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Simulation and Optimal Decision Making the Design of Technical Systems

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Abstract The paper presents a methodology for modeling and optimal decision-making in the design of the technical systems. The model is formed as a vector problem of mathematical programming. The model is intended to define the parameters of the technical system, in which the technical characteristics (criteria) are optimal. Mathematical model of the technical system is carried out in conditions of certainty (functional dependence of each characteristic and restrictions on parameters is known) and under conditions of uncertainty (there is not sufficient information on the characteristics of each of the functional dependence of the parameters). Conditions of uncertainty will be transformed to definiteness conditions, using methods of the regression analysis. The received to problems vector is solved on the basis of normalization of criteria and the principle of the guaranteed result. As a result of the decision received the optimum decision (the guaranteed result). The modeling methodology in the conditions of definiteness and uncertainty is illustrated on a numerical example of model of technical system, in the form of a vector problem of nonlinear programming with four criteria.

Keywords: *modeling technical systems, vector optimization, optimum decision-making*

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1. Introduction

The creation of new technical systems stimulated the development of mathematical models in their design. Such models adequately describe the functioning of technical systems. Therefore to a problem of mathematical modeling of technical systems as much attention is paid to a component of system of the automated design as in Russia [1-13], and abroad in theoretical [15,17,18] and applied aspects [16,19,20,21].

Functioning of technical object, system is defined by some set of the characteristics which are functionally dependent on parameters of system. Improvement of one of these characteristics leads another to deterioration. There is a problem of determination of such parameters which would improve all functional characteristics of technical system at the same time. These problems are solved now, both at technological (experimental) level, and at the mathematical (model) level. The model in this case can be created in the form of a vector problem of mathematical programming in which the vector criterion defines characteristics of technical system [5,7,9-13].

For the solution of a vector task we use the methods based on normalization of criteria and the principle of the guaranteed result which are for the first time presented [4]. Further we used these methods when modeling technical systems [5,7,9]. We use methods at the solution of vector tasks with equivalent criteria [9] and to the set priority of criterion [10]. If functional dependence of each characteristic and restrictions on parameters is known, we formulate mathematical model of technical system in the conditions of definiteness [5,8]. If functional dependence of each characteristic and restrictions on parameters isn't known, we formulate mathematical model of technical system in the conditions of uncertainty [7]. This work is in total directed on the solution of these problems.

The purpose of this work consists in creation of methodology of creation of mathematical model of technical system in the form of a vector problem of mathematical programming. Solutions of a vector task in the conditions of definiteness and uncertainty in total. We modeled processes of functioning of technical system. The method of optimum decision-making under the set conditions is presented.

For realization of a goal in work it is presented: creation of model of technical system in the form of a vector problem of mathematical programming; the methodology of creation of mathematical model of technical system conditions of definiteness and uncertainty in total is shown; decision-making realization (i.e. a choice of optimum parameters of engineering system), on the basis of the developed software. The methodology of modeling is illustrated on a numerical example of model of the technical system, in the form of a vector problem of nonlinear programming realized in Matlab [14] system. The methodology has system character and can be used as for technical, and economic tasks, [11,12].

2. Statement of a Problem. Methodology of Modeling of Technical Systems in the Conditions of Definiteness and Uncertainty

The problem of a choice of optimum parameters of technical systems according to functional characteristics arises during the studying, the analysis and design of technical systems and is connected with quality production. The problem includes the solution of the following tasks:

Creation of mathematical model which defines interrelation of each functional characteristic from parameters of technical system i.e. is formed of the vector problem of mathematical programming;

Methods of the solution of a vector task get out. In work it is offered to use the methods based on normalization of criteria and the principle of the guaranteed result. The software which realizes these methods is developed.

2.1. Creation of Mathematical Model of Technical System

The technical system which functioning depends on $\boldsymbol{N}$ - a set of design data is considered[1]: $X=\{x_1\ x_2\ \dots\ x_N\}$, N - number of parameters, each of which lies in the set limits

$$x_j^{\min} \le x_j \le x_j^{\max},\ j=\overline{1,N} \quad or\ X^{\min} \le X \le X^{\max} \tag{1}$$

где $x_j^{\min}$, $x_j^{\max}$, $\forall j\in \boldsymbol{N}$ - lower and top limits of change of a vector of parameters of technical system.

The result of functioning of technical system is defined by a set $\boldsymbol{K}$ to technical characteristics of $f_k(X)$, $k=\overline{1,K}$ which functionally depend on design data $X=\{x_j,\ j=\overline{1,N}\}$, in total they represent a vector function:

$$F(X)=\left(f_1(X)\,f_2(X)\dots f_K(X)\right)^T. \tag{2}$$

The set of characteristics (criteria) to is subdivided into two subsets $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$: $\boldsymbol{K}=\boldsymbol{K}_1\cup\boldsymbol{K}_2$

$\boldsymbol{K}_1$ is a subset of technical characteristics which numerical sizes it is desirable to receive as it is possible above: $f_k(X)\ \rightarrow \mathbf{max}$, $k=\overline{1,K_1}$.

$\boldsymbol{K}_2$ - it subsets of technical characteristics which numerical sizes it is desirable to receive as it is possible below: $f_k(X)\rightarrow \mathbf{min}$, $k=\overline{K_1+1,K}$, $\boldsymbol{K}_2\equiv \overline{K_1+1,K}$.

Mathematical model of technical system which solves in general a problem of a choice of the optimum design decision (a choice of optimum parameters), we will present in the form of a vector problem of mathematical programming.

$$Opt\ F(X)=\{\mathbf{max}F_1(X)=\{\mathbf{max}f_k(X),k=\overline{1,K_1}\} \tag{3}$$

$$\mathbf{min}F_2(X)=\{\mathbf{min}f_k(X),k=\overline{1,K_2}\}\}, \tag{4}$$

$$G(X)\le 0, \tag{5}$$

$$x_j^{\min} \le x_j \le x_j^{\max},\ j=\overline{1,N}, \tag{6}$$

where X - a vector of operated variable (design data) from (1);

$F(X)=\{f_k(X),\ k=\overline{1,K}\}$ - criterion which everyone a component submits the characteristic of technical system (2) which is functionally depending on a vector of variables X;

in (5) $G(X)=(g_1(X)\ \ g_2(X)\ \dots\ g_M(X))^{\mathrm{T}}$ – vector function of the restrictions imposed on functioning of technical system, $\boldsymbol{M}$ – a set of restrictions.

Restrictions are defined proceeding in them technological, physical and to that similar processes and can be presented by functional restrictions, for example, $f_k^{\min} \le f_k(X)\le f_k^{\max}$, $k=\overline{1,K}$.

It is supposed that the $f_k(X)$, $k=\overline{1,K}$ functions are differentiated and convex, $g_i(X)$, $i=\overline{1,M}$ are continuous, and (5)-(6) set of admissible points of $\boldsymbol{S}$ set by restrictions isn't empty and represents a compact:

$$S=\{X\in R^N\ |\ G(X)\le 0,\ X^{min}\le X\le X^{max}\}\ne\varnothing.$$

Criteria and restrictions (3)-(6) form mathematical model of technical system. It is required to find such vector of the $X^o\in\boldsymbol{S}$ parameters at which everyone a component the vector - functions $F_1(X)=\{f_k(X),\ k=\overline{1,K_1}\}$ accepts the greatest possible value, and a vector - functions $F_2(X)=\{f_k(X),\ k=\overline{1,K_2}\}$ are accepted by the minimum value.

To a substantial class of technical systems which can be presented by a vector task (3)-(6), it is possible to refer their rather large number of tasks from various branches of economy of the state: electrotechnical, aerospace, metallurgical (choice of optimal structure of material), etc[2]. In this article for technical system are considered in a statics. But technical systems can be considered in dynamics, using differential-difference methods of transformation [5], conducting research for a small discrete period $\Delta t\in T$.

2.2. Conditions of Creation of Mathematical Model of Technical System

At creation of mathematical model of technical system (3)-(6) conditions are possible: definiteness and uncertainty.

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1 Another way to write the vector $X=\{x_j,\ j=\overline{1,N}\}$

2 We mention the work of V.L. Levitskii “Simulation and Optimization of Parameters of Magnetoelectric Linear Inductor Electric Direct Current Motor” [[5], p. 50-120]. It deals with designing an augmented electric motor (AEM) with its model reduced to vector mathematical programming problem (3)–(6). The vector of design parameters $X = (X_1, \dots, X_5)$ consisted of X_1 for the air clearance δ, X_2 for the tooth pitch, X_3 for the number of teeth, X_4 for the height of the concentrator, and X_5 for the pole overlap coefficient. The vector of design criteria $F(X) = (f(X), p(X), \eta(X), \dots)$ included $f(X)$ for the nominal towing force, $p(X)$ for the nominal power, $\eta(X)$ for the nominal efficiency and so on, ten indices in total. The central orthogonal plan of the second order was used to construct the dependencies of f on the listed design parameters X [5, p. 96]. The work “…Multiobjective Optimization of Static Modes of Mass-Exchange Processes by the Example of Absorption in Gas Separation” [13] is an example from another industry. Thus, experimental data both from the AEM problem and from similar ES of other industries can be represented as theoretical (system) problem (3)-(6).

