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SPACE PHYSICS COORDINATE TRANSFORMATIONS: A USER GUIDE

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Abstract—This report presents a comprehensive description of the transformations between the major coordinate systems in use in Space Physics. The work of Russell (1971, *Cosmic Electrodyn.* **2**, 184) is extended by giving an improved specification of the transformation matrices, which is clearer in both conceptual and mathematical terms. In addition, use is made of modern formulae for the various rotation angles. The emphasis throughout has been to specify the transformations in a way which facilitates their implementation in software. The report includes additional coordinate systems such as heliocentric and boundary normal coordinates.

1. INTRODUCTION

The genesis of this report was the task of specifying a set of Space Physics coordinate transformations for use in ESA's European Space Information System (ESIS). The initial approach was to adopt the transformations specified in the paper by Russell (1971). However, as the work proceeded, it became clear that a number of improvements could be made:

(i) Most transformations could be factorized into a series of rotations about principal axes. This reduces each 3-D transformation into a sequence of 2-D operations. Thus the transformation can be specified more clearly in both conceptual and mathematical terms.

(ii) Each rotation could be specified using just two parameters: the axis of rotation and the rotation angle.

(iii) The rotation angles could be specified by reference to more modern publications such as the *Almanac for Computers*, which was first produced in 1977.

(iv) The orientation of the (centred) geomagnetic dipole axis could be specified in a time-dependent way using the International Geomagnetic Reference Field (IGRF).

(v) The rotation angle in the transformation from GSE to GSM coordinates could be determined in a straightforward manner.

In addition, the requirements for ESIS included coordinate systems not covered in Russell's paper, namely heliocentric systems (for studies of the interplanetary medium) and boundary normal systems. These are included in this report.

The improvements listed above can facilitate the implementation of the coordinate transformations in software. For example, the factorization of the transformations allows complex transformations to be performed by repeated use of simple modules, e.g. for matrix multiplication, to evaluate simple matrices. Thus the complexity of software can be reduced, thereby aiding its production and maintenance. Moreover, every attempt is made to minimize the number of trigonometric functions which are calculated.

1.1. *General background*

Coordinate transformations are required in Space Physics as many measured quantities are vectors, e.g. position, velocity, magnetic field, electric field, electric currents. They are usually represented numerically by three Cartesian components x, y and z, which depend on the coordinate system used. Thus there is a requirement for the transformation of vector quantities between different coordinate systems so that the scientist can put the data in the system which is appropriate for his or her current purpose.

Transformations may conveniently be performed using matrix arithmetic as described by Russell (1971). For each transformation we define a transformation matrix $\mathbf{T}$ such that $\mathbf{Q}_2 = \mathbf{T}\mathbf{Q}_1$, where $\mathbf{Q}_1$ is the initial vector and $\mathbf{Q}_2$ is the new vector. This may be written in full as:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}. \tag{1}$$

Thus the problem is simply to specify the nine components of matrix $\mathbf{T}$ for each possible transformation. This approach has several advantages:

(1) Speed of computation. There are only nine multiplications and six additions for each vector transformed. No trigonometric functions are evaluated repeatedly.

(2) A transformation $\mathbf{T}$ may be reversed by applying the inverse matrix $\mathbf{T}^{-1}$ which is simply the transpose of $\mathbf{T}$.

(3) A transformation from system A to system C may be performed by transforming first to an intermediate system B and then to system C. Thus we may multiply matrices to obtain the transformation matrix $\mathbf{T}_{AC} = \mathbf{T}_{BC}\mathbf{T}_{AB}$.

Note that the terms in the transformation matrix are just the components of the new principal axes in the old system. Thus, in equation (1) above, the new X, Y and Z axes can be expressed as unit vectors in the old coordinate system:

$$X = (t_{11}, t_{12}, t_{13})$$
$$Y = (t_{21}, t_{22}, t_{23})$$
$$Z = (t_{31}, t_{32}, t_{33}).$$

1.2. *Geocentric systems*

There are several geocentric coordinate systems in common use. They are listed in Table 1 below.

1.3. *Heliocentric systems*

There are several heliocentric coordinate systems. These are listed in Table 2 below. Note the existence of two heliocentric ecliptic systems (HAE and HEE) which differ only in their definition of the x axis. One system is an inertial frame fixed relative to the stars and the other is fixed relative to the Sun–Earth line.

