = -\nabla \cdot (\rho \nabla S / m) , \quad -\partial_t S = (2m)^{-1} (\nabla S)^2 \quad (68)$$

is

$$\rho(x, t) = f(x - vt) , \quad S(x, t) = mvx - mv^2 t / 2 \quad (69)$$

with  $\nabla S = v$ , that is,  $\rho$  spreads “causally”; with  $\rho(x, 0)$  of compact support it remains so for all  $t$ , being merely shifted with velocity  $v$ . This is in sharp contrast to the quantum case, i.e. after addition of  $V_q$ . We remarked in section 4.1 that we typically observe an instantaneous spreading of (quantum) information irrespectively of the details of the model (for the non-relativistic regime see in particular [45]). We emphasize again that this has, in our view, nothing to do with the relativistic non-covariance of ordinary quantum theory but represents rather a (the) pure “quantum phenomenon”.

When scrutinizing the structure of  $V_q$  and having a stochastic interpretation in the back of ones mind, two intriguing features strike the eye.

**Observation 6.10** *First, the prefactor,  $\hbar^2/2m$ , has the dimension of a diffusion coefficient, i.e.  $[l]^2/[t]$ , when energy is identified with frequency.*

*Second, if  $\rho$  is some statistical sum (or average) over a relatively large number of more primordial degrees of freedom,  $\rho^{1/2}$  may just describe the standard deviation or typical fluctuation of the additive quantity,  $\rho$ , about its average.*

Then, the occurrence of the quantum potential in the quantum-Hamilton-Jacobi equation tells us that the local momentary frequency of the undulation pattern is influenced by a diffusive and/or fluctuation contribution. We undertake to clarify the nature of this term in two steps. First, we try to explain its very existence. In a second step we try to provide an argument why it changes the local frequency.

We argued in the preceding sections that, in our view, the apparent non-locality of quantum theory has its origin in the two-storey structure of the medium  $QX/ST$ , that is, the physical vacuum and expresses itself in the (as yet not very well understood) complex superposition principle of ordinary quantum theory. The terms we have discussed so far were of a local nature and can be understood already on the level of the web of overlapping lumps, forming the “surface structure”,  $ST$ , of our network  $QX/ST$ .

We noted however, that the lumps (that is, on a level of lesser resolution, the physical points), even if they are non-overlapping (in other words, being some distance apart in macroscopic space), may nevertheless be connected in the underlying network by some (perhaps even appreciable) number of *interbonds* (cf. section 3.2). Via these interbonds information may be exchanged almost instantaneously. We expect however that this type of information exchange is less *coherent* and less *organized*, that is, more *stochastic* than the information exchange taking place among directly overlapping lumps, i.e between *infinitesimally neighboring* regions of macroscopic space. Our central conjecture is now that the mysterious quantum potential is just the remnant on scale  $[L_{qm}]$  of this kind of non-local coupling between non-neighboring lumps or points!

We may consider each given lump or the respective local cluster,  $C$ , as an *open statistical subsystem*, being embedded in the ambient space,  $QX$ , acting as kind of a *reservoir*. Each of these lumps or local clusters oscillates in its respective local coherent mode, described above, having an amplitude,  $Q_0 + \rho(x, t)$  with the deviation,  $\rho(x, t)$  varying on the larger scale  $[L_{qm}]$ . Being a sum over a relatively large number of more elementary degrees of freedom we expect this deviation of the ground oscillation,  $\rho$ , to fluctuate due to the above described correlations between the different lumps like

$$|\delta\rho| \approx \rho^{1/2} \quad (70)$$

The reason why we expect this kind of fluctuations to be more coherent than the permanent ground fluctuations, due to almost randomly changing bond-orientations among the elementary nodes, is the following. In each lump or local cluster a certain fraction of the nodes/bonds is slaved by the collective mode and behaves relatively coherently. By the same token, *interbonds* between nodes, belonging to these respective sets in the various lumps, are also expected to change their orientations more coherently than bonds, not belonging to these particular sets. The same applies then to the blobs of charge, exchanged via these particular bundles of interbonds connecting the different lumps or local clusters. On the other side, there are a great number of other lumps, our particular lump is connected with via such bundles of interbonds. That is, the incoming or outgoing blobs of charge have different phase relations as the local states in the distant lumps or local clusters, our given lump is connected with, are different. We hence expect these fluctuations neither to be completely correlated nor uncorrelated. We therefore arrive at the following conclusion

**Conclusion 6.11** *The charge fluctuations in a given lump, arising from the above described mechanism, are on the one side expected to be more coherent than the almost patternless groundfluctuations. On the other side, they should be sufficiently random on scale  $L_{qm}$  to justify the above standard fluctuation formula. We answer our first question by claiming that the quantum potential arises*

just from this particular kind of non-local information exchange between distant lumps.

