

Thermodynamics meets Special Relativity – or what is real in Physics?

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Abstract

In this paper we carefully reexamine the various frameworks existing in the field of relativistic thermodynamics. We scrutinize in particular the different conceptual foundations of notions like the relativistic work, heat force, moving heat and relativistic temperature. As to the latter notion we argue that, as in ordinary thermodynamics, relativistic absolute temperature should be introduced operationally via relativistic Carnot processes. We exhibit the more implicit or even hidden tacit preassumptions being made and point to a couple of gaps, errors and inconclusive statements in some of the existing literature. We show in particular that there is a wide-spread habit to draw general conclusions from the analysis of too restricted and special thermodynamic processes, e.g. processes with constant pressure, which is dangerous and sometimes leads to wrong results. Furthermore, we give a detailed analysis of the so-called zeroth law of relativistic thermodynamics with the help of a relativistic Carnot process. We rigorously show that, contrary to certain statements in the literature, thermodynamic systems at different relativistic temperatures, moving relative to each other, can thermally stably coexist provided that their respective temperatures obey a certain functional relation (given by the Lorentz factor). This implies however that their respective rest temperatures are the same.

1 Introduction

While relativistic thermodynamics is in principle a quite old field of research, starting almost immediately after the fundamental Einstein papers of 1905 with seminal contributions by Planck, Einstein and Planck's student v. Mosengeil (see for example [1],[2],[3]), there is nevertheless a still ongoing debate both about the overall working philosophy, certain of its basic principles and various technical details. See for example the recent [4], which, however, deals primarily with various problems of relativistic statistical mechanics (a catchword being: Jüttner distribution). In the following we will refrain from commenting on the many additional problems being inherent in the latter field, as relativistic thermodynamics is already a quite ambitious field of its own. Furthermore, while there exist of course a lot of connections between thermodynamics and statistical mechanics, the relativistic regime poses quite a few problems of its own due to the relatively rigid constraints on the class of admissible microscopic interactions if one stays within the framework of (point) particles. Conceptually it may therefore be reasonable to regard relativistic quantum field theory as the appropriate framework to develop a relativistic version of statistical mechanics with its natural possibilities of particle creation and annihilation and the interaction of fields replacing the forces between (point) particles.

The reasons for this still ongoing debate in relativistic thermodynamics are manifold. The history of the different points of view and approaches is in our opinion meanwhile so contorted and faceted because two fields had to be merged which have their own specific technical and epistemological problems. It hence appears to be reasonable to us, to try to isolate the crucial points where opinions differ and concentrate on the deeper reasons, why discussions have lasted for such a long time without coming to a final conclusion. This holds in particular so as we will show that in our view various of the common arguments do contain gaps and even errors, which we try to exhibit in the following.

Our own interest was raised anew when we came across the so-called “Einstein-Laue Discussion” as being reviewed in [5] and [6]. It is a curious but little known fact that according to the detailed analysis of the exchange of letters between Einstein and Laue, made by Liu, Einstein changed his opinion about the correct transformation properties of various thermodynamic quantities completely in the early fifties without apparently being aware of this fact. While in [2] he got results which go conform with the results of e.g. Planck, he arrived already in 1952 at transformation laws which a couple of years later were published by Ott and Arzeliès ([7],[8]).

While this may be a remarkable psychological or historical phenomenon, what is conceptually more important in our view, is the deeper reason why such eminent thinkers came to contradictory conclusions, as it can certainly be ruled out that for example Planck, Einstein, Laue, Pauli, Tolman, to mention a few, simply committed errors in their calculations. It therefore seems to be worthwhile, to analyse the steps in the reasonings of the various authors who contributed to this field and to exhibit and isolate the sometimes only tacitly made or even entirely hidden preassumptions on which the various analyses were based. It then becomes perhaps clearer, in what sense our subtitle: “or what is real in physics”, may be justified.

In our view one of the problems in the more recent discussions is in fact that the respective physical situation is frequently only incompletely described, or, on the other hand, a very special case is analysed instead of a really general move on the thermodynamic state manifold, thus leaving out important aspects or emphasizing only points which support the own point of view. We will come to this phenomenon in more detail in the following sections but mention just a typical example, i.e. the controversial discussion between Arzeliès, Gamba and Kibble ([8],[9],[10] and the respective comments and remarks in the same volume of the journal). When reading for example these papers, it becomes obvious that the authors simply talk about quite different systems and incompatible situations, while apparently being only incompletely aware of this fact. Anyhow, in our opinion the position of Kibble is the more reasonable one in the mentioned discussion.

To give an example, we think it is not helpful to actually include parts of the exterior of the confined system or the walls into the thermodynamic discussion. One should rather adopt the philosophy that thermodynamic systems are dealt with in the way they are defined in ordinary thermodynamics. Otherwise the discussion becomes very cumbersome in our view. In this context we would like to remark that our criticism applies also to certain points in the paper of Ott. We will comment on these aspects in section 6.

To begin with, we make a brief classification of the different working philosophies and opinions (see also [11]). First, there is the *classical period*, represented e.g. by Planck, Einstein, v.Laue, Pauli, Tolman ([1],[2],[12],[13],[14]), and being roughly described by the transformation laws of heat and temperature

$$\delta Q = \delta Q_0/\gamma \quad , \quad T = T_0/\gamma \tag{1}$$

with γ the *Lorentz factor*

$$\gamma = (1 - u^2/c^2)^{-1/2} \quad (2)$$

and the subscript, 0, denoting in the following the variables in the comoving inertial or rest frame (CIF) of the thermodynamic system. Its velocity relative to the laboratory frame is u and the variables in the laboratory frame are δQ and T .

For convenience we usually assume that the thermodynamic system is at rest in the IF, X' , which is in *standard position* with respect to the laboratory frame, X . That is, it moves with velocity u in the positive x-direction, its coordinate axes being parallel to the ones of X and with coordinate origins coinciding at $t = t' = 0$ ([15]).

In the sixties (and earlier in the mentioned letters of Einstein) another transformation law emerged ([7],[8]):

$$\delta Q = \delta Q_0 \cdot \gamma \quad , \quad T = T_0 \cdot \gamma \quad (3)$$

One should however remark that, while superficially being the same, the point of view of e.g. Arzeliès is quite different from the one, hold by Ott (cf. the discussion between Arzeliès, Gamba and Kibble, mentioned above). Furthermore, while we arrive at the same transformation laws in the present paper as Ott, the situation discussed by him in [7], section 2, is also only a particular case and does not! really deal with a general variation of thermodynamic variables. Therefore some of the really critical problems were not addressed by him (cf. the section about the Ott-paper).