2.2.1. Creation of Mathematical Model of Technical System in the Conditions of Definiteness

Conditions of definiteness are characterized by that functional dependence of each characteristic and restrictions on parameters of technical system [5,8] is known.

For creation of functional dependence we perform the following works.

1. We form a set of all functional characteristics of technical systems $\boldsymbol{K}$. The size of the characteristic we will designate f_k, $\forall k \in \boldsymbol{K}$. We determine a set of all parameters $\boldsymbol{N}$ on which these characteristics depend. Sizes of parameters we will present in the form of a vector of $X = \{x_j, \ j=\overline{1,N}\}$. We give the verbal description of characteristics of technical systems.

2. We conduct research of the physical processes proceeding in technical system. For this purpose we use fundamental laws of physics: modeling of magnetic, temperature fields; conservation laws of energy, movement etc. We establish information and functional relation of characteristics of technical systems and her parameters: $f_k(X)$, $k=\overline{1,K}$. The set of characteristics of $\boldsymbol{K}$ is subdivided into two subsets of $\boldsymbol{K}_1 \subset \boldsymbol{K}$, $\boldsymbol{K}_2 \subset \boldsymbol{K}$, $\boldsymbol{K}=\boldsymbol{K}_1 \cup \boldsymbol{K}_2$. $\boldsymbol{K}_1$ is a subset of technical characteristics by which it is desirable to receive numerical sizes as it is possible above: $F_1(X)=\{f_k(X) \rightarrow \mathbf{max}, \ k=\overline{1,K_1}\}$. $\boldsymbol{K}_2$ – is a subset of technical characteristics by which it is desirable to receive numerical sizes as it is possible below: $F_2(X) = \{f_k(X) \rightarrow \mathbf{min}, \ k=\overline{K_1+1,K}\}$, $\boldsymbol{K}_2 \equiv \overline{K_1+1,K}$.

3. We define functional restrictions: $f_k^{\min} \le f_k(X) \le f_k^{\max}$, $k=\overline{1,K}$ and parametrical restrictions:

$$x_j^{\min} \le x_j \le x_j^{\max}, \ j=\overline{1,N}, or \ X^{\min} \le X \le X^{\max}.$$

4. As a result we will construct mathematical model of technical system in the form of a vector problem of mathematical programming:

$$Opt \ F(X) = \left\{ \begin{array}{l} \mathbf{max} F_1(X) = \{\mathbf{max} f_k(X), k=\overline{1,K_1}\}, \\ \mathbf{min} F_2(X) = \{\mathbf{min} f_k(X), k=\overline{1,K_2}\} \end{array} \right\}$$

$$\text{at restrictions } f_k^{\min} \le f_k(X) \le f_k^{\max}, k=\overline{1,K}, \quad (7)$$

$$x_j^{\min} \le x_j \le x_j^{\max}, \ j=\overline{1,N}.$$

The task (7) is adequate tasks (3)-(6).

2.2.2. Creation of Mathematical Model of Technical System in the Conditions of Uncertainty

Conditions of uncertainty are characterized by that there is no sufficient information on functional dependence of each characteristic and restrictions from parameters [7].

Conceptual Decision Making Problem Statement. Initially, in a general form it is given in [15]. We introduce the respective designations - a_i, $i=\overline{1,M}$, for the admissible decision making alternatives and $A = (a_1 \ a_2 \ \ldots \ a_M)$ for the vector of the set of admissible alternatives.

We match each alternative $a \in A$ to K numerical indices (criteria) $f_1(a), \ldots, f_K(a)$ that characterize the system. We can assume that this set of indices maps each alternative into the point of the K-dimensional space of outcomes (consequences) of decisions made - $F(a) = (f_1(a) f_2(a) \ldots f_K(a))^T$. We use the same symbol $f_k(a)$ both for the criterion and for the function that performs estimating with respect to this criterion. Note that we cannot directly compare the variables $f_v(a)$ and $f_k(a)$, $v \neq k$ at any point $F(a)$ of the K-dimensional space of consequences since it would mostly have no sense since these criteria are generally measured in different units. Using these data, we can state the decision making problem.

The decision maker is to choose the alternative $a \in A$ so that to obtain the most suitable result, i.e., $F(a) \rightarrow \min$.

This definition means that the required estimating function should reduce the vector $F(a)$ to a scalar preference or "value" criterion. In other statement, it is equivalent to setting a scalar function V given in the space of consequences and possessing the following property

$$V\big(F(a)\big) \ge V\big(F(a')\big) \Leftrightarrow F(a) \gg F(a')$$

where the symbol >> means "no less preferable than" [2,15]. We call the function $V(F(a))$ the value function. The name of this function in publications may vary from an order value function to a preference function to a value function. Thus, the decision maker is to choose $a \in A$ such that $V(F(a))$ is maximum. The value function serves for indirect comparison of how important certain values of various criteria of the system are. That said, the matrix $F(a)$ of admissible outcomes of alternatives takes the form

$$F = \begin{bmatrix} a_1 & f_1^1 \ldots f_1^K \\ \ldots & \\ a_M & f_M^1 \ldots f_M^K \end{bmatrix}. \quad (8)$$

where $f_i^{\,j} = f_i(a_i)$ and all alternatives in it are represented by the vector of indices $F(a)$. For the sake of definiteness and without loss of generality, we assume that the first criterion (any criterion can be the first) is arranged in the increasing (decreasing) order, with the alternatives re-numbered $i=\overline{1,M}$.

The problem implies that the decision maker is to choose the alternative $a^o \in A$ such that it will yield the "most suitable (optimal) result" [15].

For the engineering system, we can represent each alternative a_i by the N-dimensional vector $X_i = \{x_{ij}, j=\overline{1,N}\}$, $i=\overline{1,M}$ of its parameters and its outcomes by the K-dimensional vector criterion $\{f_1(X_i), \ldots, f_K(X_i), \ i=\overline{1,M}\}$. Taking this into account, matrix of outcomes (8) takes the form

$$I = \begin{bmatrix} X_1 & f_1(X_1) \ldots f_K(X_1) \\ \ldots & \\ X_M & f_1(X_M) \ldots f_K(X_M) \end{bmatrix}, \quad (9)$$

where multiple criteria (characteristics) $\boldsymbol{K}$ is subdivided into two subsets $\boldsymbol{K}_1 \subset \boldsymbol{K}$, $\boldsymbol{K}_2 \subset \boldsymbol{K}$, $\boldsymbol{K}=\boldsymbol{K}_1 \cup \boldsymbol{K}_2$. $\boldsymbol{K}_1$ is a subset of the technical characteristics which numerical sizes it is desirable to receive as it is possible above:

$$I_1(X) \equiv \{\{f_k(X_i, i=\overline{1,M}\}^T \mathbf{max}, k=\overline{1,K_1}\}.$$

$\boldsymbol{K}_2$ are subsets of technical characteristics which numerical sizes it is desirable to receive as it is possible below:

$$I_2(X) \equiv \{\{f_k(X_i, i=\overline{1,M}\}^T \rightarrow \mathbf{min}, \mathrm{k}=\overline{1,K_2}\},$$

$$K_2 \equiv \overline{K_1+1,K}.$$

The task (9) decision-makers consists in a choice of such set of design data of X^o system which would allow to receive optimum result [15].

Discussion. At present, problems (8) and (9) are solved by a number of "simple" methods based on forming special criteria such as Wald, Savage, Hurwitz, and Bayes-Laplace criteria, which are the basis for decision making.

The Wald criterion of maximizing the minimal component helps make the optimal decision that ensures the maximal gain among minimal ones - $\max_{k=\overline{1,K}} \min_{i=\overline{1,M}} f_i^k$.

The Savage minimal risk criterion chooses the optimal strategy so that the value of the risk r_i^k is minimal among maximal values of risks over the columns - $\min_{i=\overline{1,M}} \max_{k=\overline{1,K}} r_i^k$. The value of the risk r_i^k is chosen from the minimal difference between the decision that yields maximal profit $\max_{i=\overline{1,M}} f_i^k$, $k=\overline{1,K}$, and the current value f_i^k, $r_i^k = (\max_{i=\overline{1,M}} f_i^k) - f_i^k$, with their set being the matrix of risks $R = \left\| r_i^k \right\|_{i=\overline{1,M}}^{k=\overline{1,K}}$.

