## FUNCTIONS AS PATTERNS, TABLES \& GRAPHS

Grade/Level : $7^{\text {th }} \& 8^{\text {th }}$ grades

## Duration/Length : 5 days (45 minutes each)

## Brief Overview:

Day I The unit begins with a function machine model for patterns.
Day II Students will work on developing a function rule for Letter Number Patterns.

Day III Students will create their own letter patterns, and represent them in tables and equation models by using NLVM website resources.

Day IV Students will represent their patterns in graphs by using NLVM website resources.

Day V Students will work in groups to make poster boards for four different representations of a pattern.

## PROCESS STANDARDS

## Problem Solving Strand

7.PS. 1 Use a variety of strategies to understand new mathematical content and to develop more efficient methods
7.PS. 4 Observe patterns and formulate generalizations
7.PS. 5 Make conjectures from generalizations
7.PS. 6 Represent problem situations verbally, numerically, algebraically, and graphically
7.PS. 14 Determine information required to solve the problem

## Reasoning and Proof Strand

7.RP. 2 Use mathematical strategies to reach a conclusion

Communication Strand
7.CM. 1 Provide a correct, complete, coherent, and clear rationale for thought process used in problem solving
7.CM.4 Share organized mathematical ideas through the manipulation of objects, numerical tables, drawings, pictures, charts, graphs, tables, diagrams, models and symbols in written and verbal form

## Connections Strand

7.CN. 1 Understand and make connections among multiple representations of the same mathematical idea
7.CN. 3 Connect and apply a variety of strategies to solve problems
7.CM.10 Use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale
Representation Strand
7.R. 1 Use physical objects, drawings, charts, tables, graphs, symbols, equations, or objects created using technology as representations
7.R. 2 Explain, describe, and defend mathematical ideas using representations
7.R. 3 Recognize, compare, and use an array of representational forms
7.R. 4 Explain how different representations express the same relationship

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CONTENT STANDARDS

## Algebra Strand

7.A. 7 Draw the graphic representation of a pattern from an equation or from a table of data
7.A. 8 Create algebraic patterns using charts/tables, graphs, equations, and expressions
7.A. 10 Write an equation to represent a function from a table of values
8.A. 15 Understand that numerical information can be represented in multiple ways: arithmetically, algebraically and graphically
8.A. 16 Find a set of ordered pairs to satisfy a given linear numerical pattern (expressed algebraically); then plot the ordered pairs and draw the line
8.A. 19 Interpret multiple representations using equation, table of values and graph

## NCTM STANDARDS

## Algebra Standard for Grades 6-8

Understand patterns, relations, and functions:

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

Use mathematical models to represent and understand quantitative relationships

- Model and solve contextualized problems using various representations, such as graphs, tables, and equations.


## WEB RESOURCES:

NLVM website
http://enlvm.usu.edu/ma/nav/toc.jsp?sid= shared\&cid=emready@patterns relations functions\&bb=course
http://nlvm.usu.edu/en/nav/frames asid 191 g 3 t 1.html

DAY I :

Objectives:

- Define linear functions in function machine model
- Identify numerical patterns
- Develop the notion of a general rule based on numerical patterns

Materials:

- Teacher's computer \& projector
- NLVM software
- 25 worksheet (HW)

Procedure: 1- Discuss with students how machines work in real life. After reviewing first two examples, ask students to complete the chart below with three more examples:

| Input | Machine | Output |
| :--- | :--- | :--- |
| Bread | Toasters | toast |
| Dirty clothes | Washer | Clean clothes |
|  |  |  |
|  |  |  |
|  |  |  |

2- Introduce function machine and discuss how it works with numbers. Asks students to find certain outputs based on the given values by using function machine software.
http://nlvm.usu.edu/en/nav/frames_asid_191_g_3 t 1.html
3- Give students the following two questions and discuss what we can do in order to find outputs for bigger inputs

> Complete the table

| X (input) | 1 | 2 | 3 | 4 | 5 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Y (output) | 4 | 8 | 12 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Complete the table

| X (input) | 2 | 3 | 4 | 5 | 6 | 30 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| Y (output) | 7 | 10 | 13 |  |  |  |



## FUNCTION MACHINE

1. Complete the table and find a rule for each pattern.

| X (input) | 1 | 2 | 3 | 4 | 6 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y (output) | 4 | 8 | 12 |  |  |  |

RULE: $\mathrm{y}=$
2-

| X (input) | 8 | 7 | 6 | 5 | 4 | 1 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| Y (output) | 5 | 4 | 3 |  |  |  |

RULE: $\mathrm{y}=$
3.

| X (input) | 3 | 4 | 5 | 6 | 10 | 30 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| Y (output) | 7 | 9 | 11 |  |  |  |

RULE: $\mathrm{y}=$
4.

| X (input) | -2 | 4 | 6 | 8 | 10 | 20 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| Y (output) | 1 | -2 | -3 |  |  |  |

