Math 430, Fall 2008

Matrix Analysis

Instructor: John Zweck

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Webpage: I will maintain a web page for the course, linked from my web page www.math.umbc.edu/∼zweck. I will also communicate with you using a class email list.

Phone: (410) 455 2424

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Lectures: MW, 2:30-3:45pm (BIOL LH1)

Text: "Matrix Analysis and Applied Linear Algebra", by Carl Meyer

Other Books: You may also find the following excellent books useful, though I will not explicitly rely on them.

1. "Applied Linear Algebra", by Peter J. Olver and Chehrzad Shakiban Prentice-Hall, Upper Saddle River, N.J., 2005
2. "Linear Algebra and Its Applications", by Gilbert Strang
3. Appendix on Matrix Theory in "An Introduction to Multivariate Statistical Analysis", 3rd edition, by T.W. Anderson
4. "Linear Algebra and its Applications", by David C. Lay (This is the Math 221 book)
5. "Matrix Computations", by G. Golub and C.F. Van Loan

Prerequisites: Math 221, Math 251 and Math 301 or permission of instructor. **Note that you will be expected to do some proofs and we will discuss proofs a lot in class.**

Your Review: I will assume that by Wed Sept 3rd you have reviewed the following sections from Meyer's book: 1.2, 1.3, 2.1-2.5, 3.2, 3.5. The material is basically the same as that covered in Sections 1.1-1.8, 2.1-2.3 of Lay's book (the Math 221 text). We will use this review material extensively in our course, but I will not discuss it in class. I will assign homework on this material.

Material Covered: In class we will cover the following sections of Meyer's book: Chapters 3.2-3.7, 4.1-4.8, 5.3-5.6, 5.8-5.11, 6.1, 6.2, 7.1-7.6.

Office Hours: M 10:00-11:00, W 11:00-12:00 *and by appointment*. I can often answer short questions immediately after class. If you cannot come to my office hours *please* contact me in class or by email to set up a time to meet. Also, you are encouraged to ask me questions by email. I rarely check my phone messages.

Course Summary and Learning Goals

In this course we study some of the basic techniques from linear algebra that undergird the analysis and simulation of a large and diverse collection of applications in science and engineering. The equations used to model these applications typically involve many variables. Even when the underlying equations are nonlinear, solution techniques—both analytical and numerical—often involve exploiting the geometric and algebraic structure of linear transformations on finite dimensional vector spaces. Since every such linear transformation is given by matrix multiplication, this involves analyzing the structure of matrices.

The main themes of the course are: (1) Matrix algebra (akin to vector algebra in Multivariable calculus); (2) Linear transformations, focusing on the rank-and-nullity and change-of-basis theorems; (3) Linear transformations that preserve the inner product between vectors, with an emphasis on the discrete Fourier transform; (4) A careful development of the theory and meanings of determinants; (5) Eigensystems with a focus on the spectral theorem for normal matrices, functions of matrices and with applications to systems of ordinary differential equations; (6) Some general decomposition theorems for matrices; (7) Calculations involving block matrices.

The course is designed to serve the needs of undergraduate Mathematics, Computer Science, and Physics majors, as well as incoming graduate students in Statistics and Mathematics. The course material is widely used in pure and applied mathematics and statistics, and in applications fields. Applications include solutions of systems of linear ordinary differential equations, methods to solve large linear systems—and therefore linear and nonlinear partial differential equations—on a computer (see Math 630), multivariate probability and statistics, signal and image processing, quantum mechanics, Hilbert space theory (linear transformations on infinite dimensional inner product spaces), and differential geometry (see Math 423).

Homework and exams will emphasize calculations for specific examples based on the theory discussed in class, as well as some more abstract proofs based on similar sorts of calculations to the ones performed in class. Exams will also test your understanding of definitions and theorems covered in class.

Students wishing to master the course material will be guided by the following **learning goals**.

1. Students will master the definitions, examples, calculations, theorems and proofs discussed in class and covered on homework.

2. Students will organize their understanding of the course material using the seven themes discussed above, and identify interconnections between them.