The Hurwitz criterion helps choose the strategy that lies somewhere between absolutely pessimistic and optimistic (i.e., the most considerable risk)

$$\max_{k=\overline{1,K}} \left(\alpha \min_{i=\overline{1,M}} f_i^k + (1-\alpha) \max_{i=\overline{1,M}} f_i^k \right),$$

where α is the pessimistic coefficient chosen in the interval $0 \le \alpha \le 1$.

The Bayes-Laplace criterion takes into account each possible consequence of all decision options, given their probabilities $\max_{i=\overline{1,M}} \sum_{k=1}^{K} f_i^k p_i$.

All these and other methods are sufficiently widely described in publications on decision making [2,15-21]. All of them have certain drawbacks. For instance, if we analyze the Wald maximin criterion, we can see that by the problem's hypothesis all criteria are in different units. Hence, the first step, which is to choose the minimal component $f_k^{\min} = \min_{i=\overline{1,M}} f_i^k$, is quite reasonable, and all $f_k^{\min}$, $k=\overline{1,K}$, are measured in different units, therefore the second step, which is to maximize the minimal component $\max_{k=\overline{1,K}} f_k^{\min}$, is pointless. Although it brings us slightly closer to the solution, the criteria measurement scale fails to solve the problem since the chosen criteria scales are judgmental.

We believe that to solve problem (8), (9), we need to form a measure that would allow evaluating any decision to be made, including the optimal one. In other words, we need to construct axiomatics that shows, based on the set of K criteria, what makes one alternative better than the other. In its turn, axiomatics can help derive a principle that helps find whether the chosen alternative is optimal. The optimality principle should become the basis for the constructive methods of choosing optimal decisions. We propose such approach for the vector mathematical programming problem that is essentially close to decision making problem (8), (9).

The vector task in the conditions of uncertainty (9) will assume in the form

$$Opt\ F(X) = \{\mathbf{max} I_1(X) \equiv \{\mathbf{max}\{f_k(X_i, i=\overline{1,M})\}^T, k=\overline{1,K_1}\}, \tag{10}$$

$$\mathbf{min} I_2(X) \equiv \{\mathbf{min}\{f_k(X_i, i=\overline{1,M})\}^T, k=\overline{1,K_2}\}\}, \tag{11}$$

at restrictions

$$f_k^{\min} \le f_k(X) \le f_k^{\max}, k=\overline{1,K}, \quad x_j^{\min} \le x_j \le x_j^{\max}, j=\overline{1,N}. \tag{12}$$

where X - a vector of operated variable (design data) equivalent (1);

$F(X)=\{I_1(X), I_2(X)\}$ - vector criterion which everyone a component submits the characteristic of technical system (2) which is functionally depending on the size of discrete value of a vector of variables X; ***M*** – set of discrete values of a vector of variables X; in (12) $f_k^{\min} \le f_k(X) \le f_k^{\max}$, $k=\overline{1,K}$ – a vector function of the restrictions imposed on functioning of technical system, $x_j^{\min} \le x_j \le x_j^{\max}$, $j=\overline{1,N}$ – parametrical restrictions.

Discussion. Using designations (10), (11), it is possible to give some assessment to dimension of uncertainty. If N - a set a component of a vector of variables X is equal to M – a set of discrete values of a vector of variables X, uncertainty linear (for example, $N = M = 2$, or $N = M = 3$ etc.). (As in two measured $R^{N=2}$ space it is possible to draw a line, in three measured R^3 - the plane, etc.). If $N < M$, uncertainty is nonlinear. If $N > M$, uncertainty is full. (For example, in three-dimensional space of R^3 through two points it is possible to carry out an infinite set of the planes). Generally, than there are more than measurements of M, that definiteness is more. Full definiteness comes when functional dependence of $f(X)$ is known. In this case the set of points of X is infinite.

Accuracy of measurements represents the second party of uncertainty. In this work accuracy isn't investigated.

2.3. Creation of Mathematical Model of Technical System in the Conditions of Definiteness and Uncertainty in Total

In real life of a condition of definiteness and uncertainty are combined. The model of technical system also has to reflect these conditions. We will unite models (7) and (10) - (13). As a result we will receive model of technical system in the conditions of definiteness and uncertainty in total:

$$Opt\ F(X) = \begin{cases} \mathbf{max} F_1(X) = \{\mathbf{max} f_k(X), k=\overline{1,K_1^{def}}\}, \\ \mathbf{max} I_1(X) \equiv \{\mathbf{max}\{f_k(X_i, i=\overline{1,M})\}^T, \\ k=\overline{1,K_1^{unc}} \end{cases}, \tag{13}$$

$$\begin{aligned}&\mathbf{min}F_2(X)=\{\mathbf{min}f_k(X), k=\overline{1,K_2^{def}}\},\\&\mathbf{min}I_2(X)\equiv\left\{\mathbf{min}\{f_k(X_i,i=\overline{1,M}),k=\overline{1,K_2^{unc}}\}\right\},\end{aligned} \quad (14)$$

at restrictions

$$f_k^{\min} \le f_k(X) \le f_k^{\max}, k=\overline{1,K}, \quad (15)$$

$$x_j^{\min} \le x_j \le x_j^{\max}, j=\overline{1,N}, \quad (16)$$

where X - a vector of operated variable (design data) equivalent (1); $F(X)$={ $F_1(X)$ $F_2(X)$ $I_1(X)$, $I_2(X)$} - vector criterion which everyone a component represents a vector of criteria (characteristics) of technical system (2) which functionally depend on discrete values of a vector of variables X where K_1^{def}, K_2^{def} (*definiteness*), K_1^{unc}, K_2^{unc} (*uncertainty*) the set of criteria of *max* and *min* created in the conditions of definiteness and definiteness; in (12) $f_k^{\min} \le f_k(X) \le f_k^{\max}$, $k=\overline{1,K}$ – a vector function of the restrictions imposed on functioning of technical system $x_j^{\min} \le x_j \le x_j^{\max}$, $j=\overline{1,N}$ – parametrical restrictions.

2.4. Transformation of a Problem of Decision-Making in the Conditions of Uncertainty into a Problem of Vector Optimization in the Conditions of Definiteness

Elimination of uncertainty consists in use of qualitative and quantitative descriptions of technical system which can be received, for example, by the principle "entrance exit". Transformation of basic data "entrance exit" to functional dependence is carried out by use of mathematical methods (the regression analysis).

The technical system in which experimental data are presented in the form of a matrix (9), is considered in the following designations:

$$I=\begin{bmatrix}X_1 & y_1(X_1) \ldots y_K(X_1)\\ \ldots & \\ X_M & y_1(X_M)\ldots y_K(X_M)\end{bmatrix}, or\ I=[X \quad Y], \quad (17)$$

where is considered: $X=\{X_i=\{x_{ij}, j=\overline{1,N}\}, i=\overline{1,M}\}$ - design data of technical system, $\boldsymbol{N}$ – a set of parameters of system, $\boldsymbol{M}$ - a set of alternatives (experiments); Y={y_{ik}, $k=\overline{1,K}$, $i=\overline{1,M}$}, $\boldsymbol{K}$ – a set of criteria (characteristics) by which each alternative is estimated, [4].

Construction a vector - function (criteria) is carried out on a method of the smallest squares

$\min\sum_{i=1}^{M}(y_i-\overline{y}_i)^2$, where by y_i, $i=\overline{1,M}$ - really observed sizes, and $\overline{y}_i$, $i=\overline{1,M}$ their estimates received for one-factorial model by means of function $\overline{y}_i = f(X_i,A)$, $X_i=\{x_i\}$. As $f(X_i, A)$ we use a polynom. In applied part of work the polynom of the second degree is used:

$$\min_A f(A,X)\equiv\sum_{i=1}^{M}\left(y_j-\begin{pmatrix}a_0+a_1x_{1i}+a_2x_{1i}^2\\+a_3x_{2i}+a_4x_{2i}^2\\+a_5x_{1i}*x_{2i}\end{pmatrix}\right)^2,$$

Result: Basic data {{$f_k(X_i$, $i=\overline{1,M}$ }$^{\mathrm{T}}$, $k=\overline{1,K_1^{unc}}$ }, {$f_k(X_i$, $i=\overline{1,M}$ }$^{\mathrm{T}}$, $k=\overline{1,K_2^{unc}}$ }} in problems of decision-making in the conditions of uncertainty (10), (11) and (13), (14) the functions - $f_k(X)$, $k=\overline{1,K_1^{unc}}$, $f_k(X)$, $k=\overline{1,K_2^{unc}}$ are transformed.

