

Robust Lead Compensator Design for an Electromechanical Actuator Based on H_∞ Theory

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Abstract In this paper, we design a robust lead compensator for a real Electromechanical Actuator (EMA) harmonic drive by introducing an approach based on H_∞ control theory. Here, we address three main topics; experimental identification, uncertainty modelling, and robust control design for a real EMA harmonic drive system. This method verifies good tradeoff between the powerful H_∞ controller and the unique features of compensators, such as: simplicity, low cost and easy implementation. The H_∞ controller and the extracted compensator are almost identical within the EMA bandwidth range. Simulation and test results prove the effectiveness of the proposed approach and the superiority of the performance of the designed robust EMA with lead compensator based on H_∞ controller over the original EMA; this preference is pertaining to its robustness to parametric uncertainties and high performance.

Keywords: Electromechanical actuator (EMA), identification, uncertainty modelling, robust control, lead compensator, H_∞ control theory

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1. Introduction

Electromechanical actuators (EMA) are finding increasing use in the robotic and aerospace applications, since they have attractive characteristics such as: simplicity, reliability, low cost, good dynamic characteristics, and easy control [1,2].

Modelling and identification of the plant properly is the most effective step in control system design procedure. A good model is the simplest model that best explains the dynamics and successfully simulates or predicts the output for different inputs [3]. However, EMA modeling is subjected to uncertainty due to several reasons including operating point changes, perpetual parametric variations because of temperature changes, aging, unmodelled dynamics, and asymmetric behaviour. Consequently, the desired EMA's performance will be unachievable and, in some cases, its stability may be lost. Usually, based on experience, this problem will not be solved by using the conventional controllers; instead robust controllers with regard to the uncertainties in the model are needed to obtain the desired performance and stabilization demands in dealing with dynamic uncertainties [4].

Much effort is devoted to design robust controllers for uncertain systems based on different robust design methods known in literature such as Kharitonov's theorem, small gain theorem, H_∞ , and Quantitative Feedback Theory (QFT) [5,6,7]. Computational methods for

determining the set of all stabilizing controllers, of a given order and structure, for linear time-invariant delay free systems are reported in [8,9]. A graphical design method of tuning the PI and PD controllers achieving gain and phase margins is developed in [10,11]. Besides, in [12], the Kharitonov theorem is exploited for characterizing all PID controllers that stabilize an uncertain plant, also PID controllers design for systems without time delay were presented in [13,14]. H_∞ theory based controller was designed to get faster and more accurate EMA system in [15,16] and to improve the tracking and resolution of a servo positioning system in [17]. But, a well-known drawback of the H_∞ controller, it is high order controllers and high computational cost.

In this paper, the EMA harmonic drive system is modelled as linear system with parametric uncertainty. In addition, the multiplicative uncertainty describing the deviation of the nominal model from the real EMA system due to the ignored nonlinearities is extracted. A method based on H_∞ theorem and Bode diagram is proposed to design a lead compensator for the EMA system. This controller verifies good tradeoff between the powerful H_∞ controller and the unique features of compensators.

After presenting this brief introduction, the reader will be informed about more details which have been organized as following: The EMA experimental set-up is described in section 2. In section 3, identification and uncertainty modelling are presented. The method of deriving the compensator from the H_∞ robust controller will be explained, and the designed EMA system

validation is carried out in section 4. Finally, conclusion is stated.

2. Experimental Set-up

Our EMA consists of a DC motor with integrated harmonic drive gearing with 300:1 reduction ratio, driven by PWM power driver, and a potentiometer as position sensor fixed on the output shaft with coefficient of 0.5 Volt/deg, as shown in Figure 1. In this figure, r is the set point voltage, v is the PWM output, δ is the output angle, and δ_v is the angle measured by the potentiometer in volts. Here, r , v , and δ_v are acquired from experimental tests by a sampling time of 1 msec.