RULE: $\mathrm{y}=$


## FUNCTION MACHINE

1. Complete the table and find a rule for the pattern.

| X (input) | 1 | 2 | 3 | 4 | 6 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y (output) | 4 | 8 | 12 | 16 | 24 | 40 |

RULE: $\mathrm{y}=4 \mathrm{x}$
2-

| X (input) | 8 | 7 | 6 | 5 | 4 | 1 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| Y (output) | 5 | 4 | 3 | 2 | 1 | -2 |

RULE: $\quad \mathrm{y}=\mathrm{x}-3$
3.

| X (input) | 3 | 4 | 5 | 6 | 10 | 30 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| Y (output) | 7 | 9 | 11 | 13 | 21 | 61 |

RULE: $\quad \mathrm{y}=2 \mathrm{x}+1$
4.

| X (input) | -2 | 4 | 6 | 8 | 10 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y (output) | 1 | -2 | -3 | -4 | -5 | -10 |

RULE: $\quad y=-x / 2$ or $-1 / 2 x$

Objectives:

- Write a function rule based on numerical patterns

Materials:

- Projector and teacher computer.
- 25 computers with internet access (computer lab)
- 25 worksheet (HW)

Procedure:

1) Discuss the tower numbers pattern below with students.

2) Express the pattern in a table and find the number of dots for $n=4,5$ Discuss with them how to find $50^{\text {th }}$ figure.

| \# position <br> (Input) | \# of dots <br> (output) |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 50 |  |

3) Using the previous question, discuss the necessity of general rule.

Come up with a function rule.

## Function Rule:

4) Have students work in a group and answer the next two questions.

## CLASS WORK

Draw the next two figures on the grid line below.
Complete the table and write a function rule.


| \# position <br> (Input) | \# of dots <br> (output) |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 20 |  |

Function Rule:

Draw the next two figures, complete the table and write a function rule.
1)


2)

| \# position <br> (Input) | \# of dots <br> (output) |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 20 |  |

Function Rule:

Draw the next two figures, complete the table and write a function rule.
1)


| \# position <br> (Input) | \# of dots <br> (output) |
| :--- | :--- |
| 1 | 7 |
| 2 | 12 |
| 3 | 17 |
| 4 | 22 |
| 5 | 27 |
| 20 | 102 |

Function Rule: $y=5 x+2$

|  |  | - |  |  |  |  |  | $\theta$ |  |  |  |  |  | - |  |  |  |  |  |  |  |  | $\bigcirc$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\bigcirc$ | 0 | 0 |  |  |  |  | - |  |  |  |  |  | 0 |  |  |  |  |  |  |  |  | 0 |  |  |  |  |
|  |  | - |  |  |  | - | - | * 0 | - | - |  |  |  | $\bigcirc$ |  |  |  |  |  |  |  |  | 0 |  |  |  |  |
|  |  | $\mathrm{n}=1$ |  |  |  |  |  | - |  |  | 0 | 0 | 0 | 0 | O | 0 | 0 |  |  |  |  |  | $\bigcirc$ |  |  |  |  |
|  |  |  |  |  |  |  |  | - |  |  |  |  |  | - |  |  |  |  | $\bigcirc$ | 0 | 0 | O | 0 | O | $\bigcirc$ | o | 0 |
|  |  |  |  |  | 0 |  |  | $\mathrm{n}=2$ |  |  |  |  |  | - |  |  |  |  |  |  |  |  | $\bigcirc$ |  |  |  |  |
|  |  |  |  |  | 0 |  |  |  |  |  |  |  |  | 0 |  |  |  |  |  |  |  |  | 0 |  |  |  |  |
|  |  |  |  |  | $\bigcirc$ |  |  |  |  |  |  |  |  | $\mathrm{n}=3$ |  |  |  |  |  |  |  |  | $\bigcirc$ |  |  |  |  |
|  |  |  |  |  | $\bigcirc$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\bigcirc$ |  |  |  |  |
|  |  |  |  |  | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | = 4 |  |  |  |  |
| 0 | 0 | 0 | 0 | 0 | 00 | 0 | 0 | 00 | 00 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | $\bigcirc$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | = 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


| \# position <br> (Input) | \# of dots <br> (output) |
| :--- | :--- |
| 1 | 5 |
| 2 | 9 |
| 3 | 13 |
| 4 | 17 |
| 5 | 21 |
| 20 | 81 |

Function Rule: $y=4 x+1$

DAY III:
Objectives:

- Create a pattern and represent it in a table.
- Write a function rule based on numerical patterns

Materials:

- Projector and teacher's computer.
- 25 computers with internet access (computer lab)
- 25 worksheet (Class work)

Procedure:

1) Review HW problems with students. Make sure they are able to write a function rule for each pattern.
2) Set students up with Make Your Own Pattern activity in the computer lab.