Remark: Note that Moeller in his beautiful book ([29]) changed his convention from the classical point of view in the earlier editions to the convention of Ott and Arzeliès in the last edition which we are citing in the references.

Somewhat later and up to quite recent times a third approach was promoted by e.g. Landsberg and coworkers ([16],[17],[18],[19],[20]; see also [21]), another reference, discussing various points of view is [22]. Landsberg et al argue that temperature and heat are *Lorentz-invariants*, i.e. behave as scalars, that is

$$\delta Q = \delta Q_0 \quad , \quad T = T_0 \quad (4)$$

This point of view is presently shared by a number of other workers in the field. It is sometimes argued that all this is rather a matter of convenience as the transformation laws are not really fixed by the condition of relativistic covariance. This is certainly correct. On the other hand, some of the

arguments advanced in favor of this latter opinion are in our view not really convincing. As a prominent reference for such an opinion (“pseudoproblem”) see for example [23], p.334.

It is e.g. argued that both the classical point of view ($T = T_0 \cdot \gamma^{-1}$) and the more modern one ($T = T_0 \cdot \gamma$) are compatible in so far as in the former heat is exchanged at constant velocity, in the latter at constant momentum. We will show in the following that this parallelism of points of view cannot be maintained. We rigorously show that the heat force as it was invoked in the classical approach does simply not exist (section 4.1), more specifically, of the three components making up the total classical heat force, only one can be granted a real existence. Furthermore, exchange of heat between system and comoving reservoir (i.e., both having the same velocity) is a transparent process. What however is the meaning of exchange of heat at equal or constant momentum between a (possibly) small system and a huge reservoir? We show below that we get the transformation laws

$$\delta Q = \delta Q_0 \cdot \gamma \quad , \quad T = T_0 \cdot \gamma \quad (5)$$

via heat exchange at constant velocity.

What, for example, frequently happens is that two, in principle different, situations are mixed up. If one inserts an empirical thermometer into a moving substance and reads off the temperature from the laboratory frame, there exists little doubt that one in fact observes the *rest temperature* T_0 . This is not! the temperature of a moving system. In this respect temperature behaves differently from length or time. Another thought experiment ([18] and elsewhere) discusses the heat exchange between bodies moving relative to each other and tries to construct a paradox unless the temperature is a scalar. This argument is also flawed as we will show in section 5 (the *relativistic zeroth law*).

At the end of this introduction we want to briefly comment on two other papers. In [24] it is for example argued that one should take the thermodynamical variables as scalars *like in general relativity*? In the first place, the building blocks of general relativity are general *tensors*. Scalars do not play any particular role. In our approach some of the variables are 4-vectors which have a very nice transformation behavior. Furthermore, it is claimed that some *grotesque situations* do arise because of the non-equivalence of simultaneity. As to this point, it is frequently overlooked that by assumption all moves on the state manifold are performed in a quasi-static way so that the problem of non-simultaneity is not really virulent.

In [25] the author introduces a new principle, claiming that thermal equilibrium between bodies in relative motion is impossible. We show however

in the section about the zeroth law of thermodynamics (section 5) that, to the contrary, this is possible and free of logical contradictions if described in the way we did it and as it was anticipated by v.Laue.

2 Notations and Standard Formulas in Relativistic Mechanics

The Minkowski metric is denoted by $\eta_{\nu\mu}$ with the convention $(+ - - -)$. Greek indices run from 0 to 3, latin indices from 1 to 3. Four-vectors carry a greek index (abstract index notation), three-vectors are in bold face. The energy of a particle or system is denoted by E , its three-momentum by \mathbf{G} (as the letter p is already used for the pressure). We write

$$ds^2 = c^2 \cdot d\tau^2 = \eta_{\nu\mu} dx^\nu dx^\mu \quad (6)$$

with $x^0 = c \cdot t$ and $d\tau$ the proper time interval measured by a comoving ideal standard clock (i.e., a clock being unaffected by acceleration; see the discussion in e.g. [15]).

We have

$$E = m \cdot c^2 \quad , \quad \mathbf{G} = m \cdot \mathbf{u} \quad , \quad m = \gamma \cdot m_0 = m_0 \cdot (1 - u^2/c^2)^{-1/2} \quad (7)$$

for a particle or system moving with the momentary velocity \mathbf{u} relative to a certain IF. m_0 is the proper mass and γ the *Lorentz factor*. In 4-vector notation this reads

$$G^\nu = (E/c, \mathbf{G}) = m_0 \cdot U^\nu \quad (8)$$

with

$$U^\nu = dx^\nu/d\tau = (c \cdot \gamma, \mathbf{u} \cdot \gamma) \quad (9)$$

the 4-velocity with $\eta_{\nu\mu} U^\nu U^\mu = c^2$.

The 3-force is conventionally defined via

$$\mathbf{F} = d/dt (m \cdot \mathbf{u}) = d/dt \mathbf{G} \quad (10)$$

Its transformation properties (and the covariance properties of other derived notions) become more transparent by finding the correct 4-dimensional generalisation. The 4-force (or Minkowski force, [26]) is defined by

$$F^\nu = (\gamma \cdot \mathbf{F} \cdot \mathbf{u}/c, \gamma \cdot \mathbf{F}) \quad (11)$$

with $F^\nu = dG^\nu/d\tau$. Note that it holds (for a *rest-mass preserving force*!)

$$\mathbf{F} \cdot \mathbf{u} = \mathbf{F} \cdot d\mathbf{x}/dt = dW/dt = dE/dt \quad (12)$$

so that in that case the 4-force can alternatively be written

$$F^\nu = (\gamma \cdot c^{-1} \cdot dE/dt, \gamma \cdot \mathbf{F}) \quad (13)$$

This subtle point is treated in more detail in subsection 4.1 where we discuss the relativistic concept of work in more detail. Note that in collisions the rest-mass may change in the moment of contact. Rindler in [27], p.92 speaks in this context of *heat-like* forces.

As in relativistic mechanics 4-momentum is conserved for a closed system of particles

$$\sum_i m_i \cdot \mathbf{u}_i = \text{const} \quad (14)$$

implies that the above definition of force guarantees that the law: *actio* = *reactio* holds. This plays a certain motivational role for conceptual generalisations being made in relativistic thermodynamics and is particularly stressed by Tolman ([14]).