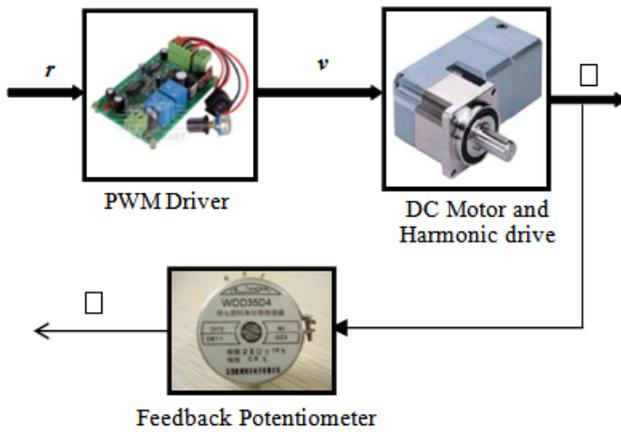


Figure 1. EMA harmonic drive scheme

3. System Identification and Uncertainty Modelling

The objective of system identification is to find the approximate models of dynamic system based on knowledge of the observed input and output data to be used in uncertain modelling.

In this section, the linear model of the harmonic drive system is identified based on the test data. The captured data, related to several test sets at different conditions, were used for model estimation and validation purposes. The data of every set were divided into two portions: first one was used in the model estimation process (identification data), and the second was utilized in the validation process (validation data). The data captured from the real plant were used in the iterative prediction-error estimation method (PEM) [18]. The discrete time model is then transformed to continuous time model to be used in the uncertainty modelling.

3.1. System Identification

The input is the voltage to motor and the output is the potentiometer output. In this case, the PRBS signal cannot be used for identification since the test takes place in the open loop form and PRBS signal transfers the position sensor to saturation condition. Hence, the input signal for identification process is chosen to be a symmetric square wave and the output angle is measured from the potentiometer. But, to satisfy the persistent excitation (PE)

property, we prepare numbers of square waves with different frequencies and amplitudes for identification. Frequencies of square waves are chosen between 0 to 20 Hz and their amplitudes are chosen between 0.1 to 27 volts. These values are chosen based on the desirable bandwidth and operational conditions of motor, respectively. Seven imported data sets corresponding to different amplitudes were used for model identification. For instance, one test set results of the EMA harmonic drive system and its identified model are depicted in Figure 2.

The average error produced by a model is encapsulated in the mean squared error (MSE), which measures the precision of the estimated model. It is calculated as:

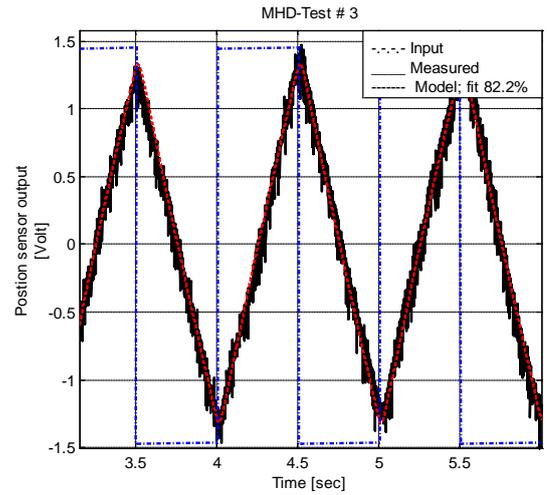


Figure 2. Measured data and identified model for the EMA harmonic drive system

$$MSE(y, y_m) = \frac{1}{n} \sum_{i=1}^n (y(i) - y_m(i))^2 \quad (1)$$

where y is the actual (measured) output, y_m is the output of the model, and n is the number of data samples. Furthermore, the model should attain best fitting to the measured data. The percentage of the output variation that is explained by the model is:

$$fit \% = \left(1 - \frac{|y_m - \bar{y}|}{|\bar{y} - \bar{y}|} \right) \times 100\% \quad (2)$$

where $\bar{y} = mean(y)$.

The identified models are tabulated with their MSEs and fitting percentages in Table 1.