As a result the vector problem (13)-(16) will be transformed into a vector problem in the conditions of definiteness:

$$Opt\ F(X)=\left\{\begin{aligned}&\mathbf{max}F_1(X)=\{\mathbf{max}f_k(X),k=\overline{1,K_1}\},\\&\mathbf{min}F_2(X)=\{\mathbf{min}f_k(X),k=\overline{1,K_2}\}\end{aligned}\right\}, \quad (18)$$

at restrictions

$$\begin{aligned}&f_k^{\min}\le f_k(X)\le f_k^{\max}, k=\overline{1,K},\\&x_j^{\min}\le x_j\le x_j^{\max}, j=\overline{1,N},\end{aligned} \quad (19)$$

where $F(X)=\{f_k(X), k=\overline{1,K}\}$ - vector criterion which everyone a component submits the characteristic of technical system which is functionally depending on a vector of variables X; subset of criteria $K_1= K_1^{def} \cup K_1^{unc}$, $K_2= K_2^{def} \cup K_2^{unc}$.

3. Vector Optimization – Mathematical Apparatus of Modeling of Technical Systems

3.1. Axiomatics of Vector Optimization

At present, theoretical studies and methods of solving vector optimization problems are held in the following directions - methods of solving vector problems based on criteria convolution; methods using restrictions on criteria; goal programming methods; methods based on searching for compromise decision and on human-machine decision making procedures. To analyze the listed methods, we compare the results of solving the test example by these methods with the method based on criteria normalization and the principle of guaranteed result [[4], pp. 9-15].

Conceptual difficulty of the solution of vector tasks consists in the formulation of axiomatics of vector optimization. Such axiomatics defines in what one solution of a vector task is better than other solution of a vector task. The principle of an optimality is output from such axiomatics.

Axiom 1. (About equality and equivalence of criteria in an admissible point of vector problems of mathematical programming)

In of vector problems of mathematical programming two criteria with the indexes $k\in\boldsymbol{K}$, $q\in\boldsymbol{K}$ shall be considered as equal in $X\in\boldsymbol{S}$ point if relative estimates on k-th and q-th to criterion are equal among themselves in this point, i.e. $\lambda_k(X) = \lambda_q(X)$, $k, q \in \boldsymbol{K}$.

We will consider criteria equivalent in vector problems of mathematical programming if in $X\in S$ point when comparing in the numerical size of relative estimates of $\lambda_k(X)$, $k=\overline{1,K}$, among themselves, on each criterion of $f_k(X)$, $k=\overline{1,K}$, and, respectively, relative estimates of λ_k(X), isn't imposed conditions about priorities of criteria.

Definition 1. The relative level λ in a vector problem represents the lower assessment of a point of $X \in S$ among all relative estimates of $\lambda_k(X)$, $k = \overline{1,K}$:

$$\forall X \in S \;\; \lambda \le \lambda_k(X), k = \overline{1,K}, \tag{20}$$

the lower level for performance of a condition (20) in an admissible point of $X \in \boldsymbol{S}$ is defined by a formula

$$\forall X \in S \;\; \lambda = \min_{k \in K} \lambda_k(X). \tag{21}$$

Ratios (20) and (21) are interconnected. They serve as transition from operation (21) of definition of min to restrictions (20) and vice versa.

The level λ allows to unite all criteria in a vector problem one numerical characteristic of λ and to make over her certain operations, thereby, carrying out these operations over all criteria measured in relative units. The level λ functionally depends on the $X \in \boldsymbol{S}$ variable, changing X, we can change the lower level - λ. From here we will formulate the rule of search of the optimum decision.

Definition 2. (Principle of an optimality).

The vector problem of mathematical programming at equivalent criteria is solved, if the point of $X^o \in S$ and a maximum level of λ^o (the top index o - optimum) among all relative estimates such that is found

$$\lambda^\circ = \max_{X \in S} \min_{k \in K} \lambda_k(X). \tag{22}$$

Using interrelation of expressions (20) and (21), we will transform a maximine problem (22) to an extreme problem

$$\lambda^\circ = \max_{X \in S} \lambda, \tag{23}$$

$$\lambda \le \lambda_k(X), k = \overline{1,K}. \tag{24}$$

The resulting problem (23)-(24) let's call the λ-problem.

λ-problem (23)-(24) has $(N+1)$ dimension, as a consequence of the result of the solution of λ-problem (23)-(24) represents an optimum vector of $\boldsymbol{X^o} \in \boldsymbol{R}^{N+1}$, $(N+1)$ which component an essence of the value of the λ^o, i.e. $\boldsymbol{X^o}=\{x_1^o, x_2^o, ..., x_N^o, x_{N+1}^o\}$, thus $x_{N+1}^o = \lambda^o$, and $(N+1)$ a component of a vector of $\boldsymbol{X^o}$ selected in view of its specificity.

The received a pair of $\{\lambda^o, X^o\} = \boldsymbol{X^o}$ characterizes the optimum solution of λ-problem (23)-(24) and according to vector problem of mathematical programming (3)-(6) with the equivalent criteria, solved on the basis of normalization of criteria and the principle of the guaranteed result. We will call in the optimum solution of $\boldsymbol{X^o}=\{X^o, \lambda^o\}$, X^o - an optimal point, and λ^o - a maximum level.

An important result of the algorithm for solving vector problems (3)-(6) with equivalent criteria is the following theorem.

Theorem 1. (The theorem of two most contradictory criteria in a vector problem of mathematical programming with equivalent criteria).

In convex vector problems of mathematical programming at the equivalent criteria which is solved on the basis of normalization of criteria and the principle of the guaranteed result, in an optimum point of $\boldsymbol{X^o}=\{\lambda^o, X^o\}$ two criteria are always - denote their indexes $q \in \boldsymbol{K}$, $p \in \boldsymbol{K}$ (which in a sense are the most contradiction of the criteria $k = \overline{1,K}$), for which equality is carried out:

$$\lambda^\circ = \lambda_q(X^\circ) = \lambda_p(X^\circ), q, p \in \boldsymbol{K}, X \in S, \tag{25}$$

and other criteria are defined by inequalities:

$$\lambda^\circ = \lambda_q(X^\circ) \forall k \in \boldsymbol{K}, q \ne p = k. \tag{26}$$

3.2. Mathematical Algorithm of the Solution of a Vector Task

For the solution of vector problems of mathematical programming (3)-(6) the methods based on axiomatics of normalization of criteria and the principle of the guaranteed result [4,8] are offered. Methods follow from an axiom 1 and the principle of an optimality 1. We will present in the form of a number of steps:

Algorithm of the solution of a vector task (3)-(6) at equivalent criteria.

Step 1. The problem (3)-(6) by each criterion separately is solved, i.e. for $\forall k \in K_1$ is solved at the maximum, and for $\forall k \in K_2$ is solved at a minimum. As a result of the decision we will receive:

X_k^* - an optimum point by the corresponding criterion, $k=\overline{1,K}$;

$f_k^* = f_k(X_k^*)$ – the criterion size k-th in this point, $k=\overline{1,K}$.

Step 2. We define the worst value of each criterion on $\boldsymbol{S}$: f_k^0, $k=\overline{1,K}$. For what the problem (3), (5)-(6) for each criterion of $k=\overline{1,K}_1$ on a minimum is solved:

$f_k^0 = \min f_k(X)$, $G(X) \le B$, $X \ge 0$, $k=\overline{1,K}_1$.

The problem (4)-(6) for each criterion on a maximum is solved:

$f_k^0 = \max f_k(X)$, $G(X) \le B$, $X \ge 0$, $k=\overline{1,K}_2$.

As a result of the decision we will receive: $X_k^0=\{x_j, j=\overline{1,N}\}$ - an optimum point by the corresponding criterion, $k=\overline{1,K}$; $f_k^0=f_k(X_k^0)$ – the criterion size k-th a point, X_k^0, $k=\overline{1,K}$.

Step 3. The analysis of a set of points, optimum across Pareto, for this purpose in optimum points of $X^*=\{X_k^*, k=\overline{1,K}\}$ are defined sizes of criterion functions of $F(X^*)=\{f_q(X_k^*), q=\overline{1,K}, k=\overline{1,K}\}$ and relative estimates

$$\lambda(X^*) = \{\lambda_q(X_k^*), q = \overline{1,K}, k = \overline{1,K}\},$$

$$\lambda_k(X) = \frac{f_k(X) - f_k^o}{f_k^* - f_k^o}, \forall k \in \boldsymbol{K}:$$

$$F(X^*) = \begin{vmatrix} f_1(X_1^*), ..., f_k(X_1^*), \\ ... \\ f_1(X_k^*), ..., f_k(X_k^*) \end{vmatrix}, \quad \lambda(X^*) = \begin{vmatrix} \lambda_1(X_1^*), ..., \lambda_k(X_1^*), \\ ... \\ \lambda_1(X_k^*), ..., \lambda_k(X_k^*) \end{vmatrix}. \tag{27}$$

As a whole on a problem of accordance with (9) $\forall k \in \boldsymbol{K}$ the relative assessment of $\lambda_k(X)$, $k=\overline{1,K}$ lies within $0 \le \lambda_k(X) \le 1$, $\forall k \in \boldsymbol{K}$.

Step 4. Creation of the λ-problem.