Defining acceleration or 4-acceleration by

$$\mathbf{a} = d\mathbf{u}/dt \quad , \quad a^\nu = dU^\nu/d\tau = \gamma \cdot dU^\nu/dt \quad (15)$$

one sees, that in general the force vector is not! parallel to the acceleration vector. We rather have

$$\mathbf{F} = m \cdot d\mathbf{u}/dt + dm/dt \cdot \mathbf{u} \quad (16)$$

The (problematic) generalisation to relativistic thermodynamics will also play a certain role in the following.

3 Some Formulas from Relativistic Continuum Mechanics

The derivation of the thermodynamic behavior of relativistic equilibrium systems is to a large extent based on concepts from continuum mechanics. A very well-written source are the chapters 6 and 7 in [29], which we recommend as a reference. As already at this stage some diverging opinions emerge, being related to various at first glance counterintuitive aspects of the theory (cf. e.g. the above mentioned dispute between Kibble, Arzeliès and Gamba), some brief remarks seem to be in order.

Treating everything within the well developed framework of field theory has a great advantage. There exists a rich source of notions and calculational

tools which are founded on well understood principles. The most important concept is the *energy-momentum-stress tensor*, $T_{\nu\mu}$, which is the starting point of most of the derivations. For closed systems (cf. [29] chapt. 6) it has the important property that it leads to conservation of energy-momentum and that its symmetry leads to the canonical identification of momentum density and energy-flow.

Furthermore, for *stressed* continuum systems it allows for the transparent derivation of somewhat counterintuitive formulas. For the momentum density we have

$$\partial_t g_i + \partial_{x_k} (g_i \cdot u_k + t_{ik}) = 0 \quad (17)$$

with

$$T_{ik} = (g_i \cdot u_k + t_{ik}) \quad (18)$$

Here \mathbf{g} is the momentum density, \mathbf{u} the local velocity (relative to some IF) and t_{ik} the (*relative*) *stress tensor*. Note that the occurrence of this latter term is at first glance perhaps a little bit unexpected, that is, stress density contributing to momentum flux. Correspondingly we have for the energy flux:

$$\partial_t \varepsilon + \partial_{x_k} (\varepsilon \cdot u_k + u_i \cdot t_{ik}) = 0 \quad (19)$$

with

$$S_k := (\varepsilon \cdot u_k + u_i \cdot t_{ik}) \quad (20)$$

the energy flux.

Remark: Note that the energy density includes the elastic contributions in addition to the translatory energy of ordinary moving matter.

One should make a remark as to the two versions of stress tensor. The above version is called the *relative stress tensor*. It has a transformation behavior which is different! from the covariance behavior of the so-called *absolute stress tensor*,

$$p_{ik} = g_i \cdot u_k + t_{ik} \quad (21)$$

The latter version occurs in the full energy momentum tensor of field theory and really has the correct transformation behavior of a 2-tensor under the Lorentz group (cf. the remarks in [29], p.184ff or [14], p.69ff). In t_{ik} forces or stresses are calculated relative to a surface element being momentarily at rest with respect to the medium!, while in the latter case a coordinate system is used which is e.g. at rest in space (Lagrange versus Euler point of view in continuum mechanics).

According to the general principles of relativistic (system) mechanics, we have the canonical identification

$$g_k = S_k/c^2 \quad (22)$$

Remark: Given the transparent and straightforward derivation of these formulas (as far as we know, being due to v. Laue), it is a little bit surprising that e.g. the stress contributions in the energy flux are called *pseudo contributions* by Arzelès.

This point becomes particularly important if a thermodynamic equilibrium system, being enclosed in a container, is treated with the stresses being reduced to a scalar pressure, p . In that case the expression for the momentum density becomes

$$g_k = \rho \cdot u_k + p \cdot u_k/c^2 \quad (23)$$

with ρ the relativistic matter density, $\varepsilon = \rho \cdot c^2$.

In the particular case of thermodynamic equilibrium systems, the pressure is constant over the volume of the system. Furthermore, we assume that it moves uniformly with velocity \mathbf{u} . We then can easily integrate the above equations and get:

$$\mathbf{G} = (E + p \cdot V)/c^2 \cdot \mathbf{u} \quad (24)$$

What is not yet known is the functional form of the energy, E .

In [14] ε or E is calculated in the following way. With the definition of 3-force, $\mathbf{F} := d\mathbf{G}/dt$, we have

$$dE'/dt = \mathbf{F}' \cdot \mathbf{u}' - \cdot dV'/dt \quad (25)$$

for intermediate values of the respective variables. We start from a system which is initially at rest, having pressure p^0 and proper volume V^0 and which is then quasi-stationary brought to the final velocity \mathbf{u} . Using the above expression for \mathbf{G} , the fact that p is a Lorentz invariant, i.e. $p = p_0$, and the change of volume by Lorentz contraction, $V = V_0 \cdot \gamma$, one can integrate the above expression and get

$$E + p \cdot V = (E_0 + p_0 \cdot V_0) \cdot \gamma \quad (26)$$

or

$$E = (E_0 + p_0 V_0 \cdot u^2/c^2) \cdot \gamma \quad (27)$$

as the desired transformation equation for energy.

Another, in our view more direct, method goes as follows. Starting again from the rest system, the work done by pressure forces due to Lorentz contraction is ($p = p_0$):

$$\Delta E_1 = -p \cdot (V - V_0) = p_0 \cdot V_0 \cdot (1 - \gamma^{-1}) = (p_0 V_0 \cdot u^2/c^2) \cdot \gamma \quad (28)$$

The translatory contribution is

$$E_2 = E_0 \cdot \gamma \quad (29)$$

hence

$$E = (E_0 + p_0 V_0 \cdot u^2/c^2) \cdot \gamma \quad \text{or} \quad E + pV = (E_0 + p_0 V_0) \cdot \gamma \quad (30)$$

Remark: In the latter derivation we have not used the definition of force. On the other hand, concerning the question: what is real in physics?, we have calculated the work done by the pressure on the volume, changing due to Lorentz contraction. Hence, people who consider this as being only apparent (e.g. Rohrlich,[28]), may have problems with the above derivations.

Observation 3.1 *We see that for non-closed systems like an equilibrium system, being confined to a vessel, $(E/c, \mathbf{G})$ does not transform like a 4-vector, as might be expected from ordinary relativistic kinematics. On the other hand, $(H/c, \mathbf{G})$ is a 4-vector, with*

$$H := E + pV \quad (31)$$

the enthalpie.