3. Students will become proficient at setting up and performing matrix algebra calculations to study the structure of matrices, and apply these technqiues throughout the course.

4. Students will understand and be able to use the algebraic and geometric structures of special classes of matrices and linear transformations, including matrices that are square, diagonal, symmetric, block, orthogonal, unitary, Fourier, diagonalizable, normal, positive definite, or stochastic.

5. Students will do a class project in which they will gain experience in (1) reading and assimilating the material in a research-level journal article; (2) gaining a higher-level understanding of a sophisticated real-world application of the course material, rather than simply tackling isolated problems; (3) formal mathematical and technical writing; (4) working indepenedently of the course instructor in collaboration with another student.

6. Students will use the experience gained in this course to prepare them for higher level mathematics courses, for research, and for professional work involving mathematics. Specifically, they will learn how how mathematics involves gaining mathematical and scientific knowledge by integrating concepts from a wide variety of subjects. For example, they will see how matrix analysis integrates ideas from linear algebra, calculus, the geometry of vectors and vector subspaces, differential equations, analysis and physics. In particular, students will learn to identify the abstract mathematical principles that are encoded into matrix analysis and which unite apparently disparate examples and applications.

7. Finally, students will be encouraged to develop an appreciation for the many applications of matrix analysis.

Academic Misconduct

I will not tolerate cheating in any form. All instances of cheating I discover will be reported to UMBC's academic integrity committee. (See http://www.umbc.edu/integrity/) In particular, in this course, giving or receiving aid on exams will result in a grade of zero for that exam. Copying of homework solutions from other students in the class, from students who have previously taken this or an equivalent course, from a solutions manual, or from the web will treated as a serious offense. At a minimum this will result in a grade of zero for that homework (which will not be counted as one of the two lowest homeworks I drop when calculating your overall homework grade). For flagrant cheating on homework I reserve the right to give a grade of zero for the homework on which the students was found to have cheated as well as on all homeworks that were turned in prior to the discovery of the offense. Also see comments below in the subsection on Homework.

Here is a summary of UMBC's official policy on academic misconduct, which I fully endorse:

> By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal. To read the full Student Academic Conduct Policy, consult the *UMBC Student Handbook*, the *Faculty Handbook*, or the UMBC Policies section of the *UMBC Directory*.

Grading

Grades: Homework 15%, Project 10%, Two Midterm Exams 20% each, Final 35%

Homework: The homework problems will be posted on the course web page for each day of class. Problems assigned on a Monday and Wednesday will be due at the *start* of class the following **Wednesday**. At least some of them will be graded. Make sure your homework paper is *stapled*. *No late homework will be accepted!* Your lowest two homework grades will be dropped. You may ask me questions about the homework and you may collaborate with another student in the class. In fact you are encouraged to do so! However the final write up is your own – *two (almost) identical solutions may both be given zero.* I do not encourage large groups of people to work together on homework. Do not miss class to complete a homework. *I will not accept homework that is handed in after the first few minutes of class.*

Project: The project is due Monday November 24th at the start of class. **You should start it now!** See below for more details.

Midterm Exams: There will be two midterm exams.

- Midterm 1: Monday September 29th.
- Midterm 2: Monday November 3rd.

Final Exam: **Friday December 12th, 1:00-3:00pm in BIOL LH1. The final will be based on the whole course.** Some questions may be more difficult than those on the midterms.

Make up exams: If you miss one of the midterms you *may* be given the chance to take a make up exam. To request a make up you should speak with me **no later than 48 hours after** the exam time. Generally speaking, you will be offered a make up if you are sick or if a close relative or friend is gravely injured/sick or dies. However I will listen to all reasonable requests. Be prepared to bring appropriate evidence in support of your request. There will be no make ups for the final exam.

Project

The project is based on the article *The $25,000,000,000 Eigenvector: The Linear Algebra Behind Google* by Kurt Bryan and Tanya Leise, SIAM review, Vol. 48, Num. 3, Sept. 2006, pp. 567-581. The paper is available on the class web page, together with some clarifying comments on some of the questions. The project is due Monday November 24th at the start of class. You are to work in a group of two. Both students are expected to contribute equally to the work. Submit ONE project write up. You must send me email stating who you are working with by Monday October 6th. However you are encouraged to start working immediately. The project only relies on an understanding of Math 221 material.