2. SPECIFICATION OF THE TRANSFORMATION MATRICES

The transformations described in the following sections are presented as matrices, which are either a simple rotation matrix (a rotation of angle ζ about one of the principal axes) or are the products of simple rotation matrices. These simple matrices have only two degrees of freedom and so only two parameters are needed to specify the nine terms in the matrix. These two terms can be the rotation angle and the name of the rotation axis: X, Y or Z. Thus we can specify a simple rotation matrix as

$$\mathbf{E} = \langle \zeta, \text{axis} \rangle$$

and specify a product matrix as

$$\mathbf{T} = \mathbf{E}_1\mathbf{E}_2 = \langle \zeta_1, \text{axis}_1 \rangle * \langle \zeta_2, \text{axis}_2 \rangle.$$

Inversion is straightforward:

$$\mathbf{E}^{-1} = \langle -\zeta, \text{axis} \rangle$$

$$\mathbf{T}^{-1} = \mathbf{E}_2^{-1}\mathbf{E}_1^{-1} = \langle -\zeta_2, \text{axis}_2 \rangle * \langle -\zeta_1, \text{axis}_1 \rangle.$$

This specification allows the transformation matrix to be calculated in a very straightforward way as follows. (1) Identify the component rotations. (2) For each component calculate the rotation matrix: (a) calculate the rotation angle ζ; (b) calculate $\sin\zeta$ and $\cos\zeta$; (c) determine the diagonal terms of the matrix: put 1 in the Nth term, where $N = 1$ if the rotation axis is X, $N = 2$ if Y and $N = 3$ if Z; put $\cos\zeta$ in the other terms; (d) determine the two off-diagonal terms in the same columns and rows as the $\cos\zeta$ values. Put $\sin\zeta$ in the term above the diagonal and $-\sin\zeta$ in the

TABLE 1. "SIMPLE" COORDINATE SYSTEMS WITH A GEOCENTRIC ORIGIN†

System		Definition of axes
Geocentric equatorial inertial	GEI	X = First Point of Aries Z = Geographic North Pole
Geographic	GEO	X = Intersection of Greenwich meridian and geographic equator Z = Geographic North Pole
Geocentric solar ecliptic	GSE	X = Earth–Sun line Z = Ecliptic North Pole
Geocentric solar magnetospheric	GSM	X = Earth–Sun line Z = Projection of dipole axis on GSE YZ plane
Solar magnetic	SM	Y = Perpendicular to plane containing Earth–Sun line and dipole axis. Positive sense is opposite to Earth's orbital motion. Z = Dipole axis
Geomagnetic	MAG	Y = Intersection between geographic equator and the geographic meridian 90° East of the meridian containing the dipole axis. Z = Dipole axis

† Only two axes have to be defined; the third axis completes a right-handed Cartesian triad.

TABLE 2. "SIMPLE" COORDINATE SYSTEMS WITH A HELIOCENTRIC ORIGIN†

System		Definition of axes
Heliocentric Aries ecliptic	HAE	X = First Point of Aries Z = Ecliptic North Pole
Heliocentric Earth ecliptic	HEE	X = Sun–Earth line Z = Ecliptic North Pole
Heliocentric Earth equatorial	HEEQ	X = Intersection between solar equator and solar central meridian as seen from Earth Z = North Pole of solar rotation axis

† Only two axes have to be defined; the third axis completes a right-handed Cartesian triad. The HAE and HEE systems are both sometimes known as heliocentric solar ecliptic or HSE. The HEEQ system is sometimes known as heliocentric solar or HS.

term below; this sign convention defines the sense of positive rotation used in this paper; (e) set the remaining off-diagonal terms to zero. (3) Multiply the component matrices to obtain the transformation matrix.

Thus for a rotation about the Z axis we obtain the matrix shown below:

$$\langle \zeta, Z \rangle = \begin{bmatrix} \cos\zeta & \sin\zeta & 0 \\ -\sin\zeta & \cos\zeta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

3. NOTES

3.1. *Time scales*

All of the fundamental transformations defined in the following sections are time dependent. To maintain a uniform style, time is there specified by modified Julian date (MJD), which is the time measured in days from 00:00 UT on 17 November 1858 (Julian date 2400000.5). In this paper we use only the integer part of MJD, i.e. the value at 00:00 UT on the day of interest. For some applications it is also necessary to give the time within the day as Universal Time in hours (UT).