Creation of λ-problem is carried out in two stages: initially built the maximine problem of optimization with the normalized criteria which at the second stage will be transformed to the standard problem of mathematical programming called λ-problem.

For construction maximine a problem of optimization we use definition - relative level $\forall X \in \boldsymbol{S}$ $\lambda = \min\limits_{k \in K} \lambda_k(X)$.

The bottom λ level is maximized on $X \in S$, as a result we will receive a maximine problem of optimization with the normalized criteria.

$$\lambda^{\circ} = \max_{x} \min_{k} \lambda_k(X), G(X) \le B, X \ge 0. \tag{28}$$

At the second stage we will transform a problem (28) to a standard problem of mathematical programming:

$$\lambda^{\circ} = \max \lambda, \qquad \lambda^{\circ} = \max \lambda, \tag{29}$$

$$\lambda - \lambda_k(X) \le 0, k = \overline{1,K}, \rightarrow \lambda - \frac{f_k(X) - f_k^o}{f_k^* - f_k^o} \le 0, k = \overline{1,K}, \tag{30}$$

$$G(X) \le B, X \ge 0, \qquad G(X) \le B, X \ge 0, \tag{31}$$

where the vector of unknown of X has dimension of N+1: $X=\{\lambda, x_1, \ldots, x_N\}$.

Step 5. Solution of λ-problem.

λ-problem (29)-(31) is a standard problem of convex programming and for its decision standard methods are used.

As a result of the solution of λ-problem it is received:

$\boldsymbol{X^o}=\{\lambda^o, X^o\}$ - an optimum point;

$f_k(X^o)$, $k=\overline{1,K}$ - values of the criteria in this point;

$\lambda_k(X^o) = \dfrac{f_k(X^o) - f_k^o}{f_k^* - f_k^o}$, $k=\overline{1,K}$ - sizes of relative estimates;

λ^o - the maximum relative estimates which is the maximum bottom level for all relative estimates of $\lambda_k(X^o)$, or the guaranteed result in relative units, λ^o guarantees that all relative estimates of $\lambda_k(X^o)$ more or are equal λ^o in X^o point to

$$\begin{aligned} &\lambda^{\circ}, \lambda_k\left(X^{\circ}\right) \ge \lambda^{\circ}, k = \overline{1,K} \\ &\text{or}\, \lambda^{\circ} \le \lambda_k\left(X^{\circ}\right), k = \overline{1,K}, X^{\circ} \in S, \end{aligned} \tag{32}$$

and according to the theorem the 2 point of $\boldsymbol{X^o}=\{\lambda^o, x_1, \ldots, x_N\}$ is optimum across Pareto.

In total we presented "Methodology of modeling of technical systems in the conditions of definiteness and uncertainty" in sections 2.1, 2.2, 2.3, 2.4 and 3.2 and adoptions of the optimum decision at equivalent criteria. Numerical realization of methodology is presented in the following section.

4. Results. Numerical Problem of Modeling of Technical System

We will consider a task "Numerical modeling of technical system" in which data on some set of functional characteristics (definiteness conditions), discrete values of characteristics (an uncertainty condition) and the restrictions imposed on functioning of technical system are known.

It is given. The technical system, which functioning is defined by two parameters $X=\{x_1, x_2\}$ – a vector (operated) variables. Basic data for the solution of a task are four characteristics (criterion) of $F(X)=\{f_1(X), f_2(X), f_3(X), f_4(X)\}$ which size of an assessment depends on a vector of X. For characteristics of $f_1(X)$, $f_2(X)$ functional dependence on parameters X (a definiteness condition) is known:

$$\begin{aligned} &f_1(X) = 67.425 + 0.02225 * x_1 + 0.00239 * x_1^2 \\ &-0.05625 * x_2 + 0.00029 * x_2^2 + 0.0021232 * x_1 * x_2, \\ &f_2(X) = 4456.3 - 2.315 * x_1 + 0.239 * x_1^2 \\ &+2.805 * x_2 - 0.037 * x_2^2 - 0.22192 * x_1 * x_2 \end{aligned} \tag{35}$$

Functional restrictions:

$$\begin{aligned} &3800 \le f_2(X) \equiv 4456.3 - 2.315 * x_1 + 0.239 * x_1^2 \\ &+2.805 * x_2 - 0.037 * x_2^2 - 0.22192 * x_1 * x_2 \le 5500 \end{aligned} \tag{36}$$

Parametrical restrictions:

$$25 \le x_1 \le 100, \; 25 \le x_2 \le 100. \tag{37}$$

Table 1. Numerical values of parameters and characteristics of technical system

x_1	x_2	$y_3(X)$→max	$y_4(X)$→min
25	25	1148	490.9
25	50	1473	483.1
25	75	1798	557.3
25	100	2122	521.5
50	25	725	498.1
50	50	968	521.5
50	75	1212	549.9
50	100	1456	578.3
75	25	440	507.3
75	50	572	549.9
75	75	734	592.5
75	100	897	635.1
100	25	202	521.5
100	50	284	578.3
100	75	385	635.1
100	100	446	691.9

For the third and fourth characteristic results of experimental data are known: sizes of parameters and corresponding characteristics (uncertainty condition). Numerical values of parameters X and characteristics of $y_3(X)$, $y_4(X)$ are presented in Table 1.

In the made decision, assessment size of the first, second and the third characteristic (criterion) is possible to receive above (max), for the fourth characteristic is possible below (min). Parameters $X=\{x_1, x_2\}$ change in the following limits: $x_1, x_2 \in$ [25. 50. 75. 100.].

It is required. To make the best decision (optimum).

Methodology *of modeling of technical system in the conditions of definiteness and uncertainty.*

1. Creation of mathematical model of technical system.

1.1. ***Construction in the conditions of definiteness*** is defined by functional dependence of each characteristic and restrictions on parameters of technical system. In our example two characteristics (35) and restrictions (36)-(37) are known:

$$f_1(X) = 67.425 + 0.02225 * x_1 + 0.00239 * x_1^2$$
$$-0.05625 * x_2 + 0.00029 * x_2^2 + 0.0021232 * x_1 * x_2, \quad (38)$$
$$f_2(X) = 4456.3 - 2.315 * x_1 + 0.239 * x_1^2$$
$$+2.805 * x_2 - 0.037 * x_2^2 - 0.22192 * x_1 * x_2,$$

Functional restrictions:

$$3800 \le \begin{pmatrix} f_2(X) \equiv 4456.3 - 2.315 * x_1 \\ +0.239 * x_1^2 + 2.805 * x_2 \\ -0.037 * x_2^2 - 0.22192 * x_1 * x_2 \end{pmatrix} \le 5500 \quad (39)$$

Parametrical restrictions:

$$25 \le x_1 \le 100, \; 25 \le x_2 \le 100. \quad (40)$$

These data are used further at creation of mathematical model of technical system.

1.2. Construction in the conditions of uncertainty consists in use of the qualitative and quantitative descriptions of technical system received by the principle "entrance exit" in Table 1. Transformation of information (basic data of $y_3(X)$, $y_4(X)$) to a functional type of $f_3(X)$, $f_4(X)$ is carried out by use of mathematical methods (the regression analysis).

Basic data of Table 1 are created in Matlab system in the form of a matrix

$$I = |X, Y| = \{x_{i1} x_{i2} y_{i3} y_{i4}, i = \overline{1, M}\}. \quad (41)$$

For each set experimental these y_k, $k=\overline{3,4}$ function of regression on a method of the smallest squares in Matlab system is formed. A_k,- polynom defining interrelation of factors of $X_i = \{x_{1i}, x_{2i}\}$ (41) and functions $\overline{y}_{ki} = f(X_i, A_k)$, $k=\overline{3,4}$ is constructed.

As a result of calculations we received system of coefficients of $A_k = \{A_{0k}, A_{1k}, A_{2k}, A_{k3}, A_{4k}, A_{5k}\}$ which define coefficients of a polynom (function):

$$f_k(X, A) = A_{0k} + A_{1k} x_1 + A_{2k} x_1^2$$
$$+A_{3k} x_2 + A_{4k} x_2^2 + A_{5k} x_1 * x_2, k = \overline{3,4} \quad (42)$$

As a result of calculations of coefficients of A_k, k=3, we received the $f_3(X)$ function:

$$f_3(X) = 1273.5 - 19.919 * x_1 + 0.0854 * x_1^2$$
$$+16.071 * x_2 + 0.001 * x_2^2 - 0.13034 * x_1 * x_2, \quad (43)$$

The graphical representation of the $f_3(X)$ (X) function is shown in Figure 1.

We showed in Figure 1 X_3^*, X_3^0 the best (maximum) and worst (minimum) decision, according to $f_3(X_3^*)$, $f_3(X_3^0)$ – sizes of functions.