3.2. Model Uncertainty Estimation

Considering the transfer functions for the identified models tabulated in Table 1, the parametric uncertain model was built to be used in H_∞ robust controller design.

$$G_{MHD}(s) = \frac{K}{s(s+q)} \quad (3)$$

The uncertain parameters of the model and their intervals are:

$$K \in [582, 1310]; K_{nominal} = 961$$

$$q \in [56.5, 79]; q_{nominal} = 61.4$$

The nominal model can be written as follows:

$$G_{nom}(s) = \frac{961}{s(s+61.4)} \quad (4)$$

Table 1. Identification Results for the EMA HarmonicDrive System

Test set No.	Model	MSE	Fitting (%)
1	$\frac{1310}{s(s+72.55)}$	0.0079	96.8
2	$\frac{1187.4}{s(s+63.86)}$	0.0034	95.7
3	$\frac{636}{s(s+64.85)}$	0.0079	82.2
4	$\frac{582.6}{s(s+56.5)}$	0.0072	86.2
5	$\frac{1236}{s(s+79)}$	0.0033	94.4
6	$\frac{1308}{s(s+76.91)}$	0.0032	96
7	$\frac{655}{s(s+57.45)}$	0.049	72

4. Robust Controller Design

In this section, a novel approach to design a robust lead compensator based on H_∞ robust controller theory will be detailed. The overall philosophy in the design procedure presented here is to design a robust H_∞ controller, then reduce its order maintaining its characteristics in the working frequency range, next it will be re-arranged to get the final compensator form.

4.1. System requirements

The required performance of the EMA system to be designed is a deadbeat response as shown in Table 2.

Table 2. Time Domain System Performance Requirements

Parameter	Value
Rise time	$t_r < 20$ msec
Settling time	$t_s < 35$ msec
Steady-state position error	0
Overshoot	$< 1\%$

Consequently, the frequency domain system requirements may be derived from Table 2 as follows: The settling time constraint leads to minimum bandwidth 10 Hz. The overshoot constraint ($< 1\%$) assigns the minimum damping ratio to 0.826, and the minimum phase margin should be 40 deg.

4.2. H_∞ Robust Controller Design

H_∞ theory presents powerful frequency-domain framework for capturing control design requirements such as bandwidth, response speed, robust stability, and disturbance rejection [19]. The EMA system in H_∞ framework is illustrated in Figure 3.

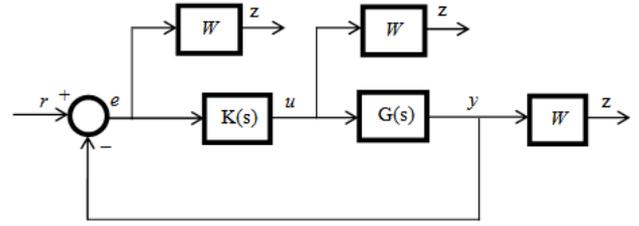


Figure 3. EMA system in H_∞ framework

K is the robust controller, G : Motor and harmonic drive system, r : command reference signal, u : control signal, e : tracking error, and y : plant output. Define the error sensitivity function $S = (1 + GK)^{-1}$, the control signal sensitivity function $U = K(1 + GK)^{-1}$, and the complementary sensitivity function $T = GK(1 + GK)^{-1}$.

$$\text{Robust performance constraint: } \|W_e S\|_\infty \leq 1$$

$$\text{Robust Stability constraint: } \|WT\|_\infty \leq 1$$

$$\text{Control signal (saturation) constraint: } \|W_u U\|_\infty \leq 1$$

W , W_u and W_e are frequency dependent weighting functions.

The objective is to find controller K that stabilizes G_{nom} and satisfies H_∞ norm of a transfer matrix consisting of weighting functions is smaller than one, i.e.:

$$\begin{bmatrix} \|W_e S\|_\infty \\ \|W_u U\|_\infty \\ \|WT\|_\infty \end{bmatrix} \leq 1. \text{ This problem is called mixed sensitivity}$$

problem [19].