Note that Universal Time is different from coordinated Universal Time (UTC) which is the time scale usually used for data recording. UTC is atomic time adjusted by an integral number of seconds to keep it within 0.6 s of UT. For our purposes the difference between UT and UTC may be neglected. The apparent position of the Sun is calculated using yet another time scale: terrestrial dynamical time or TDT. The difference between UT and TDT ($\sim$57 s in 1991) may also be neglected for our purposes. Note that TDT superseded a time scale called ephemeris time at the beginning of 1984.

3.2. *Style*

To maintain uniform style in the specification of times and angles, it was necessary to modify some of the formulae used in the next section compared with their specifications in the reference documents.

All angles are specified in the simplest way which maintains an accuracy of 0.001° up to the year 2100. This has allowed the simplification of some formulae, e.g. by the deletion of small terms and a reduction in the number of significant digits.

3.3. *Latitude and longitude*

In some coordinate systems, especially GEO and HEEQ, position is often specified in terms of latitude ϕ, longitude λ and radial distance R. These are related to Cartesian components using:

$$x = R\cos\phi\cos\lambda$$
$$y = R\cos\phi\sin\lambda$$
$$z = R\sin\phi$$

and

$$R = (x^2+y^2+z^2)^{1/2}$$

$$\phi = \arctan\frac{z}{(x^2+y^2)^{1/2}}$$

$$\lambda = \begin{cases} \arccos\dfrac{x}{(x^2+y^2)^{1/2}} & \text{if } y > 0 \\ 360^\circ - \arccos\dfrac{x}{(x^2+y^2)^{1/2}} & \text{otherwise.} \end{cases}$$

3.4. *Units*

The transformations are independent of the units used for x, y, z and R provided that the same units are used throughout any set of transformations. All rotation angles are specified in degrees.

Table 3. Breakdown of geocentric coordinate transformations

To	From GEI	From GEO	From GSE
GEI	$\mathbf{1}$	$\mathbf{T}_1^{-1}$	$\mathbf{T}_2^{-1}$
GEO	$\mathbf{T}_1$	$\mathbf{1}$	$\mathbf{T}_1\mathbf{T}_2^{-1}$
GSE	$\mathbf{T}_2$	$\mathbf{T}_2\mathbf{T}_1^{-1}$	$\mathbf{1}$
GSM	$\mathbf{T}_3\mathbf{T}_2$	$\mathbf{T}_3\mathbf{T}_2\mathbf{T}_1^{-1}$	$\mathbf{T}_3$
SM	$\mathbf{T}_4\mathbf{T}_3\mathbf{T}_2$	$\mathbf{T}_4\mathbf{T}_3\mathbf{T}_2\mathbf{T}_1^{-1}$	$\mathbf{T}_4\mathbf{T}_3$
MAG	$\mathbf{T}_5\mathbf{T}_1$	$\mathbf{T}_5$	$\mathbf{T}_5\mathbf{T}_1\mathbf{T}_2^{-1}$
	GSM	SM	MAG
GEI	$\mathbf{T}_2^{-1}\mathbf{T}_3^{-1}$	$\mathbf{T}_2^{-1}\mathbf{T}_3^{-1}\mathbf{T}_4^{-1}$	$\mathbf{T}_1^{-1}\mathbf{T}_5^{-1}$
GEO	$\mathbf{T}_1\mathbf{T}_2^{-1}\mathbf{T}_3^{-1}$	$\mathbf{T}_1\mathbf{T}_2^{-1}\mathbf{T}_3^{-1}\mathbf{T}_4^{-1}$	$\mathbf{T}_5^{-1}$
GSE	$\mathbf{T}_3^{-1}$	$\mathbf{T}_3^{-1}\mathbf{T}_4^{-1}$	$\mathbf{T}_2\mathbf{T}_1^{-1}\mathbf{T}_5^{-1}$
GSM	$\mathbf{1}$	$\mathbf{T}_4^{-1}$	$\mathbf{T}_3\mathbf{T}_2\mathbf{T}_1^{-1}\mathbf{T}_5^{-1}$
SM	$\mathbf{T}_4$	$\mathbf{1}$	$\mathbf{T}_4\mathbf{T}_3\mathbf{T}_2\mathbf{T}_1^{-1}\mathbf{T}_5^{-1}$
MAG	$\mathbf{T}_5\mathbf{T}_1\mathbf{T}_2^{-1}\mathbf{T}_3^{-1}$	$\mathbf{T}_5\mathbf{T}_1\mathbf{T}_2^{-1}\mathbf{T}_3^{-1}\mathbf{T}_4^{-1}$	$\mathbf{1}$