As a result of calculations of coefficients of A_k, k =4, we received the $f_4(X)$ function:

$$f_4(X) = 481.7 - 0.6915 * x_1 + 0.0047 * x_1^2 +$$
$$0.3535 * x_2 - 0.0023 * x_2^2 + 0.021808 * x_1 * x_2, \quad (44)$$

The graphical representation of the $f_4(X)$ function is shown in Figure 2.

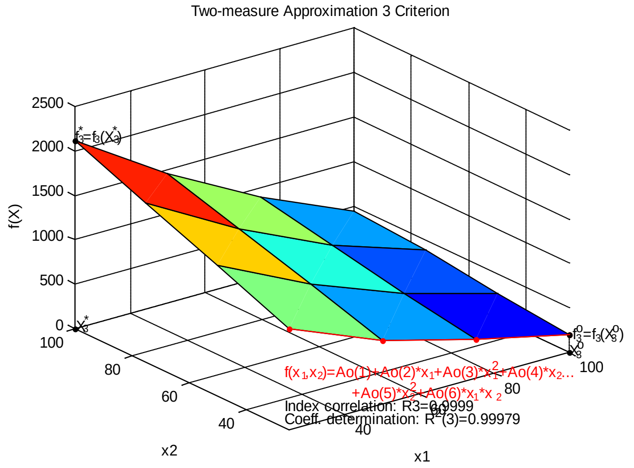

Figure 1. The function $f_3(X)$ in two-dimensional system of coordinates of $X = \{x_1, x_2\}$

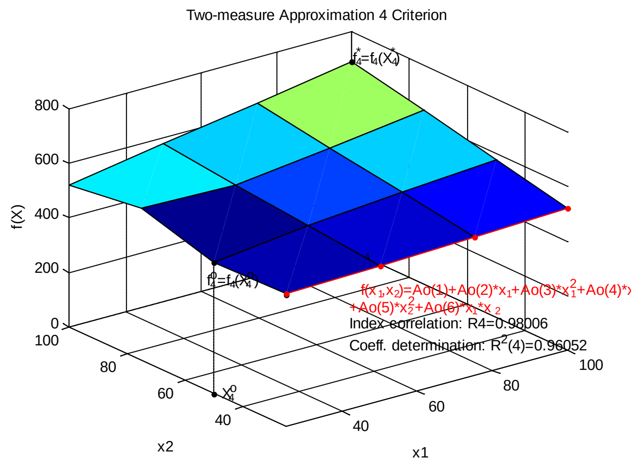

Figure 2. The function $f_4(X)$ in two-dimensional system of coordinates of $X = \{x_1, x_2\}$

We showed in Figure 2 X_4^*, X_4^0 the best (minimum) and worst (maximum) decision, according to $f_4(X_4^*)$, $f_4(X_4^0)$ – sizes of functions.

Parametrical restrictions are similar (40): $25 \le x_1 \le 100$, $25 \le x_2 \le 100$.

1.3. Creation of mathematical model of technical system (The general part for conditions of definiteness and uncertainty).

For creation of mathematical model of technical system we used:

the functions received conditions of definiteness (38) and uncertainty (43), (44);

functional restrictions (39);

parametrical restrictions (40).

We considered functions (38) and (43), (44) as the criteria defining focus of functioning of technical system. A set of criteria ***K***=4 included three criteria of $f_1(X)$, $f_2(X)$, $f_3(X)$ →max and $f_4(X)$ →min. As a result model of functioning of technical system was presented a vector problem of mathematical programming:

$$opt\ F(X) = \{max\ F_1(X)$$
$$= \{max\ f_1(X) \equiv 67.425$$
$$+0.02225 * x_1 + 0.00239 * x_1^2 - 0.05625 * x_2 \quad (45)$$
$$+0.00029 * x_2^2 + 0.0021232 * x_1 * x_2$$

$$\max f_2(X) \equiv 4456.3 - 2.315 * x_1 + 0.239 * x_1^2 + 2.805 * x_2 - 0.037 * x_2^2 - 0.22192 * x_1 * x_2, \quad (46)$$

$$\max f_3(X) \equiv 1273.5 - 19.919 * x_1 + 0.0854 * x_1^2 + 16.071 * x_2 + 0.001 * x_2^2 - 0.13034 * x_1 * x_2\}, \quad (47)$$

$$\min F_2(X) = \{\min f_4(X) \equiv 481.7 - 0.6915 * x_1 + 0.0047 * x_1^2 + 0.3535 * x_2 - 0.0023 * x_2^2 + 0.021808 * x_1 * x_2\}\} \quad (48)$$

at restrictions

$$3800 \le \begin{pmatrix} f_2(X) \equiv 4456.3 - 2.315 * x_1 \\ +0.239 * x_1^2 + 2.805 * x_2 \\ -0.037 * x_2^2 - 0.22192 * x_1 * x_2 \end{pmatrix} \le 5500 \quad (49)$$

$$25 \le x_1 \le 100, \; 25 \le x_2 \le 100. \quad (50)$$

The vector problem of mathematical programming represents model of adoption of the optimum decision in the conditions of definiteness and uncertainty in total.

2. The solution of a vector problem of mathematical programming - model of technical system.

The solution of a vector task (45)-(50) with equivalent criteria was submitted as sequence of steps.

Step 1. Problems (45)-(50) were solved by each criterion separately, thus used the function *fmincon* (…) of *Matlab* system [14], the appeal to the function *fmincon* (…) is considered in [8].

As a result of calculation for each criterion we received optimum points: X_k^* and $f_k^* = f_k(X_k^*)$, $k=\overline{1,K}$ – sizes of criteria in this point, i.e. the best decision on each criterion:

$$X_1^* = \{x_1 = 100, x_2 = 100\}, \; f_1^* = f_1(X_1^*) = -112.06;$$

$$X_2^* = \{x_1 = 97.16, x_2 = 48.09\}, \; f_2^* = f_2(X_2^*) = -5500.0;$$

$$X_3^* = \{x_1 = 25.0, x_2 = 100.0\}, \; f_3^* = f_3(X_3^*) = -2120.15;$$

$$X_4^* = \{x_1 = 25.0, x_2 = 25.0\}, \; f = f_4(X_4^*) = 488.38.$$

Restrictions (50) and points of an optimum in coordinates $\{x_1, x_2\}$ are presented on Figure 3.

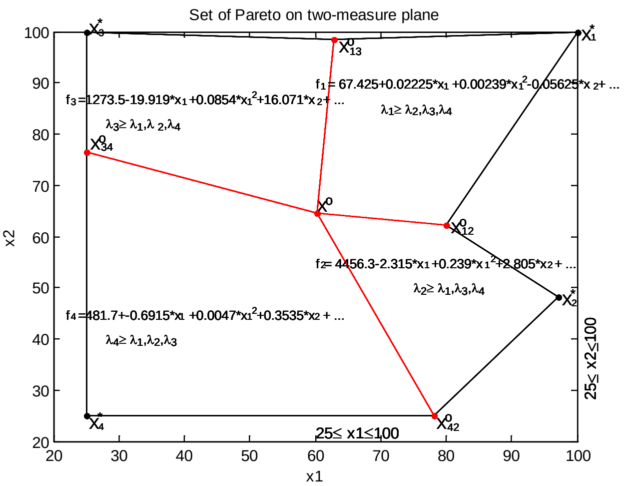

Figure 3. Pareto's great number, $S^o \subset S$ in two-dimensional system of coordinates

Step 2. We defined the worst unchangeable part of each criterion (anti-optimum):

$$X_1^0 = \{x_1 = 25.0, x_2 = 25.0\}, \; f_1^0 = f_1(X_1^0) = 69.57;$$

$$X_2^0 = \{x_1 = 37.32, x_2 = 98.88\}, \; f_2^0 = f_2(X_2^0) = 3800;$$

$$X_3^0 = \{x_1 = 83.1, x_2 = 25.0\}, \; f_3^0 = f_3(X_3^0) = 339.7;$$

$$X_4^0 = \{x_1 = 100, x_2 = 100\},$$

$$f_4^0 = f_2(X_4^0) = -689.9. \; (\text{Top index zero}).$$

Step 3. We made the analysis of a set of points, optimum across Pareto. In points of an optimum of $X^* = \{X_1^*, X_2^*, X_3^*, X_4^*\}$ sizes of criterion functions of $F(X^*) = \left\| f_q(X_k^*) \right\|_{q=\overline{1,K}}^{k=\overline{1,K}}$ determined. Calculated a vector of $D = (d_1 \; d_2 \; d_3 \; d_4)^T$ - deviations by each criterion on an admissible set of $\boldsymbol{S}$: $d_k = f_k^* - f_k^0$, $k=\overline{1,4}$, and matrix of relative estimates of

$$\lambda(X^*) = \left\| \lambda_q(X_k^*) \right\|_{q=\overline{1,K}}^{k=\overline{1,K}},$$

$$\text{where } \lambda_k(X) = (f_k^* - f_k^0) / d_k.$$

$$F(X^*) = \begin{vmatrix} -112.1 & -4306.1 & -449.3 & 690.0 \\ -100.0 & -5500.0 & -310.5 & 572.5 \\ -72.1 & -3903.5 & -2120.2 & 534.2 \\ -69.6 & -4456.1 & -1149.8 & 488.4 \end{vmatrix}, D = \begin{vmatrix} 42.48 \\ -1700. \\ 1780.5 \\ -201.6 \end{vmatrix},$$

$$\lambda(X^*) = \begin{vmatrix} 1.0000 & 0.2977 & 0.0616 & 0 \\ 0.7170 & 1.0000 & -0.0164 & 0.5829 \\ 0.0584 & 0.0609 & 1.0000 & 0.7726 \\ 0 & 0.3859 & 0.4550 & 1.0000 \end{vmatrix}$$

Discussion. The analysis of sizes of criteria in relative estimates showed that in points of an optimum of $X^* = \{X_1^*, X_2^*, X_3^*, X_4^*\}$ the relative assessment is equal to unit. Other criteria there is much less than unit. It is required to find such point (parameters) at which relative estimates are closest to unit. The step 4 is directed on the solution of this problem.