The reason is that work is done by e.g. the walls of the vessel or the exterior, which is not included in E . The other possibility is to include all! contributions in the system under discussion, for example the contributions of the walls, and use a total energy-momentum tensor as in the case of a closed system. This would however become a very nasty enterprise in our view and should be avoided.

4 The First and Second Law of Relativistic Thermodynamics

In this section we describe on what fundamental laws we want to base relativistic thermodynamics. We begin with the first and second law, because the zeroth law is more complicated to formulate. It is useful to divide the set

of mechanical or thermodynamical variables into *invariants* and *covariants*, respectively. We regard entropy and pressure as invariants under Lorentz transformations.

$$S = S_0 \quad , \quad p = p_0 \quad (32)$$

This can be directly calculated for the pressure via its definition as force per area. In case of the entropy, S , one can also provide a calculational argument (as already Planck did). We prefer however to invoke the statistical nature of entropy as a measure of the width of the distribution relevant microstates, being represented by a particular macrostate. This property will not change if systems are quasi stationarily accelerated while maintaining their interior state (see also [12] chaps.4.e and 23).

Remark: As to the definition of pressure as an invariant, the following subtle point should be kept in mind. This holds for a concept of pressure being defined with respect to a surface element being momentarily at rest in the medium! The pressure, occurring e.g. in the energy-momentum-stress tensor is of course not! a scalar but it contains an additional kinematical term (cf. e.g. [29]). It is defined with respect to a coordinate system being fixed in space or space-time.

We assume the first and second law of thermodynamics to hold also for moving equilibrium systems. For the rest system we have

$$dE_0 = \delta Q_0 + \delta W_0 = T_0 \cdot dS_0 - p_0 \cdot dV_0 \quad (33)$$

where for reasons of simplicity we assume all processes to be reversible. We assume that corresponding laws exist for the moving system, whereas some of the variables have yet to be scrutinized in more detail in this latter case. That is,

$$dE = \delta Q + \delta W \quad , \quad \delta Q = T \cdot dS \quad (34)$$

in the reversible case.

The meaning of E, p, V is clear. More problematical is the meaning of $\delta Q, T, \delta W$. A very naive first guess can immediately be ruled out. One may be tempted to assume that all contributions in the first law do transform in the same way under Lorentz transformation (as they are of the same nature). Assuming for example that dE is the zeroth component of a 4-vector (as in ordinary relativistic mechanics; but see the preceding section), and that δW has the same form as in the rest system, i.e. $\delta W = -p \cdot dV$, we would get:

$$dE = dE_0 \cdot \gamma \quad , \quad \delta W = -p dV_0 / \gamma = \delta W_0 / \gamma \quad (35)$$

We immediately can infer from this observation that the whole matter must be more complicated.

4.1 The Relativistic Concept of Work

In relativistic mechanics the 3-force was canonically defined by $\mathbf{F} = d\mathbf{G}/dt$, and this identification was taken over unchanged and, apparently without much hesitation, by all workers of the classical period (e.g. Planck, Einstein, v.Laue, Pauli, Tolman) to the regime of relativistic thermodynamics. This is a little bit funny, because we will see immediately that it leads to strange consequences, which were however fully accepted by the above mentioned scientists. They even found strong arguments why these strange consequences are in fact entirely natural.

With

$$\mathbf{F} = d\mathbf{G}/dt = d/dt(m_0 \cdot \gamma \cdot \mathbf{u}) = \dot{m}_0 \cdot (\gamma \mathbf{u}) + m_0 \cdot d/dt(\gamma \mathbf{u}) \quad (36)$$

there may be a non-vanishing contribution even if the velocity remains constant, provided $\dot{m}_0 \neq 0$. This extra and counter intuitive term was greeted by e.g. Tolman in chapt. 25 of [14] as a contribution which may come from a possible influx of heat at constant velocity.

In adiabatic or purely mechanical processes, where the rest mass or rather rest energy remains constant, we would have the ordinary (Newtonian) interpretation of 3-force

$$\dot{E} = \dot{W} = \mathbf{F} \cdot \mathbf{u} \quad (37)$$

with $dW = \mathbf{F} \cdot d\mathbf{r}$ the element of mechanical work. This comes about as follows:

$$\begin{aligned} \dot{W} &= d/dt(m\mathbf{u}) \cdot \mathbf{u} = m\dot{\mathbf{u}} \cdot \mathbf{u} + \dot{m}\mathbf{u}^2 = m_0\gamma\dot{\mathbf{u}} \cdot \mathbf{u} + m_0\gamma^3\mathbf{u}^2/c^2 \dot{u} \cdot \mathbf{u} \\ &= m_0\gamma^3\dot{\mathbf{u}} \cdot \mathbf{u} \cdot (1 - \mathbf{u}^2/c^2 + m\dot{u}^2/c^2) = m_0\gamma^3\dot{\mathbf{u}} \cdot \mathbf{u} = d/dt(m \cdot c^2) = dE/dt \end{aligned} \quad (38)$$

Such a type of force is called by Rindler ([15], p.124 ff) a *rest-mass preserving force*. In situations where the rest mass is allowed to vary, we would get an extra contribution, $\dot{m}_0\gamma\mathbf{u}^2$, which cannot be incorporated in $dE/dt = d/dt(m \cdot c^2)$. So, in this case, the identification of $\mathbf{F} \cdot \mathbf{u}$ and \dot{E} does no longer hold. In the classical papers this distinction is, as far as we can see, not made, i.e. the classical work term is simply assumed to be

$$\delta W := \mathbf{u} \cdot d\mathbf{G} \quad (39)$$

as the element of work, done on the system.

In non-relativistic thermodynamics work can be done on the system by pressure forces, $-pdV$, and by adiabatic, translatory forces. In the classical framework of relativistic thermodynamics a third form of work is assumed to occur, i.e. a contribution of the above type, $\delta W := \mathbf{u} \cdot d\mathbf{G}$, which can be different from zero even if we have only an influx of heat at constant velocity, \mathbf{u} . This increases the internal energy and hence, by the relativistic identification of mass and energy, the rest-mass of the system and thus the momentum. The corresponding force according to this philosophy is then

$$\mathbf{F}_{heat} = \dot{m}_0 \cdot (\gamma \cdot \mathbf{u}) \quad (40)$$

It is in our view difficult to understand why the classical authors emphatically justified the occurrence of such a work term. It is perhaps noteworthy that Moeller in his first editions of [29] also followed this line of reasoning, while in his last edition he changed to the Ott-Arzeliès convention. Even more recently Rindler in [27], p.92 explicitly states that an object, being heated in its rest frame, experiences a force in every other IF of the type described above. On the other hand, Einstein in his mentioned letters to v.Laue argued against such *virtual* force terms and also Ott provides some arguments. However, we think that our above observation (formula (38)) is perhaps the most convincing from a theoretical point of view.