4. GEOCENTRIC SYSTEMS

Transformations between six geocentric systems defined in Table 1 can be broken down into five fundamental transformations which are described in the following section. The remaining 25 transformations can then be calculated by matrix operations as shown in Table 3 above.

4.1. *GEI to GEO*

$$\mathbf{T}_1 = \langle \theta, Z \rangle. \tag{2}$$

This matrix corresponds to a rotation in the plane of the Earth's geographic equator from the First Point of Aries to the Greenwich meridian. The rotation angle θ is the Greenwich mean sidereal time. This can be calculated using the following formula (U.S. Naval Observatory, 1989):

$$\theta = 100.461 + 36000.770\ T_0 + 15.04107\ \mathrm{UT},$$

where

$$T_0 = \frac{\mathrm{MJD} - 51544.5}{36525.0}. \tag{3}$$

Note that T_0 is the time in Julian centuries (36525 days) from 12:00 UT on 1 January 2000 (known as epoch 2000.0) to the previous midnight.

4.2. *GEI to GSE*

$$\mathbf{T}_2 = \langle \lambda_\odot, Z \rangle * \langle \varepsilon, X \rangle. \tag{4}$$

These two matrices correspond to:

(1) rotation from the Earth's equator to the plane of the ecliptic;

(2) rotation in the plane of the ecliptic from the First Point of Aries to the Earth–Sun direction.

These two angles are calculated as follows (U.S. Naval Observatory, 1989). First ε, the obliquity of the ecliptic:

$$\varepsilon = 23.439 - 0.013\ T_0$$

and then $\lambda_\odot$, the Sun's ecliptic longitude:†

$$M = 357.528 + 35999.050\ T_0 + 0.04107\ \mathrm{UT}$$

$$\Lambda = 280.460 + 36000.772\ T_0 + 0.04107\ \mathrm{UT}$$

$$\lambda_\odot = \Lambda + (1.915 - 0.0048\ T_0) \sin M + 0.020 \sin 2M, \tag{5}$$

where T_0 is defined in equation (3) above. Note that, strictly speaking, TDT should be used here in place of UT, but the difference of about a minute gives a difference of $\sim 0.0007°$ in $\lambda_\odot$.

4.3. *GSE to GSM*

$$\mathbf{T}_3 = \langle -\psi, X \rangle, \tag{6}$$

where ψ is the angle between the GSE Z axis and projection of the magnetic dipole axis on the GSE YZ plane (i.e. the GSM Z axis) measured positive for rotations towards the GSE Y axis. It can be calculated thus:

$$\psi = \arctan (y_e / z_e),$$

where ψ lies between $+90$ and $-90°$ and the values of y_e and z_e are obtained from:

† M is the Sun's mean anomaly and Λ its mean longitude.

$$\mathbf{Q}_e = \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} \quad (7)$$

which is a unit vector describing the dipole axis direction in the GSE coordinate system. Unfortunately, this direction is usually defined in the GEO coordinate system as:

$$\mathbf{Q}_g = \begin{bmatrix} \cos\phi\cos\lambda \\ \cos\phi\sin\lambda \\ \sin\phi \end{bmatrix},$$

where ϕ and λ are the geocentric latitude and longitude of the dipole North geomagnetic pole. These may be derived from the first order (i.e. dipole) coefficients of the IGRF, g_1^0, g_1^1 and h_1^1, adjusted to the time of interest. Longitude is given by:

$$\lambda = \arctan\frac{h_1^1}{g_1^1}, \quad (8)$$

where, in practice, λ must lie in the fourth quadrant. The latitude is given by:

$$\phi = 90.0 - \arcsin\frac{g_1^1\cos\lambda + h_1^1\sin\lambda}{g_1^0}. \quad (9)$$