Guidelines for writing up the project:

Your grade for the project will be based on the correctness of your solutions to the 17

problems posed in the paper as well as on the following requirements for the project write up.

- Present your solutions to the 17 exercises given in the paper.
- You should also write a half page introduction which summarizes the main points of the paper.
- In addition, read one of the references given on page 581 of the paper and give a one page summary of the main point of that reference and how it relates to the paper by Bryan and Leise.
- Finally, write a short conclusion that shows you have understood the point of the exercises you have done.
- Make sure any figures you include are labeled, have a caption, and are referred to in the main text of the write up.
- You may write as much as you like by hand rather than using word/math processing software. If you put mathematical derivations separately from the main text ut them in an Appendix and refer to them from the main text.
- Your audience is a typical student just starting Math 430. In addition, your write up should convince me that you have a deep understanding of the material discussed in the paper.

Start the project early! Let me know if you have questions or you get stuck!

How I assign final grades

For each exam I work out how many points I expect a student who has a solid understanding of the material to get. I tend to put the bottom B near this score. Then I work out where to place the bottom A,C,D using the grade distribution and by looking at individual exams. I also work out the bottom A,B,C,D for the homework, and project(s). Then I take an imaginary student who got the bottom B (say) for each component of the course and calculate their score. If your score is higher than the imaginary student's you get a B. If it is a little less than the imaginary student's score I look carefully at your work to decide whether you deserve a B or a C. Most importantly I look at your final exam. In particular, students on the borderline between two grades and who show mastery of the material on the final are more likely to receive the higher grade. *However, students who do very poorly on the final might find that their course grade is lower than they had expected!* In short I reward "strong finishers" who can show me they have a solid understanding of the entire course.

You can calculate your current overall grade from the cut-offs I will provide after each exam and project.

Study Tips

1. See the *Study Habits* survey of past students in my classes and the *Guidelines for Math/Stat Course Study Groups* information on my web page.
2. On the course web page I will post the sections that we will cover each day. **You are expected to read the section ahead of time**, so as to be familiar with the material.
3. It is very important to keep the main definitions, statements of theorems, and simpler examples on the forefront of your minds throughout the course, since we will refer back to them many times. You will need to digest the material several times to master it — before class, in class, reading through material after class, rederiving for yourself without any aid results discussed in class, and doing the assigned problems.
4. This is a fast paced course. Do not get behind. Do not miss class. Ask for help well before you are totally lost.
5. **Ask questions**. If you are dazed and confused your class mates will be too!
6. In class I call on people by name to answer questions mostly to help me find out whether you are understanding what's going on. **If you do not feel comfortable being called on in class, please come and talk with me.**
7. Come and talk with me in my office. Talk math with your fellow students, don't work in isolation.
8. Learn the art of taking good notes. My lectures will often present a somewhat complementary perspective on the subject to that in the textbook.
9. Do *all* the hwk problems. Work out what your mistakes are on the graded hwk and learn from them.

Advice for Homework

1. Never start your homework the day before it is due!!
2. Begin each hwk assignment *the same day* that we cover the material in class.
3. Write up your homework so that you'll easily understand it in a month's time when you're studying for the exam!
4. The only way to learn math is to do it: Struggle to solve problems for yourself.
5. However, if you get stuck on a problem for too long get help and get it before you waste too much time!! Here are some places you can go for help.
    - Carefully read your notes from lectures and the book (again!).
    - Draw a schematic picture to help you think about the problem.
    - Ask me for help by email or in person.
    - Ask a fellow class member – often two heads are better than one! **I encourage you to find a study partner for this class.** First attempt the hwk yourself, then discuss them with your study partner, and finally carefully write the solutions up in your own words.
    - Sleep on it. Some of my best ideas come when I wake up in the morning.
6. Some of the homework problems will be harder than others. Don't expect to solve them all on the first try!

Advice for Exams

Study in small groups for exams and learn from each other. Presenting material to someone else is often the best way to work out whether you really know it yourself.