Observation 4.1 *As only a rest-mass preserving force can be associated with a true work term in the ordinary mechanical sense and a fortiori with a corresponding energy increase, there is no evidence that other forms of energy increase should be associated with a force.*

Some other arguments are as follows:

- In case some systems had different velocities, heat transfer between them, which is ultimately the effect of e.g. a large number of random interactions between the respective surface constituents, would also involve the transfer of some net momentum etc. However, in case the systems have the same velocity, these random exchanges are on average undirected so that no momentum transfer should be involved.
- In our view the phenomenon is not even a relativistic one. If the combination of two subsystems of the same velocity does form a compound system, one can transfer heat or matter from one subsystem to the other one, without any effect on the respective velocities. It is difficult to see any force being involved.

However, this does not! mean that the term $\mathbf{u}d\mathbf{G}$ has always taken to be zero in relativistic thermodynamics. The situation is in fact more complicated (which was, in our view, not even fully realised in the detailed analysis by Ott; see below). We have, according to our previous formulas

$$\mathbf{u}d\mathbf{G} = u^2/c^2 \cdot d(E + pV) = u^2/c^2 \cdot \gamma \cdot (E_0 + p_0V_0) \quad (41)$$

We have argued above, as also Ott did, that dE_0 does not! generate a contribution in the work, induced by some *heat force*. Therefore, the term

$$u^2/c^2 \cdot \gamma \cdot (dp_0V_0 + p_0dV_0) \quad (42)$$

remains to be discussed.

The contribution $u^2/c^2 \cdot \gamma \cdot (p_0dV_0)$ is a work term, coming solely from the Lorentz contraction of the volume element, dV_0 , in the rest system, when observed in the laboratory frame (cf. formula (28)). Such a contribution occurs also in the energy transformation law and a similar one plays the role of explicit pressure work, $-pdV$. We do not see, that an additional force is involved with this pure contraction effect. So we decide to delete this term. There remains the term

$$u^2/c^2 \cdot \gamma \cdot (dp_0V_0) = u^2/c^2 \cdot \gamma^2 \cdot (dpV) \quad (43)$$

This term occurs also in the total variation of the (internal) energy of a moving system.

A changing pressure means also a changing applied force (in addition to a change in internal energy, dE) as equilibrium has to be maintained in quasi-static processes. So it seems reasonable to associate the term really with an applied *moving force*. So we finally conclude

Statement 4.2 *In relativistic thermodynamics the only work terms we are taking into account are i) work of pressure, $-pdV$, ii) adiabatic translatory work, $\mathbf{u}d\mathbf{G}$ but with $E_0 = m_0c^2 = \text{const}$, iii) no work is involved in the exchange of heat between comoving systems, but we include a work term coming from dp_0V_0 . I.e. we assume*

$$\delta W = -pdV + u^2/c^2 \cdot \gamma^2 \cdot (dpV) + E_0/c^2 \cdot d(\gamma\mathbf{u}) \quad (44)$$

4.2 The Relativistic Concept of Heat

Heat is a subtle concept even in non-relativistic phenomenological thermodynamics. The most straightforward way of introducing it is, in our view,

to regard it as the stochastic, disordered and non-coherent contribution in the energy conservation law (in contrast to e.g. the highly organized form of work, which is, in effect, some averaged and integrated form of the individual effects of many microscopic events). So it is reasonable to define it simply by

$$\delta Q = dE - \delta W \quad (45)$$

that is, as the difference between the increase of internal energy and applied work.

From our above line of observations and arguments, its functional dependence on the proper or rest variables can now be inferred from the expressions for E and δW . In formula (30) we have got

$$E = (E_0 + p_0 V_0 \cdot u^2/c^2) \cdot \gamma \quad (46)$$

Furthermore we have

$$p \cdot dV = (p_0 \cdot dV_0) \cdot \gamma^{-1} \quad (47)$$

If the full $\mathbf{u}d\mathbf{G}$, is included in the work term (as it is done in the classical papers), one can proceed as follows:

$$dE = (dE_0 + d(p_0 V_0) \cdot u^2/c^2) \cdot \gamma \quad (48)$$

$$\delta W = -p_0 dV_0 \cdot \gamma^{-1} + u^2/c^2 \cdot (dE_0 + d(p_0 V_0) \cdot \gamma) \quad (49)$$

that is, we can keep the differential, d , outside of the product $(p_0 V_0)$, as the respective terms cancel each other. We then arrive at (see e.g. [13] or [14]):

$$\delta Q = \delta Q_0 \cdot \gamma^{-1} = \delta Q_0 \cdot (1 - u^2/c^2)^{1/2} \quad (50)$$

We learned in the preceding subsection that some of these contribution in δW are presumably inexistent. We have

$$d(p_0 V_0) = dp_0 V_0 + p_0 dV_0 \quad (51)$$

The $(dp_0 V_0)$ -contribution in the variation of the energy is compensated by the corresponding term in δW and we end up with the formula

$$\begin{aligned} \delta Q &= (dE_0 + p_0 \cdot dV_0 \cdot u^2/c^2) \cdot \gamma + p_0 \cdot dV_0 \cdot \gamma^{-1} \\ &= \gamma \cdot (dE_0 + p_0 \cdot dV_0 (u^2/c^2 + \gamma^{-2})) = \gamma \cdot (dE_0 + p_0 \cdot dV_0) \end{aligned} \quad (52)$$

We thus get

Statement 4.3 *The infinitesimal element of heat transforms under a Lorentz transformation like*

$$\delta Q = \delta Q_0 \cdot \gamma = \delta Q_0 \cdot (1 - u^2/c^2)^{-1/2} \quad (53)$$

That is, in contrast to energy and work, it transforms as the zero component of a 4-vector. A related observation was made in [29] after a long and involved calculation!