Using values given in the current IGRF for 1985 we can derive the following approximations:

$$\phi = 78.8 + 4.283\times10^{-2}\,\frac{\mathrm{MJD}-46066}{365.25}$$

$$\lambda = 289.1 - 1.413\times10^{-2}\,\frac{\mathrm{MJD}-46066}{365.25}.$$

To obtain $\mathbf{Q}_e$ we simply apply matrix arithmetic thus:

$$\mathbf{Q}_e = \mathbf{T}_2\mathbf{T}_1^{-1}\mathbf{Q}_g$$

using the matrices defined in equations (2) and (4) and so ψ and $\mathbf{T}_3$ can be determined.

4.4. *GSM to SM*

$$\mathbf{T}_4 = \langle -\mu, Y\rangle, \quad (10)$$

where μ is the dipole tilt angle, i.e. the angle between the GSM Z axis and the dipole axis. It is positive for the North dipole pole sunward of GSM Z. It is calculated using:

$$\mu = \arctan\frac{x_e}{\sqrt{y_e^2 + z_e^2}},$$

where x_e, y_e and z_e are defined in equation (7) and μ must lie between $+90$ and $-90°$.

4.5. *GEO to MAG*

$$\mathbf{T}_5 = \langle \phi - 90°, Y\rangle * \langle \lambda, Z\rangle. \quad (11)$$

The two rotations are: (i) rotation in the plane of the Earth's equator from the Greenwich meridian to the meridian containing the dipole pole; (ii) rotation in that meridian from the geographic pole to the dipole pole.

The angles ϕ and λ are given by equations (8) and (9).

5. HELIOCENTRIC SYSTEMS

5.1. *HAE to HEE*

$$\mathbf{S}_1 = \langle \lambda_\odot + 180°, Z\rangle. \quad (12)$$

This matrix corresponds to a rotation in the ecliptic plane from the First Point of Aries to the Sun–Earth line. The angle $\lambda_\odot$ is the Sun's ecliptic longitude determined by equation (5).

5.2. *HAE to HEEQ*

$$\mathbf{S}_2 = \langle \theta, Z\rangle * \langle \iota, X\rangle * \langle \Omega, Z\rangle. \quad (13)$$

These three matrices correspond to:

(1) rotation in the plane of the ecliptic from the First Point of Aries to the ascending node of the solar equator;

(2) rotation from the plane of the ecliptic to the solar equator;

(3) rotation in the plane of the solar equator from the ascending node to the central meridian.

The three rotation angles are taken from pp. 307 and 308 of the *Explanatory Supplement* (Nautical Almanac Office, 1961):

$$\Omega = 73.6667 + 0.013958\,\frac{\mathrm{MJD}+3242}{365.25}$$

$$\iota = 7.25$$

$$\theta = \arctan(\cos\iota\tan(\lambda_\odot - \Omega)),$$

where $\lambda_\odot$ is taken from equation (5). The quadrant of θ is opposite that of $\lambda_\odot - \Omega$.

6. CONVERSION BETWEEN GEOCENTRIC AND HELIOCENTRIC SYSTEMS

In some specialized cases it may be necessary to convert between these two systems, e.g. during Earth encounters by interplanetary missions such as *Giotto*, *Galileo*, etc. This transformation is best defined between the GSE and the HEE systems as these are very simply related as shown in equation (14) below.

If transformation between other systems is required, it should first be reduced to a GSE–HEE transformation using the equations presented in previous sections.

$$\mathbf{Q}_{\mathrm{HEE}} = \mathbf{R} + \langle 180^{\circ}, Z \rangle \mathbf{Q}_{\mathrm{GSE}}, \tag{14}$$

where $\mathbf{Q}_{\mathrm{HEE}}$ and $\mathbf{Q}_{\mathrm{GSE}}$ are the vectors in HEE and GSE coordinates, respectively, and $\mathbf{R}$ is a constant vector. The inverse transformation is defined simply by interchanging $\mathbf{Q}_{\mathrm{HEE}}$ and $\mathbf{Q}_{\mathrm{GSE}}$.