Step 4. Creation of λ-problem is carried out in two stages: originally the maximine problem of optimization with the normalized criteria is under construction:

$$\lambda^\circ \max_{X} \min_{k} \lambda_k(X), G(X) \le 0, X \ge 0,$$

which at the second stage was transformed to a standard problem of mathematical programming (λ-problem):

$$\lambda^\circ = \max \lambda, \quad (51)$$

at restrictions

$$\lambda - \frac{\begin{pmatrix} 67.4 + 0.02225 * x_1 + 0.0024 * x_1^2 \\ -0.05625 * x_2 + 0.00029 * x_2^2 \\ +0.002123 * x_1 * x_2 - f_1^o \end{pmatrix}}{f_1^* - f_1^o} \le 0, \quad (52)$$

$$\lambda - \frac{\begin{pmatrix} 4456.3 - 2.315 * x_1 + 0.239 * x_1^2 \\ +2.605 * x_2 - 0.037 * x_2^2 \\ -0.222 * x_1 * x_2 - f_2^o \end{pmatrix}}{f_2^* - f_2^o} \le 0, \quad (53)$$

$$\lambda - \frac{\begin{pmatrix} 1273.5 - 19.92 * x_1 + 0.0854 * x_1^2 \\ +16.07 * x_2 + 0.01 * x_2^2 \\ -0.1303 * x_1 * x_2 - f_3^o \end{pmatrix}}{f_3^* - f_3^o} \le 0, \quad (54)$$

$$\lambda - \frac{\begin{pmatrix} 481.7 - 0.6915 * x_1 \\ +0.0047 * x_1^2 + 0.3535 * x_2 \\ -0.0023 * x_2^2 + 0.0218 * x_1 * x_2 - f_4^o \end{pmatrix}}{f_4^* - f_4^o} \le 0, \quad (55)$$

$$\begin{aligned} & 3800 \le f_2(x) \le 5500; \\ & 0 \le \lambda \le 1, 25 \le x_1 \le 100, 25 \le x_2 \le 100, \end{aligned} \quad (56)$$

where the vector of unknown had dimension of N+1: $\boldsymbol{X}=\{x_1, \ldots, x_N, \lambda\}$. Appeal to function fmincon(), [14]:

$$[\mathrm{Xo}, \mathrm{Lo}] = \mathrm{fmincon}\begin{pmatrix} \text{'Z\_TehnSist\_4Krit\_L'}, \mathrm{X0}, \\ \mathrm{Ao}, \mathrm{bo}, \mathrm{Aeq}, \mathrm{beq}, \mathrm{lbo}, \mathrm{ubo}, \\ \text{'Z\_TehnSist\_LConst'}, \mathrm{options} \end{pmatrix}.$$

As a result of the solution of a vector problem of mathematical programming (45)-(50) at equivalent criteria and λ-problem corresponding to it (51)-(56) received:

$\boldsymbol{X^o}=\{X^o, \lambda^o\}=\{X^o=\{x_1=60.36, x_2=64.52, \lambda^o=0.3236\}$ - an optimum point – design data of technical system, point $\boldsymbol{X^o}$ is presented in Figure 3;

$f_k(X^o)$, $k=\overline{1,K}$ - sizes of criteria (characteristics of technical system): $\{f_1(X^o)=83.3$, $f_2(X^o)=4350.1$, $f_3(X^o)=915.8$, $f_4(X^o)=555.2\}$;

$\lambda_k(X^o)$, $k=\overline{1,K}$ - sizes of relative estimates$\{\lambda_1(X^o)=0.3236$, $\lambda_2(X^o)=0.3236$, $\lambda_3(X^o)=0.3236$, $\lambda_4(X^o)=0.6683\}$;

$\lambda^o=0.3236$ is the maximum lower level among all relative estimates measured in relative units: : $\lambda^o=min$ $(\lambda_1(X^o), \lambda_2(X^o), \lambda_3(X^o), \lambda_4(X^o))=0.3236$. A relative assessment - λ^o call the guaranteed result in relative units, i.e. $\lambda_k(X^o)$ and according to the characteristic of technical $f_k(X^o)$ system it is impossible to improve, without worsening thus other characteristics.

Discussion. We will notice that according to the theorem 1, in $\boldsymbol{X^o}$ point criteria 1, 2, 3 are contradictory. This contradiction is defined by equality of $\lambda_1(X^o)=\lambda_2(X^o)=\lambda_3(X^o)=\lambda^o=0.3236$, and other criteria an inequality of $\lambda_4(X^o)=0.6683>\lambda^o$.

Thus, the theorem 1 forms a basis for determination of correctness of the solution of a vector task. In a vector problem of mathematical programming, as a rule, for two criteria equality is carried out:

$\lambda^o = \lambda_q(X^o) = \lambda_p(X^o)$, q, $p \in \boldsymbol{K}$, $X \in S$, (in our example of such criteria three) and for other criteria is defined as an inequality:

$$\lambda^o \le \lambda_k\left(X^o\right) \forall k \in \boldsymbol{K}, q \ne p \ne k.$$

In an admissible set of points of $\boldsymbol{S}$ formed by restrictions (56), optimum points X_1^*, X_2^*, X_3^*, X_4^*, united in a contour, presented a set of points, optimum across Pareto, to $\boldsymbol{S^o} \subset \boldsymbol{S}$. For specification of border of a great number of Pareto calculated additional points: X_{12}^o, X_{13}^o, X_{34}^o, X_{42}^o which lie between the corresponding criteria.

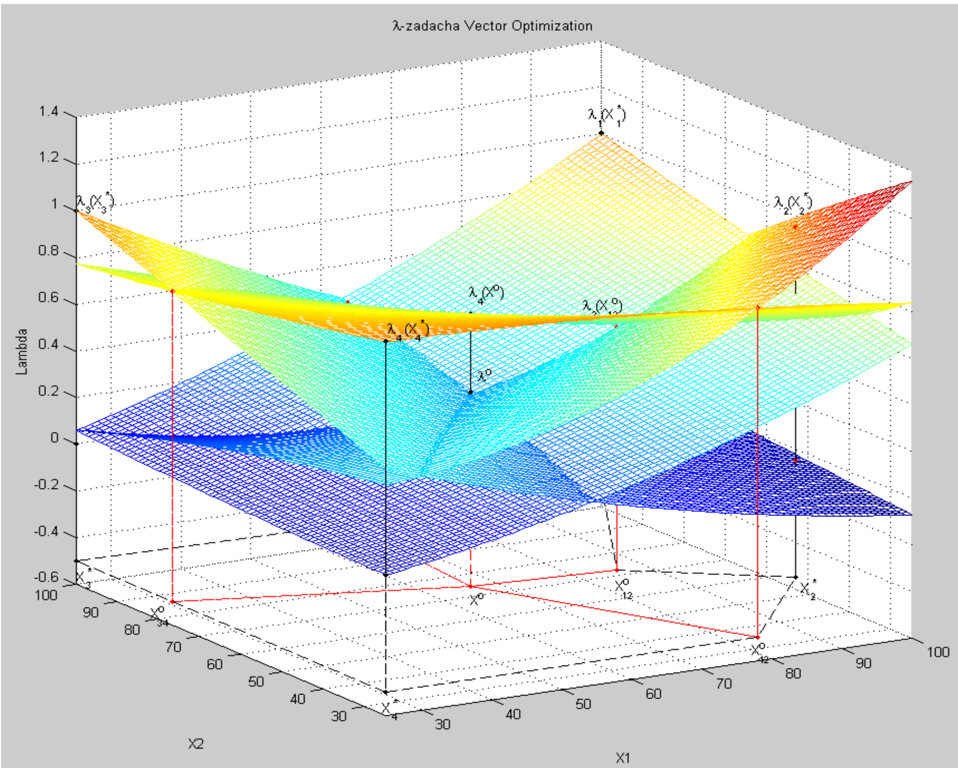

Figure 4. The solution of λ-problem in three-dimensional system of coordinates of x_1, x_2 and λ

For definition of a point of X_{12}^{o} the vector problem was solved with two criteria (51), (52), (53), (56).