We want to add two remarks. First, we see that a very subtle compensation has to happen between contributions which transform very differently under a Lorentz transformation, in order that a coherent transformation behavior of central quantities like e.g. the heat does occur. I think, this is one of the main difficulties in this business if one is really willing to treat the problems in full generality. Second, we think, the Lorentz covariance of the heat (in contrast to energy and work) is due to a subtle effect. Work is somehow the summation over microscopic transfers of energy. Neither (moving) walls or other external mechanical processes are really involved. So we think, that each of these elementary (statistical) contributions transforms as the zero component of an energy-momentum vector. The same does then hold for the sum of such contributions. This is completely different for macroscopic (internal) energy or work.

4.3 The Relativistic Concept of Temperature and the Relativistic Carnot Cycle

The notion of temperature is presumably the most problematical and subtle one in relativistic thermodynamics and there exists a wide range of different opinions. We first mention some frequent errors as to this notion. In the older literature the opinion is sometimes suggested that an observer in the laboratory system really observes a higher or lower temperature in a moving body compared to its rest temperature by somehow reading off a comoving thermometer. This opinion is almost certainly incorrect as what he will observe is simply the rest temperature. As a consequence, many of the newer thought experiments, trying to show that temperature is actually an *invariant*, are somewhat beside the point (see the section about the so-called zeroth law of thermodynamics).

To give an example, fixing a certain definite point on the temperature scale, e.g. the coexistence point of ice and water in the rest system, nothing spectacular will happen if the system is set into motion in a quasi-stationary process. That is, ice will not start to melt or water to freeze. What an

observer sees is simply the rest temperature even if the system moves. This is not some kind of *moving temperature*.

We have in fact to remember that in thermodynamics the *absolute temperature* is playing the fundamental role in the thermodynamic relations. Absolute temperature, on the other hand, is introduced and defined via the *Carnot process* or plays the role of an *integrating factor* in the relation between entropy and heat. Therefore, to begin with, we should concentrate on this notion and its generalisation.

The corresponding structural relation now fixes the transformation properties of absolute temperature. From $dS = \delta Q/T$ for a reversible process and $dS = dS_0$, $\delta Q = \delta Q_0 \cdot \gamma$, it follows

Observation 4.4 *T transforms under a Lorentz transformation as*

$$T = T_0 \cdot \gamma \quad (54)$$

Furthermore, as e.g. described in [30], the Carnot cycle allows us to define and measure absolute temperature via the universal relation

$$\eta = (Q_1 - Q_2)/Q_1 =: (T_1 - T_2)/T_1 \quad (55)$$

and η the *Carnot efficiency*. On the other hand, heat can be measured independently of the concept of temperature as is beautifully described in e.g. [31], chapt.1.7, via purely mechanical processes.

In order to show that all this is not a purely theoretical construct, one can analyse the *relativistic Carnot cycle*, as discussed in different realisations in e.g. [12] or [14].

Remark: Note that both Laue and Tolman belong to the classical period. I.e., the temperature of the moving system is lower than the rest temperature. As a consequence, some of the occurring plus or minus signs are different from our treatment below.

Consider now a simple system (the engine), working at constant pressure, $p = p_0$ (i.e. terms, containing a $dp_0 V_0$, do not! contribute), over the whole reversible cycle and operating between a reservoir, R_1 , being at rest in the laboratory frame, having temperature T_1 , and a reservoir, R_2 , moving with the velocity u , and having the temperature $T_2 = T_1 \cdot \gamma$. The system may initially be at rest in a state, described by energy E_a , volume V_a and temperature T_1 . Let it now absorb the amount of heat Q_1 from R_1 at constant pressure and doing the work $p \cdot (V_b - V_a)$. We then have

$$Q_1 = (E_b - E_a) + p \cdot (V_b - V_a) = H_b - H_a \quad (56)$$

(cf. observation 3.1).

The second step consists of a reversible adiabatic acceleration to the velocity u of reservoir R_2 , with the internal conditions unaltered. The work done by the system is

$$W_2 = E_b - E_c \quad (57)$$

with E_c given by e.g. formula (27). In the third step the amount of heat being released to the reservoir R_2 is Q_2 and the amount of work done is

$$W_3 = p \cdot (V_d - V_c) \quad (58)$$

We assume now that the amount of released heat in the third step is just sufficient so that the system can be returned to its initial state by a reversible deceleration. This is exactly the case (see below) if

$$Q_{2,0} = (E_d - E_c)_0 + p(V_d - V_c)_0 = -Q_1 = -Q_{1,0} \quad (59)$$

holds, with the subscript 0 denoting the respective proper values of the quantities, i.e. for $Q_{2,0}$ it is the amount of heat measured by a comoving observer. The work done by the system is

$$W_4 = E_d - E_a \quad (60)$$

The first law of thermodynamics tells us that

$$Q_1 + Q_2 = W_1 + W_2 + W_3 + W_4 \quad (61)$$

with Q_i the heat absorbed by the system and W_i the work done by the system, or

$$\begin{aligned} Q_2 &= (pV_d + E_d) - (pV_c + E_c) = H_d - H_c = \\ &= (pV_a + E_a) \cdot \gamma - (pV_b + E_b) \cdot \gamma = (H_a - H_b) \cdot \gamma = \\ &= - (p(V_b - V_a) + (E_b - E_a)) \cdot \gamma = -Q_1 \cdot \gamma \end{aligned} \quad (62)$$

(according to formula (30)). We see that in each cycle there is a heat transport from reservoir R_1 to reservoir R_2 and a negative amount of work done by the system on the environment, i.e.

$$\Delta W = Q_1 \cdot (1 - \gamma) < 0 \quad (63)$$

It is clear that by reversing the direction of the process we can extract a positive work out of the compound system (i.e., system plus reservoirs) while now heat is transported from R_2 to R_1 .

Observation 4.5 *It is important to note that nowhere in the calculations it was really used that the reservoir R_2 has a higher temperature. Only the first law of thermodynamics was exploited. What we however implicitly assumed is that system and reservoirs coexist thermodynamically at relative velocity zero, so that the ordinary laws of thermodynamics can be applied (e.g. quasi-static heat exchange).*

The second law now tells us that

$$Q_2/Q_1 = -T_2/T_1 = -\gamma \quad (64)$$

i.e.

$$T_2 = T_1 \cdot \gamma \quad (65)$$

That is, the relativistic Carnot cycle allows us to give an operationalistic definition of absolute temperature as in ordinary thermodynamics and exhibits the internal consistency of the framework.

Remark: In the classical framework, i.e. with a work term coming from the influx of heat, the work done by the system is positive if heat is transported from R_1 to R_2 , and vice versa.