6.1. *Position vectors*

For transformations of position vectors there is a shift of origin and so:

$$\mathbf{R} = \begin{bmatrix} R_{\odot} \\ 0 \\ 0 \end{bmatrix},$$

where $R_{\odot}$ is the distance between the Earth and the Sun, which is given by the formula (Duffett-Smith, 1979):

$$R_{\odot} = \frac{r_0(1-e^2)}{1+e\cos v},$$

where r_0 is the mean distance of the Sun from the Earth, e is the eccentricity of the Sun's apparent orbit around the Earth, $\bar{\omega}$ is the longitude of perigee of that orbit and v is the true anomaly. r_0 and v are taken from Duffett-Smith (1979) while e and $\bar{\omega}$ are taken from p. 98 of the *Explanatory Supplement* (Nautical Almanac Office, 1961).

$$r_0 = 1.495985 \times 10^8\,\mathrm{km}$$

$$e = 0.016709 - 0.0000418\,T_0$$

$$\bar{\omega} = 282.94 + 1.72\,T_0$$

$$v = \lambda_{\odot} - \bar{\omega}.$$

T_0 and $\lambda_{\odot}$ are defined in equations (3) and (5).

6.2. *Other vectors*

To transform other types of vector data, e.g. magnetic fields, there is no shift of origin and so $\mathbf{R}$ is a zero vector.

7. BOUNDARY NORMAL COORDINATES (LMN)

Boundary normal coordinates, as their name implies, are defined relative to some natural boundary such as the magnetopause or the bow shock. They allow the data to be ordered in a way which is related to that boundary. The L and M axes, equivalent to x and y, lie in a plane tangential to the boundary and the N axis, equivalent to z, is normal to the boundary. There is no universal convention to resolve the L and M axes. The relationship between LMN and other systems such as GSE is dependent on position.

Boundary normal coordinates are best defined through the analysis of data as presented below. Alternatively, they may be derived through use of a model.

The transformation to boundary normal coordinates may be defined through a minimum variance analysis of magnetometer data (Sonnerup and Cahill, 1967; Sonnerup, 1976). First, select a set of magnetometer data spanning a boundary crossing of interest. Then using these data in the initial coordinate system (e.g. GSE), construct a "covariant" matrix $\mathbf{M}$ where:

$$m_{ij} = \overline{B_i B_j} - \bar{B}_i \bar{B}_j, \tag{15}$$

where i and j can each represent the x, y and z components. Thus we obtain the total of nine terms m_{ij} required in $\mathbf{M}$.

The overbars in equation (15) indicate the taking of a mean over all N records in the dataset, i.e.

$$\overline{B_i B_j} = \frac{\sum_{\alpha=1}^{N} B_i B_j}{N}.$$

We now determine the eigenvalues λ_i and eigenvectors $\mathbf{V}_i$ of the matrix $\mathbf{M}$, i.e. we find values satisfying the equation:

$$\mathbf{M}\mathbf{V}_i = \lambda_i \mathbf{V}_i.$$

There should be three solutions. We select the eigenvalue with the lowest absolute value (λ_N); its associated eigenvector defines the boundary normal direction N. However, it is necessary to check that N points outward, i.e. away from the object (planet, comet) which supports the boundary. If not, the signs of all components in N are reversed to achieve this.

The L direction must lie within the plane defined by the other two eigenvectors but otherwise the choice of direction is arbitrary. One choice is to select the projection of solar magnetospheric Z direction onto this plane (Russell and Elphic, 1978). Another choice is to select the eigenvalue with the largest absolute value (λ_L) and let its associated eigenvector define the L direction. If the sign of N has been reversed it will be necessary to reverse the sign of L. In both cases the M direction is defined so as to complete a right-handed Cartesian system, e.g. as a vector product $M = N \wedge L$.

This method only works well if the minimum eigenvalue can clearly be distinguished, e.g. the ratio $\lambda_M/\lambda_N > 1.5$ (Sonnerup and Cahill, 1967). If this condition is not satisfied, the data are unsuitable and the analysis should not be carried further.

If the analysis is successful, the transformation matrix can be defined as

$$\mathbf{T} = \begin{bmatrix} L_x & L_y & L_z \\ M_x & M_y & M_z \\ N_x & N_y & N_z \end{bmatrix},$$

where L_x is the x component, in the initial coordinate system, of the vector describing the L direction in the boundary normal coordinate system. $L_y, \ldots, N_z$ are defined in a similar way.

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