Here, we consider the multiplicative uncertainty to describe the deviation of the nominal model from the real system due to the ignored nonlinearities in real system. Assume G_{MHD} is the models of the motor and harmonic drive, identified from the test sets tabulated in Table 1, and G_{nom} is the nominal model. Using the multiplicative uncertainty definition:

$$G_{MHD}(s) = (1 + \Delta(s)W(s))G_{nom}(s) \quad (5)$$

where $W(s)$ is the uncertainty weighting function, and Δ is an operator with infinity-norm less than unity [20], i.e. $\|\Delta(s)\|_\infty < 1$, so

$$\left| \frac{G_{MHD}(j\omega) - G_{nom}(j\omega)}{G_{nom}(j\omega)} \right| \leq |W(j\omega)|, \forall \omega \quad (6)$$

By plotting the frequency response of $\left| \frac{G_{MHD}(j\omega) - G_{nom}(j\omega)}{G_{nom}(j\omega)} \right|$, the multiplicative uncertainty weighting function can be extracted from the upper bound of different uncertainty frequency plots of the identified uncertain system; they are shown in Figure 4. The multiplicative uncertainty weighting function is approximated by:

$$W(s) = \frac{s+110}{275}$$

which illustrates that robustness boundary provides 8 dB of gain below 30 rad/sec and suppress to more than -10 dB above 600 rad/sec. See Figure 5.

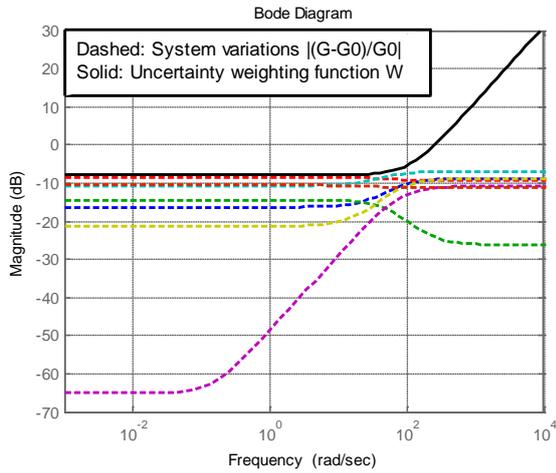


Figure 4. Uncertainty and weighting function frequency responses; simulation

It may be concluded from Figure 4 that the uncertainty at low frequencies is small (about -7.5 dB), which allow robust control of the EMA system within this bandwidth. W is the robustness weighting function to be used in H_∞ robust controller design. Use of this identified uncertainty model in the robust control design procedure decreases the deviations between designed model performance and experimental results [21].

The sensitivity weighting function is assigned to:

$$W_e(s) = 1.2 \times \frac{0.1s + 50}{s + 50 \times 10^{-3}}$$

which indicates sensitivity reduction of 100:1 (-40 dB) for frequencies up to 1 rad/sec (i.e. at low freq. the closed loop should reject disturbance at the output by a factor of 100 to 1 or the steady state tracking errors due to step input should be less than 1%), as depicted in Figure 5.

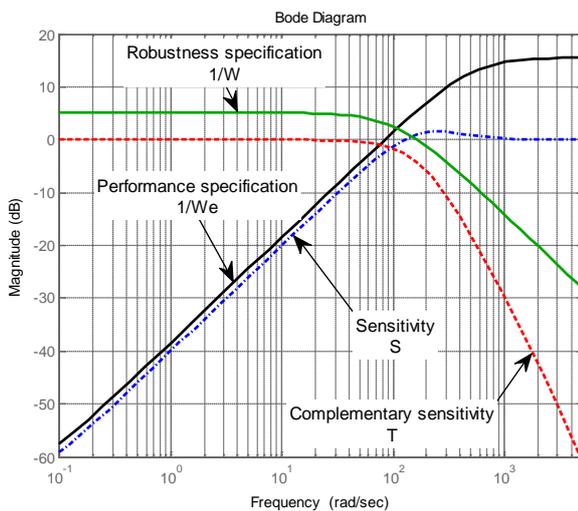


Figure 5. Closed loop sensitivity, complementary sensitivity, robustness and performance functions with H_∞ controller; simulation

The control signal sensitivity weighting function W_u (the actuator saturation weighting function) is chosen considering the physical limitation of the EMA. Thus, it can be considered to be a constant, so that the maximum expected input r (2 volt in this case) never saturates the actuator (saturation voltage 27 volt). Its value is estimated to be: $W_u = 0.02$.