Given that there exists such a type of fluctuation, with  $|\delta\rho| \approx \rho^{1/2}$ , the next question is, how does this fluctuation pattern effect a change of local frequency

$$\delta(-\partial_t S(x, t)) = -(\hbar^2/2m) \cdot \Delta(\rho^{1/2}) \cdot \rho^{-1/2} \quad (71)$$

One may at first make up ones mind about how a mechanism can change at all the local frequency with the corresponding amplitude,  $Q_0 + \varphi(x, t)$  being more or less kept fixed. The period of the ground cycle is of order  $t_{pl}$ . This time interval consists of a consecutive number of *clock-time intervals*,  $\tau$ ,

$$t_{pl} = N \cdot \tau \quad (72)$$

In each clock-time interval,  $\tau$ , the charge in a given lump,  $P$ , is changed by the amount

$$\Delta Q(t, P) = q(N_{in}(t; P) - N_{out}(t; P)) = q\left(\sum_{in} J_{ik}(t; P) - \sum_{out} J_{ik}(t; P)\right) \quad (73)$$

(cf. section 3.2). If this change per clock-time interval is increased locally by some dynamical mechanism which results in a local change of the network state in and around the lump under discussion, more specifically, bigger positive  $\Delta Q$  in the ascending part of the cycle, bigger negative jumps in the descending part, with  $Q_{max}$  kept essentially fixed by some other stabilizing mechanism, the local frequency will increase and vice versa, since the necessary  $Q_0$  is filled up in a shorter or longer clock-time interval. In this sense one may envisage how amplitude and frequency can vary more or less independently.

The quantum potential, and by the same token, its effect on the local frequency, would vanish locally if  $\Delta\rho^{1/2} = 0$ . On the other side, we still expect the local amplitude to fluctuate on average by the amount  $\rho^{1/2}$ . A locally constant  $\rho$  means, according to our interpretation, that the neighboring grains experience the same amount of average fluctuation. We have now to remember that both the ground frequency,  $\omega_{pl}$  or the modulated frequency,  $\omega_{pl} + \partial_t S(x, t)$ , and the corresponding amplitudes are considered to be emergent quantities, being created by an autonomous process of self-organisation within the network  $QX/ST$ . In other words, these particular values within or around some lump or local cluster,  $C$ , are the result of the local network state as a whole. If this local environment is changed, we have to expect the same for the local values of these collective quantities.

If  $\Delta\rho^{1/2} \lessgtr 0$  around a given lump, the charge fluctuation within the lump is greater/smaller on average compared to the surrounding lumps (which can be inferred from Gauss-law). According to our primordial network laws discussed in section 3.1 and its implications on the level of the web of lumps, described in section 3.2, higher charge fluctuation in a lump means a higher level of reorientation of so-called *interbonds* during a cycle of the collective undulation.

### Observation/Definition 6.12

$$\Delta\rho^{1/2} \leq 0 \tag{74}$$

leads to a higher/lower bond-fluctuation rate in the lump or the local cluster under discussion compared to the neighborhood. We call this bond-fluctuation rate the bond-volatility. It is another example of an emergent collective quantity.

We conjecture now that such a higher/lower *bond-volatility* (relative to the surrounding lumps or local clusters) will enhance or diminish the height of the jumps we were talking about above and thus increase or reduce the local frequency. We frankly agree that this is, so far, only a qualitative analysis, but we know of related effects in other fields of physics. See e.g. [64] for a *stochastic triggering or enhancement* of various *resonance phenomena*. We expect a similar mechanism to be at work in our complex dynamical system.

A last point to mention is the physical meaning of the “normalisation” of the term by  $\rho^{1/2}$  in the denominator. Such small deviations from the huge vacuum values, we are talking about, are so-called “*weak-field phenomena*”. We therefore expect that there exists a *linear* relation between the fluctuation of local charge,  $\rho^{1/2}$ , and the number of nodes, being involved in this fluctuation. One would hence get the change of frequency by following the charge variation at a generic node over one cycle. One has hence to divide by the amount of total charge fluctuation in the lump to get the change in frequency. This explains the occurrence of the denominator in the quantum potential. The proportionality of the respective quantities is, on the other side, encoded in the prefactor,  $\hbar^2/2m$ .