# LETTER PATTERNS ACTIVITY SHEET 

Directions: Go to the link below. (Send an email attachment)
$\mathrm{http}: / /$ enlvm.usu.edu/ma/nav/toc.jsp?sid=
atterns relations functions\&bb=course
A. Click on Choose a pattern

Patterns, Relations and Functions
Introduction
Choose a Pattern
Make Your Own Patterns
Graphing Patterns
Student Feedback Questionnnaire
Background Survey
B. Select the following patterns from choose a problem тепи.

C. Select the following patterns and answer the questions below.

F numbers 1
X numbers 1
L numbers 1
H numbers 1

1. F numbers 1

| \# position <br> input | \# of dots <br> output |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 20 |  |

Function Rule:

15
2. $\mathbf{X}$ numbers $\mathbf{1}$

| \# position <br> input | \# of dots <br> output |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 40 |  |

Function Rule:
3. L numbers 1

| \# position <br> input | \# of dots <br> output |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 50 |  |

Function Rule:
4. H numbers 1

| \# position <br> input | \# of dots <br> output |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 60 |  |

Function Rule:

16
DAY IV:

Objectives:

- Draw the graphic representation of a pattern from an equation or a table.

Materials:

- Projector and teacher computer.
- 25 computers with internet access (computer lab)
- 25 worksheet (HW)

Procedure:

1) Discuss how table data can be written as ordered pairs,

| \# position <br> input | \# of dots <br> output | Ordered pairs |
| :--- | :--- | :--- |
| 1 | 3 | $(1,3)$ |
| 2 | 5 | $(2,5)$ |
| 3 | 7 | $(3,7)$ |
| 4 | 9 | $(4,9)$ |
| 5 | 11 | $(5,11)$ |

2) Discuss how the function rule can be represented by graphing.

| \# position <br> input $(\mathrm{X})$ | \# of dots <br> output $(\mathrm{Y})$ | Ordered pairs |
| :--- | :--- | :--- |
| 1 | 3 | $(1,3)$ |
| 2 | 5 | $(2,5)$ |
| 3 | 7 | $(3,7)$ |
| 4 | 9 | $(4,9)$ |
| 5 | 11 | $(5,11)$ |

Function Rule: $\mathrm{Y}=2 * \mathrm{X}+1$
3) Introduce the Graph Patterns activity below.

## Directions: Go to the link below (send an email attachment)

http://enlvm.usu.edu/ma/nav/toc.jsp?sid= shared\&cid=emready@p atterns relations functions\&cf=activity

## A. Click on Graphing Patterns

Patterns, Relations and Functions
Introduction
Choose a Pattern
Make Your Own Patterns
Graphing Patterns
Student Feedback Questionnnaire
Background Survey
Teacher Feedback Questionnnaire
Select Lessons to Copy
B. Click on light grid lines

C. Write the patterns from previous day in ordered pairs.

Then enter them in the section called data and click on plot

## EXAMPLE:

| pattern | \# of <br> dots |
| :--- | :--- |
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
| 4 | 9 |


D. Type the function rule from the same data in the section called

Function. Then click on Plot/Upgrade.

E. Explain how the plotted points and the graph of the function are related.
F. Apply to the same process for the patterns from Letter Patterns Activity Sheet.

DAY V:
Objectives:

- Create algebraic patterns using charts/tables, graphs, equations, and expressions on a poster board.

Materials:

- Poster boards ( Grid line)
- Markers, ruler
- Multiple Representation worksheets
- Blank Multiple Representation sheets (attached)

Procedure:

1) Divide the class into groups of four.
2) Provide each group with materials to make Multiple

Representations for Patterns poster.

## MULTIPLE REPRESENTATIONS FOR PATTERNS

Directions: We have learned five different representations of patterns so far which are as follows:

- Geometric Model (letter patterns)
- Table Chart Model (input/output)
- Function Rule Model (equations)
- Graphic Model (Graph of the equation)
- Verbal expression Model

Each group will be given one model and be asked to represent it in four other models on their poster board.

Students can use attached blank sheets for different models.

## Group I

Geometric Model:


Group II (Hint: use V numbers 1 for geometric model)
Table:

| \# position <br> input | \# of dots <br> output |
| :--- | :--- |
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
| 4 | 9 |
| 5 | 11 |

Group III (Use L numbers 1 for geometric model)
Equation:

$$
Y=4 * X
$$

Group IV (Use X numbers 1 for geometric model)

Verbal Expressions:
Y is one more than four times X

Group V (Use Tower numbers for geometric model)
Graph:


## PATTERN MODELS

GEOMETRIC MODEL (BLANK)


## VERBAL EXPRESSION (BLANK)



EQUATION (BLANK)


TABLE/CHART (BLANK)

| \# POSITION (INPUT) <br> X | \# OF DOTS (OUTPUT) <br> Y |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

GRAPH (BLANK)