The H_∞ controller, designed by Matlab and verifying H_∞ cost $\gamma = 0.721$, is:

$$K = \frac{163938.5s(s + 61.4)}{(s + 4603)(s + 342.4)(s + 0.05)} \quad (7)$$

Since the H_∞ -norm of the closed-loop system is less than one, the condition: $\|WT\|_\infty \leq 1$ is satisfied in the given case. This may be checked by computing the sensitivity function of the closed-loop system and comparing it with the inverse of the performance weighting function. The result of the comparison is shown in Figure 5. It is seen that the sensitivity function lies below $1/W_e$.

Similarly, the condition $\|WT\|_\infty \leq 1$ may be checked by computing the complementary sensitivity function of the closed-loop system and comparing it with the inverse of therobustness weighting function. The result of the comparison is shown in Figure 5, where the complementary sensitivity function lies beneath $1/W$.

4.3. Compensator Derivation

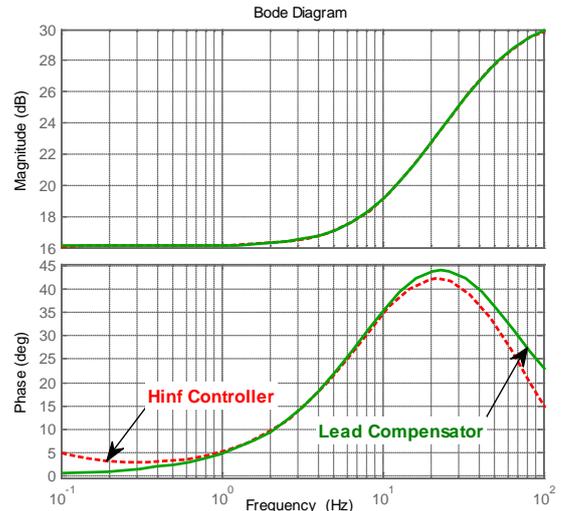


Figure 6. H_∞ controller and the reduced one; simulation

Low order controllers are normally preferred over high order controllers in control system since they are computationally less demanding; easier to implement and they have higher reliability due to fewer things to go wrong in the hardware [22]. The dc-gain of EMA system plays an important role in assessing its performance, for this reason it should be maintained. The 3rd order designed H_∞ controller will be reduced in a manner to keep its characteristics in the working frequency range using balanced residualisation (singular perturbation approximation) method which gives best approximation at low frequencies [23].

$$K_{red}(s) = \frac{35.6(s + 61.4)}{(s + 342.3)} = A \frac{(1 + \alpha Ts)}{(1 + Ts)} \quad (8)$$

which is a lead compensator with $A = 35.6$, $\alpha = 5.58$, and $T = 2.92 \times 10^{-3}$. Figure 6 shows that the H_∞ and the reduced order controllers are almost identical in the range below 100 Hz which is sufficiently wider than our application's range (10 Hz). This controller can be easily realized using passive and active components consisting

of resistors, capacitors, transistors, diodes and op-amps [24].

4.4. Compensator Realization

The lead compensator is realized by active components circuit shown in Figure 7.

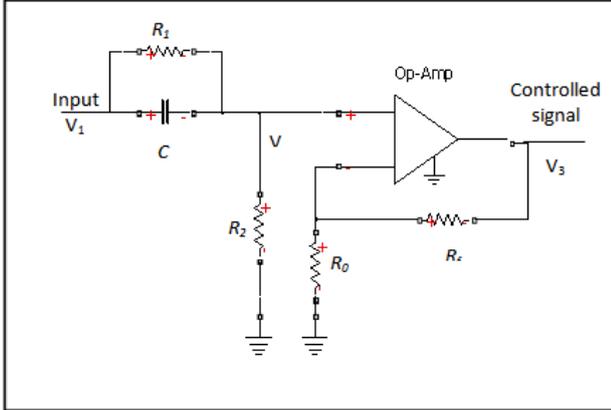


Figure 7. Designed compensator circuit

The compensator transfer function is:

$$K_{red}(s) = \frac{35.6(s + 61.4)}{(s + 342.3)} \quad (9)$$

From the circuit diagram shown in Figure 7, R_1 , R_2 and C form the compensator network where as R_f and R_0 form the compensator gain (A). The transfer function from the input voltage to the controller V_1 to the output voltage V_2 is:

$$\frac{V_2}{V_1} = \frac{s + \frac{1}{R_1 C}}{s + \frac{R_1 + R_2}{R_1 R_2 C}} \quad (10)$$