Results of the decision:

$$X_{12}^{o} = \{79.99 \;\; 62.31\}, \lambda^{o}(X_{12}^{o}) = 0.5445;$$
$$\mathrm{F}_{12} = \{92.7 \;\; 4725.6 \;\; 582.1 \;\; 578.3\};$$
$$\mathrm{L}_{12} = \{0.5445 \;\; 0.5445 \;\; 0.1362 \;\; 0.5541\}.$$

Other points were similarly defined:

$$X_{13}^{o} = \{62.94 \;\; 98.44\}, \lambda^{o}(X_{13}^{o}) = 0.4507;$$
$$F_{13} = \{88.7 \;\; 3800.0 \;\; 1142.2 \;\; 604.4\};$$
$$L_{13} = (0.4507 \;\; 0.0 \;\; 0.4507 \;\; 0.4243\};$$

$$X_{34}^{o} = \{25.0 \;\; 76.52\}, \lambda^{o}(X_{34}^{o}) = 0.\,83;$$
$$F_{34} = \{70.9 \;\; 4121.2 \;\; 1815.2 \;\; 52.27\};$$
$$L_{34} = \;\{0.0319 \;\; 0.1890 \;\; 0.83 \;\; 0.83\};$$

$$X_{42}^{o} = \{78.14 \; 25.0\}, \lambda^{o}(X_{42}^{o}) = 0.9108;$$
$$F_{42} = \{86.7 \;\; 5348.3 \;\; 386.2 \;\; 506.4\};$$
$$L_{42} = \{0.4026 \;\; 0.9108 \;\; 0.0261 \;\; 0.9108\}.$$

Points: X_{12}^{o}, X_{13}^{o}, X_{34}^{o}, X_{42}^{o} are presented in Figure 3. Pareto's great number of ***S*** lies between points of an optimum of $X_1^* X_{12}^o X_2^* X_{13}^o X_3^* X_{34}^o X_4^* X_{42}^o$.

Coordinates of these points, and also characteristics of technical system in relative units of $\lambda_1(X)$, $\lambda_2(X)$, $\lambda_3(X)$, $\lambda_4(X)$ are shown in Figure 4 in three measured space, where the third axis of λ - a relative assessment.

In the course of modeling parametrical restrictions (30), functional restrictions (29) can be changed, i.e. some set of optimum decisions is received. Choose a final version which in our example included from this set of optimum decisions:

- parameters of technical system $X^o=\{x_1=60.36, x_2=64.52\}$;
- in point X^o of the first characteristic of $f_1(X)$ will assume to the look presented in Figure 5;

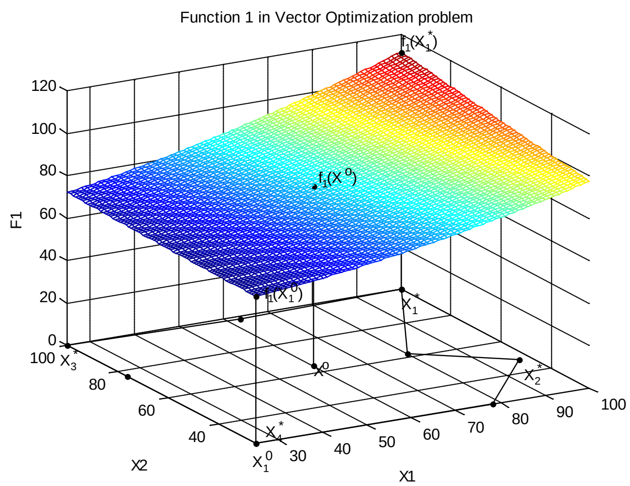

Figure 5. The first characteristics of $f_1(X)$ of technical system in natural indicator

- in point X^o of the second characteristic of $f_2(X)$ will assume to the look presented in Figure 6;

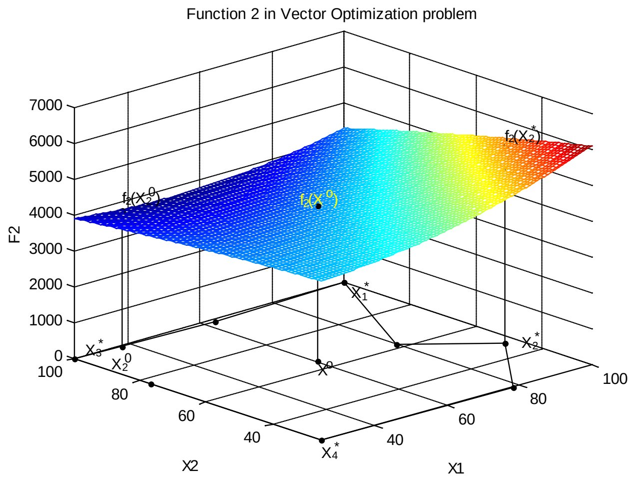

Figure 6. The second characteristics of $f_2(X)$ of technical system in natural indicator

- in point X^o of the third characteristic of $f_3(X)$ will assume to the look presented in Figure 7;

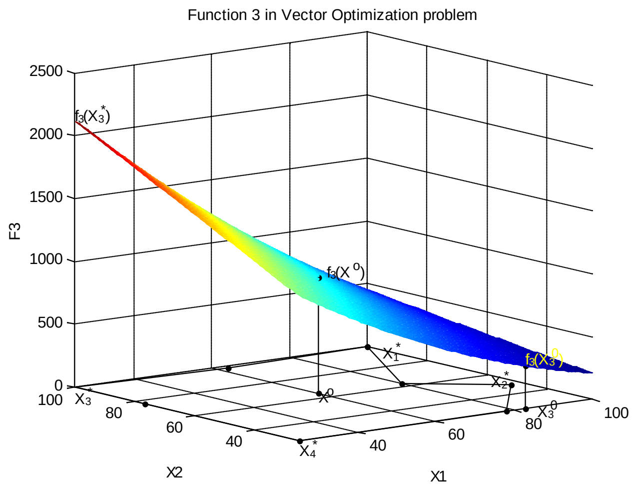

Figure 7. The third characteristics of $f_3(X)$ of technical system in natural indicator

- in point X^o of the fourth characteristic of $f_4(X)$ will assume to the look presented in Figure 8;

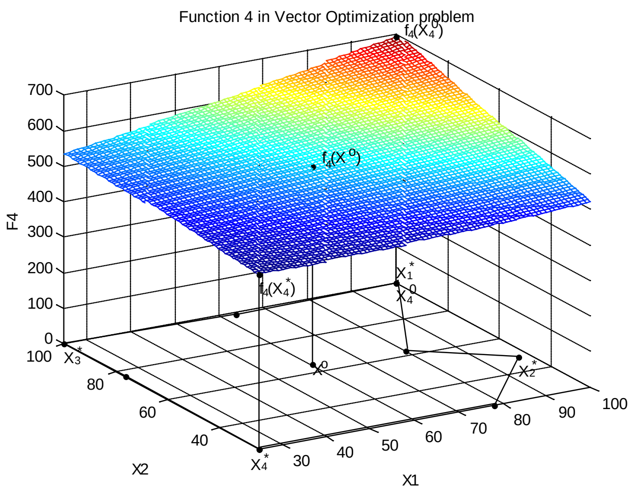

Figure 8. The fourth characteristics of $f_4(X)$ of technical system in natural indicator

Collectively, the submitted version:

• point - X^o; characteristics of $f_1(X^o)$, $f_2(X^o)$, $f_3(X^o)$, $f_4(X^o)$;

• relative estimates of $\lambda_1(X^o)$, $\lambda_2(X^o)$, $\lambda_3(X^o)$, $\lambda_4(X^o)$;

• maximum λ^o relative level such that $\lambda^o \leq \lambda_k(X^o)\ \forall k \in \boldsymbol{K}$

- there is an optimum decision at equivalent criteria (characteristics), and procedure of receiving is adoption of the optimum decision at equivalent criteria (characteristics).

5. Conclusions

The problem of adoption of the optimum decision in difficult technical system on some set of functional characteristics is one of the most important tasks of the system analysis and design. In work the new technology (methodology) of creation of mathematical model of technical system in the conditions of definiteness and uncertainty in the form of a vector problem of mathematical programming is presented. At creation of characteristics in the conditions of uncertainty regression methods of transformation of information are used. The methodology of modeling and adoption of the optimum decision is based on normalization of criteria and the principle of the guaranteed result (maxmin). This methodology has system character and can be used when modeling both technical, and economic systems. Authors are ready to participate in the solution of vector problems of linear and nonlinear programming.

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