## 7 A Brief Commentary on Several-Particle Systems, the Transition towards Macroscopic Systems and State Vector Reduction

In the preceding sections we have mainly discussed the *one-particle quantum theory*. We have omitted so far *several-* and *many-body systems*, the transition to the *macroscopic regime* and the notorious and highly faceted quantum mechanical *measurement problem*. One reason for this restriction was to keep the paper within reasonable length, since some of the above mentioned topics have a long and venerable history of their own and need a separate treatment. We want in the following to only briefly indicate how we plan to proceed in future work, in order to cope with these problems.

In a first step we have to discuss the necessary changes which occur in connection with several quantum objects, roaming through our network. As to this set of questions we made already some preliminary remarks in 4.7. Note also the critical attitude of Heisenberg expressed in the utterance preceding our own conjecture about the so-called *problem of polydimensions*. What we try to accomplish



below is exactly to supply an interpretation of *the results from polydimensions in three dimensions*, the possibility which Schroedinger had in his mind and which Heisenberg considered to be impossible. It becomes however apparent that the situation is much more subtle than Schroedinger probably expected and that this task cannot be accomplished in a naive sense (which Heisenberg rightly criticized). It again turns out, that our concept of the two storeys of space-time or the vacuum is crucial for solving this puzzle, a concept, which Schroedinger did not yet have at his disposal!

Before going into the more technical details, we want to scrutinize the above cited critical dictum of Heisenberg. Is it really impossible to envisage the quantum mechanical several-particle situation in the ordinary 3-dimensional coordinate space we are living in? Let us take a classical  $N$ -particle system with  $N$  sufficiently large and describe it within the framework of classical statistical mechanics. It has, on the one hand, several features which are similar to the quantum mechanical wave-function representation, as both systems have many elementary degrees of freedom. On the other hand, it has the advantage that it can be completely understood!

Again, the so-called *one-particle distribution function*,  $\rho_1(x)$  is both a local probability and (as a *particle density*) a local classical observable, that is, it has an immediate interpretation in 3-dimensional coordinate space. The meaning of the *pair distribution function* or (modulo appropriate normalisation) the *two-particle correlation*,  $\rho_2(x_1, x_2)$  is a little bit more abstract. In some loose sense it may be compared with the  $\rho(x_1, x_2)$  of two-particle Schroedinger theory. On the one side, it figures as a function over abstract  $\mathbb{R}^3 \times \mathbb{R}^3$ . On the other hand, it describes a concrete feature of our *compound system* as a whole, that is, a global property of our aggregate of  $N$  particles, living in the ordinary 3-dimensional coordinate space. More specifically, it encodes the mutual influence of the relative positions of the elementary constituents of our system on each other. To put it succinctly: While formally being a function over  $\mathbb{R}^3 \times \mathbb{R}^3$ , it nevertheless encodes a concrete property of our system living in three dimensions.

We now come to quantum theory. In the situation of a single quantum excitation it was not necessary to go into the possible details of the fine-structure of e.g. the charge modulations within or among the lumps or local clusters, as only the total charge modulation,  $\delta Q_0(P, t) = a(P, t)$ , entered in the coarse-grained equations of low-energy quantum theory. In the several-particle theory (we discuss in the following for convenience only the two-particle case)  $(\rho, S)$  depend now on two coordinates (see equations (34),(35)), in other words, they are now of an openly *non-local* nature in contrast to the one-particle case, where the non-locality is more hidden, as has been described above.

We may now envisage that we have in general two (or several) *entangled* excitation patterns, consisting of two more or less distinguishable modulations of the high-frequency ground-wave. These modulations are, as was already explained above, *weak-field excitations* on the comparatively large scale  $[L_{qm}] \gg [L_{pl}]$ . As

only a small fraction of the elementary charges or nodes is involved in these weak deformation patterns, there is ample space in the lumps (or local clusters) for approximately individual pulse patterns to coexist and interact or influence each other only weakly. They are however *entangled* in general unless the two-particle state is a *product-state*, that is

$$\phi(x_1, x_2) \neq \phi_1(x_1) \cdot \phi_2(x_2) \quad (75)$$

We now try to understand the mysterious phenomenon of entanglement.