Op-Amp: TL074, $R_1 = 162 \text{ K}\Omega$, $C = 0.1 \mu\text{F}$ and $R_2 = 36 \text{ K}\Omega$. The constant gain $A = 35.6$ is verified by the used compensator circuit in Figure 7; this is given by:

$$A = 1 + \frac{R_f}{R_0} = 35.6; R_f = 34.6 \text{ K}\Omega; R_0 = 1 \text{ K}\Omega$$

4.5. Performance Validation of the Robust EMA with H_∞ derived Compensator

Table 3 illustrates the time response values for the H_∞ controller and the derived lead compensator, which clarify that there is no considerable performance retreat.

Table 3. Characteristics of Robust EMAs with H_∞ Controller and Derived Compensator

Model	t_r (msec)	t_s (msec)	Over-shoot(%)	S. S. Error
Robust EMA with H_∞ controller	15.7	26	0.08	0
Robust EMA with reduced order H_∞ (lead compensator)	16.2	26.9	0.05	0

The step responses and bode plots of the uncertain closed loop EMA system with the derived lead compensator are depicted in Figure 8 and Figure 9, respectively.

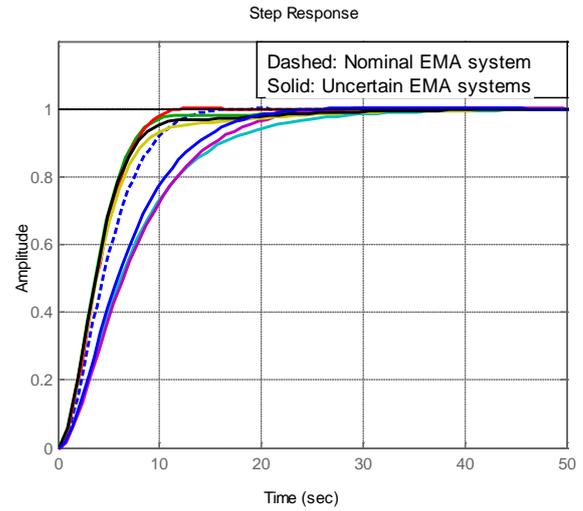


Figure 8. Step responses of nominal and uncertain closed loop EMA with the derived lead compensator; simulation

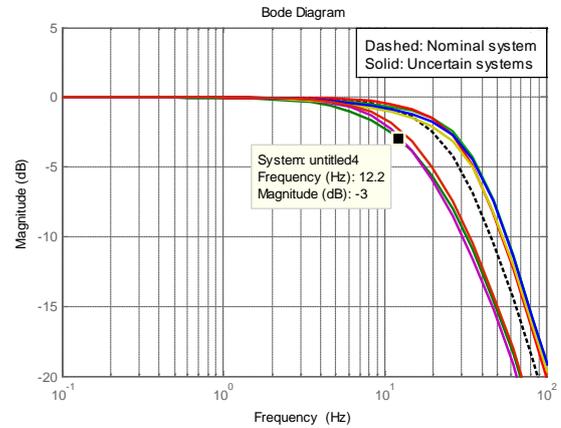


Figure 9. Closed loop frequency response for nominal and uncertain EMA with the lead compensator; simulation

The new designed EMA system with lead compensator derived from the H_∞ controller, in addition to its robustness to parametric uncertainties, has almost the same dynamics of the H_∞ controller and the simplicity advantages of compensators.

In order to verify that other system requirements in Table 2 are met with the derived compensator, the margins and bandwidths were tabulated in Table 4.