The Carnot efficiency in our reversed cycle is

$$\eta = 1 - Q_1/Q_2 = 1 - T_1/T_2 = 1 - \gamma^{-1} = 1 - \sqrt{1 - u^2/c^2} > 0 \quad (66)$$

We hence can conclude that our reversed Carnot process does work on the environment in an objective sense, the reason being mainly the Einstein-equivalence of matter and energy and not! so much a particular assumption about *moving temperature*. The energy or mass which is decelerated is larger than the mass which is accelerated due to the additional absorption of heat.

Remark: It is important to note that the above Carnot cycle is of a very special type. This will be discussed in connection with the so-called zeroth law of thermodynamics, see below.

4.4 The Covariant Expressions of Heat and Temperature

We have seen from our preceding discussions that heat and temperature transform as

$$\delta Q = \delta Q_0 \cdot \gamma \quad , \quad T = T_0 \cdot \gamma \quad (67)$$

if the initial system is the rest system. For general moving systems, moving with the velocities $\pm \mathbf{U}$ relative to each other, we thus have to conclude that the transformation laws are those of 4-vectors.

It is straightforward to build 4-vectors from scalars in the rest frame. For temperature and heat the covariant generalisations are hence

$$T^\mu = (T_0 \cdot \gamma, T_0 \cdot \mathbf{u}/c \cdot \gamma) \quad , \quad \delta Q^\mu = (\delta Q_0 \cdot \gamma, \delta Q_0 \cdot \mathbf{u}/c \cdot \gamma) \quad (68)$$

i.e.

$$T^\mu = T_0/c \cdot U^\mu \quad , \quad \delta Q^\mu = \delta Q_0/c \cdot U^\mu \quad (69)$$

with U^μ the 4-velocity.

The covariant generalisation of the second law of thermodynamics is then

$$dS \geq \beta^\mu \delta Q_\mu = \beta^0 \delta Q^0 - \beta \cdot \delta \mathbf{Q} \quad (70)$$

A certain warning should be spelled out. In the rest system we have of course that $\beta^0(0) = (T^0(0))^{-1}$, with the velocity of the system relative to the respective IF given in braces. In a IF moving with the velocity \mathbf{u} , we however have

$$\beta^0(u) = \gamma \cdot \beta^0(0) = \gamma \cdot (T^0(0))^{-1} = \gamma^2 \cdot T^0(u)^{-1} \quad (71)$$

Observation 4.6 *In a moving IF the relation between β^0 and T^0 is different from the standard form, we have in the rest system.*

Remark: See in this connection e.g. formulas (4.19),(4.20) in [32], where it is concluded that $T^0(u)^{-1} = \beta^0(u)$, which would lead to $T = \gamma^{-1} \cdot T_0$ instead of $T = \gamma \cdot T_0$. We have however seen that both β^μ and T^μ transform as 4-vectors.

5 The Covariant Expression of the Zeroth Law of Thermodynamics

A better understanding of the relativistic zeroth law (which in this context we take to be a statement about the respective temperatures of thermally coexisting subsystems) is particularly important, as it is, so to speak one of the pillars of ordinary thermodynamics. Furthermore, its seeming violation was invoked by e.g. Landsberg (as was already mentioned in the introduction; [18], p.334) to construct a paradoxon in case one is willing to assume that temperature is not! a scalar. We will now show that this conclusion is ill-founded and that, perhaps coming as a surprise, the zeroth law looks different in relativistic thermodynamics.

More specifically, we show that the transformation law, $T = T_0 \cdot \gamma$, we are favoring in this paper, is not! in contradiction with an appropriately formulated zeroth law.

Statement 5.1 (*Entropy-maximum principle*) *An isolated system being at rest (i.e. having constant rest energy) and possibly consisting of several subsystems which can e.g. exchange energy with each other, is in thermal equilibrium if its entropy is maximal.*

In ordinary thermodynamics equally of the temperatures, T_1, T_2, \dots , of the subsystems can immediately be derived from this principle.

The discussion of the relativistic case is more subtle and we proceed as follows. We employ the results of the discussion of the relativistic Carnot cycle in the preceding section. Note that the cycle is very special in several respects if compared with the ordinary Carnot cycle. We found that both

$$Q_{2,0} = -Q_{1,0} \quad , \quad Q_2 = -\gamma \cdot Q_1 = -\gamma \cdot Q_{1,0} \quad (72)$$

and

$$T_2 = T_1 \cdot \gamma = T_{1,0} \cdot \gamma \quad (73)$$

have to hold. Such strong constraints do not exist in the ordinary Carnot cycle.

We can now use this relativistic Carnot cycle in the discussion of the zeroth law. We treat the reservoirs as large but finite subsystems (as compared to the engine, which can be chosen infinitesimal if necessary). The compound system is assumed to be closed in the sense of formula (74). The engine is now used to reversibly transport a small amount of heat or internal energy from the one system to the other under the proviso

$$E_{1,0} + E_{2,0} = \text{const} \quad (74)$$

This is exactly the kind of variation which is used in the ordinary derivation of the zeroth law, where now E_i are the internal energies of the (finite) reservoirs (or subsystems) while we now write ΔQ_i or $\Delta Q_{i,0}$ for the exchanged amounts of heat with

$$\Delta Q_{2,0} = -\Delta Q_{1,0} \quad , \quad \Delta Q_2 = -\gamma \cdot \Delta Q_1 \quad (75)$$

We have thermal coexistence of the two subsystems if such an infinitesimal exchange of heat does not change the total entropy. Note in this respect that quasi stationary acceleration or deceleration of the engine does

not change the entropy. By definition, the entropy (as a state function) of the engine does not change in a complete cycle. This implies

$$Q_2/T_2 + Q_1/T_1 = 0 \quad (76)$$

(cf. formula (64)). But exactly the same formula holds for the two reservoirs (subsystems). I.e., we have

$$\Delta S_2 + \Delta S_1 = 0 \quad (77)$$

for the total variation of the entropy of the compound system.

Conclusion 5.2 *The two subsystems (of the closed compound system) are in thermal equilibrium if*

$$T_{2,0} \cdot \gamma = T_2 = T_1 = T_{1,0} \cdot \gamma \quad (78)$$

that is, if the rest temperature of the moving system is identical to the rest temperature of the other system. But observed from the laboratory frame, the temperatures differ in just the above sense.

Remark: The only place where we found this point discussed in a however qualitative manner is in [12]. V.Laue provides also another thought experiment which is equally convincing (see p.178).