**Conjecture 7.1 (Entanglement)** *What is called entanglement in ordinary quantum theory, is the non-local interaction (respectively, exchange of elementary charges) among the different excitation modes (describing the different quantum objects) via the bundles of interbonds connecting the different lumps or local clusters. That is, as in the above example of classical statistical mechanics,  $\rho(x_1, x_2), S(x_1, x_2)$  describe no longer completely local properties of the system but express a complex non-local entanglement-pattern, permeating through our complicated two-storey substrate,  $QX/ST$ .*

$$\rho_1(x_1) := \int \rho(x_1, x_2) d^3 x_2 \quad (76)$$

collects e.g. all the different contributions, resulting both from the entanglement with the components of the second excitation in the other lumps and coming from the first excitation itself (as in the one-particle situation). In the particular case of a *free* Hamiltonian and a product initial-state both excitations move independently of each other and remain non-entangled, that is

$$\rho(x_1, x_2) = \rho_1(x_1) \cdot \rho_2(x_2) \quad (77)$$

In this case each particle receives *non-local information* only from its own distant components in the other lumps as described above.

Remark: We assume, for reasons of simplicity (as this section is only of a cursory and preparatory nature), that the particles are distinguishable.

Note however that a *non-product state*,  $\rho(x_1, \dots, x_n)$ , cannot be uniquely decomposed into a product state and a contribution encoding the *entanglement* with the other particle-excitations in the distant lumps. In any case for fixed  $(x_1, \dots, x_n)$ , in the entangled situation, a certain fraction of the elementary charges belongs, so to say, to all the involved lumps at the same time as they oscillate between them via the bundles of respective interbonds. These and related phenomena have to be analyzed more carefully in future work.

**Conclusion 7.2** *The above cursory analysis shows, that “the results from poly-dimensions” can be understood in a satisfactory way in three dimensions under the proviso that we accept the two-storey structure of space-time or the physical vacuum.*

In a next step one can envisage what happens if the number of these coexisting excitations become too large in a given volume of space. In a *many-body wave-function*

$$\psi(x_1, \dots, x_N) \quad N \gg 1 \quad (78)$$

each of the approximately individual excitation modes has to be entangled *non-locally* with the other modes and/or with its own components in the distant lumps. If  $N$  exceeds a certain critical range of values, a complete quantum-entanglement may not longer be possible. The *weak-field picture* may begin to break down as all the available information channels (that is, the interbonds among the different lumps) are occupied or overcrowded. The wave-function will start to decay and goes over into a (partial mixture), that is, an incompletely entangled state.

A last point to mention is the infamous *measurement problem*. The seemingly instantaneous *state-reduction* by a measurement interference (in fact the contact with a peculiarly tuned macroscopic apparatus), while being on the surface of a quasi-local nature, will nevertheless spread its corresponding (decoherence-)information all over the microsystem almost instantaneously via the network of existing interbonds. It is thus yet another manifestation of the peculiar non-local character of the vacuum, described in the preceding sections.

Remark: An, in our view, quite up to date discussion of some of the impending problems may be found in section 2 of [65].

## References

- [1] M.Requardt: “Cellular Networks as Models for Planck-Scale Physics”, J.Phys.A:Math.Gen. 31(1998)7997, hep-th/980613
- [2] T.Nowotny, M.Requardt: “Dimension Theory on Graphs and Networks”, J.Phys.A:Math.Gen. 31(1998)2447, hep-th/9707082
- [3] T.Nowotny, M.Requardt: “Pregeometric Concepts on Graphs and Cellular Networks”, invited paper J.Chaos, Solitons and Fractals 10(1999)469, hep-th/9801199
- [4] M.Requardt: “Spectral Analysis and Operator Theory on (Infinite) Graphs ...”, math-ph/0001026, under consideration by JPA
- [5] M.Requardt: “(Quantum) Space-Time as a Statistical Geometry of Lumps in Random Networks”, Class.Quant.Grav. 17(2000)2029, gr-qc/9912059
- [6] N.Rosen: “Kann man die quantenmechanische Beschreibung der Wirklichkeit als vollstaendig betrachten?”, in P.C.Aichelburg, R.U.Sexl (eds.) “Albert Einstein, ...”, Vieweg, Braunschweig 1979