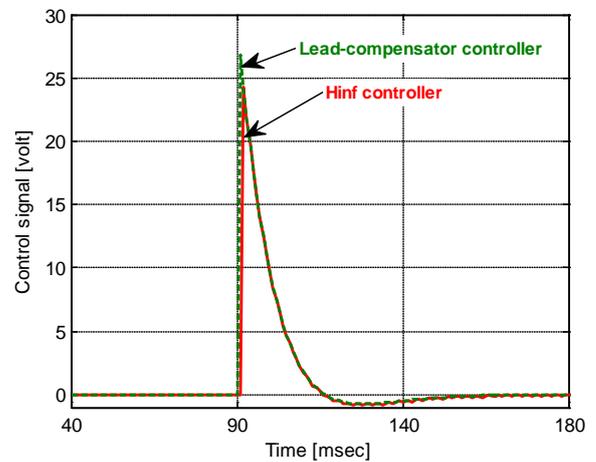


Figure 10. Control signals with H_∞ controller and the derived lead compensator; simulation

Table 4. Margins of Uncertain Plants with the Compensator

TF	Gain margin (dB)	Phase margin (deg)	Bandwidth (Hz)
G_{nom}	33.9	73.1	21
G_1	31.5	72.7	26.9
G_2	32.1	70.8	25
G_3	37.6	80.4	14.2
G_4	38.1	76.6	13.5
G_5	32.2	76.6	25.1
G_6	31.6	74.5	26.5
G_7	37.1	76	15

It is noted that system requirements are verified.

The control signal may be checked so that the actuator will not be saturated during operation. The control signals to the actuator corresponding to maximum input command (2 volts) with the designed robust controllers were plotted (as illustrated in Figure 10).

Finally, the performance of the robust EMA with H_∞ derived compensator was compared with the original classic controller EMA performance. It proves, in addition to its robustness to parametric uncertainties, better dynamic performance, as shown in Figure 11.

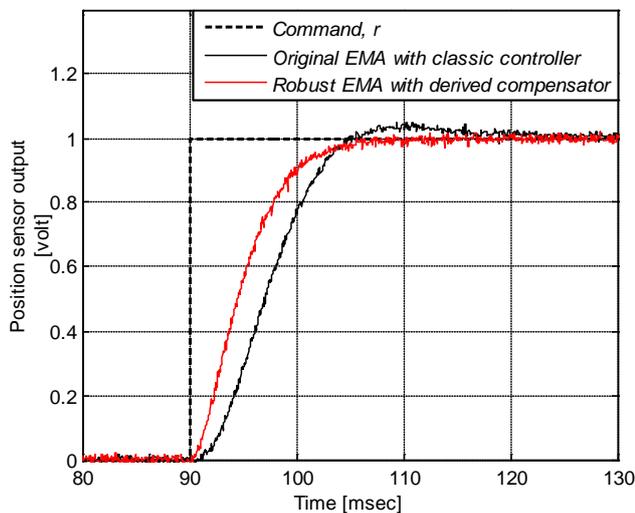


Figure 11. Measured step for original and robust EMA systems; experiment

5. Conclusion

An approach to design robust compensator, based on H_∞ theory, was applied on an EMA harmonic drive system. It led to a robust EMA system which is, in addition to its robustness, simple and easily implementable.

The robust controller design and validation procedure is studied by simulation and experiments, and its effectiveness is proven by comparing the performance of compensator with the robust H_∞ controller and the original EMA system with classic controller. The comparison has demonstrated the effectiveness of the proposed robust EMA system with lead compensator.

The proposed approach applied on the EMA system, may be applied on other control systems operating over a relatively low range of frequencies.

References

[1] Ristanović, M., Čojbašić, Ž. and Lazić, D., "Intelligent control of DC motor driven electromechanical fin actuator," *J. Control Engineering Practice*, 20 (6). 610-617. Mar. 2012.

[2] Liscouët, J., Maréchal, C. and Budinger, M., "An integrated methodology for the preliminary design of highly reliable electromechanical actuators: Search for architecture solutions," *Aerospace Science and Technology*, 22 (1). 9-18. Oct-Nov. 2012.

[3] Walter, E., Pronzato, L. and Norton, J., *Identification of Parametric Model from Experimental Data*, Springer, GB, 1997.

[4] Lu, H., Li, Y. and Zhu, C., "Robust synthesized control of electromechanical actuator for thrust vector system in spacecraft," *J. Computers and Mathematics with Applications*, 64 (5). 699-708. Sept. 2012.

[5] Toscano, R., "A simple robust PI/PID controller design via numerical optimization approach," *J. of process control*, 15 (1). 81-88. Feb. 2005.

