

Modave summerschool in mathematical physics - 2010

July 16, 2010

1 Lectures

- **Symplectic manifolds, constraints and quantization - Fredrik Ohlsson**

Abstract: ‘I plan to give a short introduction to symplectic manifolds, and talk a little bit about submanifolds and symplectic reduction/restriction. Then I will discuss the role of symplectic manifolds in the description of a classical system through the cotangent bundle. I also plan to discuss the relation between the Poisson bracket and the symplectic structure, Dirac’s description of constraints and the derivation of the Dirac bracket and illustrate with the example of quantization of electrodynamics.’

- **Lie groups in physics- Josef Lindman Hörnlund**

Abstract: ‘In these lectures I plan to do an elementary introduction of some techniques in using Lie groups in theoretical physics. I will start with a brief history of the subject and continue with discussing the basics with emphasis on the defining property of Lie groups, being both groups and smooth manifolds. We will learn how to do Riemannian geometry on these manifolds to derive for example curvatures and invariant vector fields. We will also discuss the Lorentz group, the compact groups and global decompositions such as the Iwasawa decomposition in some detail, among other things’

Outline (6h):

1. History of Lie groups in mathematics and physics
2. Basics of Lie groups
3. Riemannian geometry on Lie group manifolds
4. Classical Lie groups
5. Compact groups, spin groups

6. Global decompositions and some cosets

References:

The motivated student might feel it rewarding to freshen up some co-ordinate free Riemannian geometry and the basics of Lie algebras. A nice reference on Lie groups for physicists is available online at [arXiv:math-ph/0005032v1](https://arxiv.org/abs/math-ph/0005032v1) .

- **Hamiltonian formalism - Francois Dehouck and Cedric Troessaert**

Abstract: ‘We would like to give an hint at the power of Hamiltonian formalism applied to gauge theories. In a first part, we will review the basics of Hamiltonian formalism and illustrate it with some basic examples. We will then apply it to General Relativity and explain for example how charges, and the relations they satisfy, can be defined in this setup. We will then study some interesting examples where this formalism revealed itself useful such as AdS_3 and the Brown-Henneaux central charge and the bosonic string. We will then try to cover some more recent topics such as gravitational duality, bi-Hamiltonian systems and Wald’s conserved charges.’

- **From black holes to fluid dynamics, and back- Marco Caldarelli**

Abstract:

- Black holes in general relativity, a short review
- Phases, thermodynamics and instabilities of higher dimensional black holes
- Approximate methods to construct new black hole solutions
- Black hole perturbations and relativistic fluid dynamics’

- **M-Theory: Bagger-Lambert, ABJM and D-branes - Neil Copland**

Abstract: ‘I will start off with an introduction to M-theory focusing on the extended objects of the theory; membranes and fivebranes. I will cover their worldvolume theories before moving on to the more recent description of multiple membranes via the BLG and ABJM theories.’

Outline (4h):

1. Introduction to M-theory and 11-dimensional Supergravity
2. Branes in M-Theory
3. BLG
4. ABJM

In each case I'll be looking at the relation to string theory in 10 dimensions.

References:

1. There is a lot to summarise, a nice review is [1] by Townsend, and you could also look at sections 8.3 and 8.4 of Becker² Schwarz.
2. I'll cover membranes and fivebranes, their supergravity solutions and worldvolume actions and anything else I can fit in. An overview can be found in the earlier sections of the Review by Berman [2] .
3. I'll discuss the Bagger-Lambert-Gustavsson theory of multiple membranes based on three algebras. The original Bagger-Lambert papers are the place to start [3, 4].
4. The ABJM paper is [5], a review is [6]

References

- [1] Townsend, P. K., *Four lectures on M-theory*, 1996, hep-th/9612121
- [2] Berman, David S. *M-theory branes and their interactions*, Phys. Rept., **456** 2008, 89-126, arXiv: 0710.1707 [hep-th]
- [3] Bagger, Jonathan and Lambert, Neil, *Gauge Symmetry and Supersymmetry of Multiple M2-Branes*, Phys. Rev. **D77**, 2008, 065008, arXiv: 0711.0955, [hep-th]
- [4] Bagger, Jonathan and Lambert, Neil, *Comments On Multiple M2-branes*, JHEP **02** 2008, 105, arXiv: 0712.3738 [hep-th]
- [5] Aharony, Ofer and Bergman, Oren and Jafferis, Daniel Louis and Maldacena, Juan, *N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, JHEP **10** 2008, 091, arXiv: 0806.1218 [hep-th]
- [6] Klebanov, Igor R. and Torri, Giuseppe, *M2-branes and AdS/CFT*, Int. J. Mod. Phys. **A25** 2010, 332-350, arXiv: 0909.1580 [hep-th]