- [7] C.F.v.Weizsäcker: “Quantum Theory and Space-Time” in Symposium on the Foundations of Modern Physics, eds. P.Lahti,P.Mittelstaedt, World Scientific Singapore 1985
- [8] J.A.Wheeler: “Superspace and the Nature of Quantum Geometro Dynamics”, Battelle Rencontres 1967 p.253, eds. B.DeWitt,J.A.Wheeler, Lectures in Mathematics and Physics, Benjamin Inc., N.Y.1968
- [9] J.Stachel: “Einstein and Quantum Mechanics” in Conceptual Problems of Quantum Gravity, eds. A.Ashtekar,J.Stachel, Einstein Studies Vol. 2, Birkhäuser Boston 1991
- [10] A.Einstein: “Einleitende Bemerkungen über Grundbegriffe ” in Louis De Broglie und die Physiker, Claassen Hamburg 1955
- [11] A.Einstein: Commentary in “Albert Einstein Philosopher Scientist” ed. P.A.Schilp, 3rd edition Cambridge Univ. Pr. 1970
- [12] W.Moore: “Schrödinger, Life and Thought”, Cambridge Univ. Pr. 1989
- [13] H.Kragh: “Dirac, A Scientific Biography”, Cambridge Univ. Pr. Cambridge 1990
- [14] P.A.M.Dirac: “Is there an aether?”, Nature 168(1951)906, 169(1952)146,702
- [15] J.Bell: in “The Gost in the Atom”, eds. P.C.Davis,J.R.Brown, Cambridge Univ. Pr. Cambridge 1986
- [16] T.D.Lee: “Is the Physical Vacuum a Medium?”, CU-TP-170, talk given at Columbia University Sept.28 1979
- [17] “Mach’s Principle, From Newton’s Bucket to Quantum Gravity”, eds. J.Barbour,H.Pfister, Einstein Studies vol.6, Birkhäuser Boston 1995
- [18] S.Weinberg: “Dreams of a final Theory”, Vintage London 1993
- [19] L.Smolin: “Space and Time in the Quantum Universe” in [9] p.228ff
- [20] L.Smolin: “Stochastic mechanics, hidden variables and gravity” in “Quantum Concepts in Space and Time” p. 147ff, eds. R.Penrose,C.J.Isham, Oxford Univ. Pr. Oxford 1986
- [21] G.’t Hooft: “Equivalence Relations between Deterministic and Quantum Mechanical Systems”, J.Stat.Phys. 53(1988)323
- [22] G.’t Hooft: “Quantization of Discrete Deterministic Theories ...”, Nucl.Phys.B 342(1990)471
- [23] G.’t Hooft: “Quantum Gravity and Dissipative Deterministic Systems”, gr-qc/9903084

- [24] A.Einstein:“Die Grundlagen der Allgemeinen Relativitätstheorie”, Annalen der Physik 49(1916)769
- [25] J.Barbour:“Mach’s Mach’s Principle”, in “Grundlagenprobleme der modernen Physik”, eds. J.Nitsch,J.Pfarr,E.-W.Stachow, BI-Wissenschaftsverlag, Zürich 1981
- [26] H.Goenner:“Machsches Prinzip und Theorien der Gravitation”, in [25]
- [27] K.Westpfahl:“Heuristische Betrachtungen im Umkreis des Machschen Prinzips”, Convivium Cosmologicum p. 197, Birkhaeuser, Basel 1973
- [28] D.W.Sciama,P.Candelas,D.Deutsch:“quantum field theory, horizons and thermodynamics”, Adv.Phys. 30(1981)327
- [29] E.Nelson:“Dynamical theories of Brownian motion”, Princeton Univ. Pr., Princeton 1967
- [30] Ph.Blanchard,Ph.Combe,W.Zheng:“Mathematical and Physical Aspects of Stochastic Mechanics”, Lect.Not.Phys. 281 Springer, Berlin 1987
- [31] M.Requardt:“A New Stochastic-Geometric Conception of Microdynamics and Force”, Preprint, Göttingen 1981
- [32] R.P.Feynman:“Simulating Physics with Computers”, Int.J.Theor.Phys. 21(1982)467
- [33] K.Zuse: “The Computing Universe”, Int.Journ.Theor.Phys. 21(1982)589
- [34] L.Smolín:“The Future of Spin Networks” sect. 8, in “The Geometric Universe”, Oxford Univ.Pr., Oxford 1998 or gr-qc/9702030
- [35] C.D.Froggatt,H.B.Nielsen:“Origin of Symmetries”, World Scientific, Singapore 1991
- [36] C.J.Isham:“Canonical Quantum Gravity and the Problem of Time”, Lectures presented at the NATO Advanced Study Inst. “Recent Problems in Math.Phys.”, Salamanca June 1992, gr-qc/9210011
- [37] J.Butterfield,C.J.Isham:“On the Emergence of Time in Quantum Gravity”, gr-qc/9901024, to appear in “The Argument of Time”, ed. J.Butterfield, Oxford Univ.Pr., Oxford 1999
- [38] Th.Nowotny: Diploma Thesis, Goettingen 1998, a compilation of some of the extensive computer simulations can be found on the web site: <http://www.physik.uni-leipzig.de/nowotny/research.html> (commentary in german)
- [39] M.M.Waldrop: “Complexity, the Emerging Science at the Edge of Chaos and Order”, Viking, London 1993