[6] Valério D. and Costa, J., "Tuning of Fractional Controllers Minimising H_2 and H_∞ Norms," *Acta Polytechnica Hungarica*, 3 (4). 55-70. 2006.

[7] Yogesh, V. H., Gupta, J. R. P. and Choudhury, R., "Kharitonov's Theorem and Routh Criterion for Stability Margin of Interval Systems," *International Journal of Control, Automation, and Systems*, 8 (3). 647-654. June 2010.

[8] Saadaoui, K. and Özgüler, A. B., "A new method for the computation of all stabilizing controllers of a given order," *International Journal of Control*, 78 (1). 14-28. Jan. 2005.

[9] Tantarís, R. N., Keel, L. H. and Bhattacharyya, S. P., "Stabilization of discrete-time systems by first-order controllers," *IEEE Trans. Automat Control*, 48 (5). 858-861. May 2003.

[10] Tan, N., "Computation of stabilizing PI-PD controllers," *International Journal of Control, Automation, and Systems*, 7 (2). 175-184. April 2009.

[11] Yeroglu, C. and Tan, N., "Design of Robust PI Controller for Vehicle Suspension System," *Journal of Electrical Engineering & Technology*, 3 (1). 135-142. 2008.

[12] Huang, Y. J. and Wang, Y.-J., "Robust PID tuning strategy for uncertain plants based on the Kharitonov theorem," *ISA Transactions*, 39 (4). 419-431. Sept. 2000.

[13] Li, X. H., Yu, H.B., Yuan, M.Z. and Wang, J., "Design of robust optimal proportional-integral-derivative controller based on new interval polynomial stability criterion and Lyapunov theorem in the multiple parameters' perturbations circumstance," *IET Control Theory Appl.*, 4 (11). 2427-2440. Nov. 2010.

[14] Rigatos, G. and Siano, P., "Design of robust electric power system stabilizers using Kharitonov's theorem," *Mathematics and Computers in Simulation*, 82 (1). 181-191. Sept. 2011.

[15] Yoo, C.-H., Lee, Y.-C. and Lee, S.-Y., "A Robust Controller for an Electro-Mechanical Fin Actuator," in *Proceeding of the American Control Conference*, Boston, Massachusetts, 5, June 30-July 2, 2004, 4010-4015.

[16] Benderradji, H., Chrifi-Alaoui, L., Mahieddine-Mahmoud S. and Makouf, A., "Robust Control of Induction Motor with H_∞ Theory based on Loopshaping," *Journal of Electrical Engineering & Technology*, 6 (2). 226-232. 2011.

[17] Raafat, S. M., Akmeliawati, R. and Abdulljabaar, I., "Robust H_∞ Controller for High Precision Positioning System, Design, Analysis, and Implementation," *Intelligent Control and Automation*, 3 (3). 262-273. Aug. 2012.

[18] Ljung, L., *System Identification: Theory for the User*, 2nd edition, Prentice-Hal PTR, Upper Saddle River, NJ, 1999.

[19] Bhattacharyya, S. P., Chappellat, H. and Keel, L. H., *Robust Control: The Parametric Approach*, Prentice Hall, 1995.

[20] Tavakoli, M., Taghirad, H. D., and Abrishamchian, M., "Identification and Robust H_∞ Control of the Rotational/Translational Actuator System," *International Journal of Control, Automation, and Systems*, 3 (3). 387-396. Sept. 2005.

[21] Lim, K. B., Cox, D. E., Balas, G. J. and Juang, J.-N., "Validation of an Experimentally Derived Uncertainty Model," *Journal of Guidance, Control, and Dynamics*, 21 (3). 485-492. May-June 1998.

[22] Zhou, K., Doyle, J. C. and Glover, K., *Robust and Optimal Control*, Prentice Hall, Englewood Cliffs, New Jersey, 1996.

[23] Oh, D. C., Bang, K. H. and Park, H. B., "Controller order reduction using singular perturbation approximation," *Automatica*, 33 (6). 1203-1207. June 1997.

[24] Kuo, C. B., *Automatic Control System*, 5th edition. Prentice Hall of Indian Private Ltd, New Delhi, 1989.