6 Some Remarks on a Paper by Ott

As we already remarked in the introduction, we think that also the analysis by Ott is not really complete and contains gaps. The reason is however understandable as some of the problems are in fact well-hidden. Typically (as is the case in quite a few other investigations), it turns out that the processes being analysed are too restricted and special. This holds in particular for his version of Carnot-cycle (cf. section 2 of [7]).

For one, as usual the pressure is kept constant. By making this assumption he avoided to discuss the critical term, $dp_0 V_0$, we discussed in our analysis of the first law of thermodynamics and which is both a contribution in the problematical work term, $\mathbf{u} d\mathbf{G}$, and the internal energy. For another (and strangely enough), the volume is not! changed in his Carnot cycle during the step in which the engine absorbs heat from a reservoir. He says that the occurring work is somehow stored within the system? This is in our view very funny and difficult to understand.

A thermodynamical state describing the coexistence of e.g. fluid and steam allows for moves on the state manifold in which the pressure remains constant but then the volume changes if fluid is transformed into steam or vice versa. Otherwise the temperature will change, but in this part of the cycle the engine is in contact with a heat reservoir! Therefore the only pressure work in Ott's Carnot cycle occurs in the adiabatic parts where the engine is accelerated or decelerated. But in our view it is inconsistent to have a state change with all! state variables kept fixed.

For all these reasons Ott gets an extremely simple (almost trivial) version of the first law (cf. his formulas (13), (16) and (17)) which reads

$$\delta Q = dE = dE_0 \cdot \gamma \quad (79)$$

In short, Ott gets the same transformation laws as we did but only for a very incomplete situation.

As we have argued above, the crucial problem is to deal with terms like

$$pdV = p_0 dV_0 \cdot \gamma^{-1} \quad (80)$$

with $dV_0 \neq 0$ in the rest system in view of the completely different transformation behavior of e.g.

$$d(E + pV) = d(E_0 + p_0 V_0) \cdot \gamma \quad (81)$$

This led to a deeper analysis of the nature of work in relativistic thermodynamics. This problem is completely lost sight of if one deals only with very simplified processes (the same remark applies to the approach of Rohrlich being dealt with in the following section).

A further problematic point can be found on p.76 in formula (9). Ott obviously takes for granted that the thermodynamic energy is the zero component of a 4-vector. We think it is common knowledge that for non-closed systems as in relativistic thermodynamics this is not! the case (cf. our preceding sections). Consequently Ott concludes from his simplified (but presumably wrong!) version of the first law (formula (79)) that δQ is also the zero component of a 4-vector.

This however follows in a complete analysis only from a series of not entirely trivial steps (cf. the preceding sections or the analysis given by Moeller in his book [29]). This assumption, made by Ott was also criticized by Balescu in [34]. As far as we can see, Arzeliés makes a similar assumption. This is only correct if parts of the exterior are included in the system, which is in our view however not desirable.

7 Some Remarks on a Paper by Rohrlich

We also want to briefly comment on an approach, developed by Rohrlich ([28]). The general tenor is that there are *true* transformation properties and, on the other hand, only *apparent* ones in special relativity. We must confess that we do not appreciate very much this distinction. For example, the Lorentz contraction of length or volume is only an apparent one in the philosophy of Rohrlich and he rather suggests a volume transformation which is 4-vector like.

It is not the place here, to dwell in more detail on the philosophy, underlying Rohrlich's paper. While it is possibly shared by other physicists, we would like to make our own point of view clear, namely we regard these phenomena as real (as also e.g. Rindler does in his book [15]). A nice thought experiment as to this question is developed by Bell in his essay about special relativity ([33]), which clearly shows that Lorentz contraction is real.

There is however in section 3 of Rohrlich's paper a treatment of the transformation law of relativistic temperature which is a little bit different from the ordinary one and arrives at a result which at first glance differs from our own findings. We think this point is a good example to learn how tricky calculations in this field actually are.

Rohrlich writes the first law in the rest system slightly differently as

$$T_0 dS = dH_0 - V_0 dp_0 \quad (82)$$

with again, S , p , treated as invariants and $H_0 = E_0 + p_0 V_0$. He then makes the seemingly innocent assumption that the first law in this form holds also in an arbitrary IF. I.e., he assumes

$$TdS = dH - Vdp \quad (83)$$

and concludes from this that

$$T = T_0 \cdot \gamma^{-1} \quad (84)$$

must hold, where he treated only the special case, $dH = dH_0 = 0$ anyhow. I.e., he gets the classical transformation law. The inconsistency of his assumption possibly escaped Rohrlich because he really treated only the case $TdS = -Vdp$.

We showed however above that H transforms as the zero-component of a 4-vector, i.e. we actually have

$$H = H_0 \cdot \gamma \quad (85)$$

Furthermore, we showed that the complete work term is in fact more complicated and consists of more than only pressure work. We in fact get for the general case

$$\gamma^{-1} \cdot T dS = T_0 dS = dH_0 - V_0 dp = \gamma^{-1} \cdot dH - \gamma \cdot V dp \quad (86)$$

hence

$$T dS = dH - \gamma^2 \cdot V dp \quad (87)$$

that is

Conclusion 7.1 *The first law of thermodynamics in the above form, given by Rohrllich, does not transform in a Lorentz invariant way. Therefore the classical transformation result of temperature does not follow either.*

8 Commentary

We hope that it has become clear from our discussion that various of the papers existing in the field contain gaps or arguments which are not conclusive. We also emphasized that the wide spread habit to discuss only very particular and restricted thermodynamic processes is dangerous and leads sometimes to wrong conclusions concerning the correct transformation laws of thermodynamic variables.

This holds in particular for the various contributions entering in the (infamous) work term $\mathbf{u} \cdot \mathbf{G}$. We showed that some of them are in fact zero but there is one contribution which survives. This term has been overlooked in the past because most of the authors treated only processes at constant pressure.

Furthermore, a detailed analysis of the so-called zeroth law showed that, in contrast to what some groups claimed, thermodynamic systems, moving with a non-vanishing relative velocity, u , can thermally coexist, provided their (moving) temperatures fulfill

$$T_2 = T_1 \cdot \gamma(\mathbf{u}) \quad (88)$$

which however implies that their respective rest temperatures are equal, i.e. $T_{2,0} = T_{1,0}$.

It should again be emphasized that our guiding principle was that the first and second law of relativistic thermodynamics remain form invariant with respect to the transformed variables. While heat turns out to be a 4-vector, this was not the case for (internal) energy and work.

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