- [40] S.Kauffman: “The Origins of Order”, Oxford Univ.Pr., Oxford 1993
- [41] P.Bak: “how nature works”, Springer, N.Y. 1996
- [42] N.Rosen: “Statistical Geometry and Fundamental Particles”, Phys.Rev. 72(1947)298  
S.Roy: “Statistical Geometry and Applications to Microphysics and Cosmology”, Kluver Acad.Publ. Dordrecht 1998  
B.Schweizer, A.Sklar: “Probabilistic Metric Spaces”, North-Holland N.Y. 1983
- [43] P.A.M.Dirac: Fields and Quanta 3(1972)139
- [44] M.Requardt: “Spectrum condition, analyticity, Reeh-Schlieder and cluster properties ...”, J.Phys.A, Math.Gen. 15(1982)3715
- [45] M.Requardt: “Reeh-Schlieder-type density results ...”, J.Math.Phys. 27(1986)1571
- [46] G.K.Pedersen: “ $C^*$ -algebras and their automorphism groups”, Academic Pr., London 1979
- [47] D.Kastler: “Equilibrium States of Matter and Operator Algebra”, Symp.Math. 20(1976)49
- [48] D.Bohm, B.J.Hiley, P.N.Kaloyerou: “An Ontological Basis for the Quantum Theory”, Phys.Rep. 144(1987)321
- [49] D.Bohm, B.J.Hiley: “Non-Locality and Locality in the Stochastic Interpretation of Quantum Mechanics”, Phys.Rep. 172(1989)93
- [50] D.Dürr, S.Goldstein, N.Zanghi: “Quantum Equilibrium and the Origin of Absolute Uncertainty”, J.Stat.Phys. 67(1992)843
- [51] G.C.Ghirardi, A.Rimini, T.Weber: “Unified Dynamics for Microscopic and Macroscopic Systems”, Phys.Rev.D 34(1986)470
- [52] P.Pearle: “Combining Stochastic Dynamical State-vector Reduction...”, Phys.Rev.A 39(1989)2277
- [53] Ph.Blanchard, A.Jadczyk: “Event-enhanced quantum theory ...”, Annalen der Physik 4(1995)583
- [54] H.Haken: “Synergetics”, Springer, N.Y. 1978
- [55] H.Haken: “Information and Self-Organization”, Springer, N.Y. 1988
- [56] A.S.Mikhailov: “Foundation of Synergetics I”, Springer, N.Y. 1990
- [57] A.S.Mikhailov, A.Yu.Loskutov: “Foundation of Synergetics II”, Springer, N.Y. 1991

- [58] E.Ott: “Chaos in Dynamical Systems”, Cambridge Univ.Pr., Cambridge 1994
- [59] I.P.Cornfeld,S.V.Fomin,Ya.G.Sinai: “Ergodic Theory”, Springer, N.Y. 1982
- [60] Ch.B.Thorn: “Substructure of Strings”, hep-th/9607204 or “Supersymmetric Quantum Mechanics for String-Bits”, Phys.Rev.D 56(1999)6619
- [61] D.Bigatti,L.Susskind: “Review of Matrix Theory”, Talk given at Cargese ASI 1997, hep-th/9712072
- [62] J.Polchinski: “Quantum Gravity at the Planck-Length”, Lecture presented at the 1998 SLAC Summer Institute, hep-th/9812104
- [63] A.Ashtekar,T.A.Schilling: “Geometrical Formulation of Quantum Mechanics”, gr-qc/9706069
- [64] L.Gammaitoni,P.Haenggi,P.Jung: “Stochastic resonance”, Rev.Mod.Phys. 70(1998)223
- [65] R.Haag: “Questions in quantum physics: a personal view”, hep-th/0001006