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# COMPARISON OF SOLAR CONCENTRATORS

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**Abstract**—Even though most variations of solar concentrators have been studied or built at some time or other, an important class of concentrators has been overlooked until very recently. These novel concentrators have been called ideal because of their optical properties, and an example, the compound parabolic concentrator, is being tested at Argonne National Laboratory. Ideal concentrators differ radically from conventional instruments such as focussing parabolas. They act as radiation funnel and do not have a focus. For a given acceptance angle their concentration surpasses that of other solar concentrators by a factor of two to four, but a rather large reflector area is required. The number of reflections varies with angle of incidence, with an average value around one in most cases of interest. In order to help provide a rational basis for deciding which concentrator type is best suited for a particular application, we have compared a variety of solar concentrators in terms of their most important general characteristics, namely concentration, acceptance angle, sensitivity to mirror errors, size of reflector area and average number of reflections.

The connection between concentration, acceptance angle and operating temperature of a solar collector is analysed in simple intuitive terms, leading to a straightforward recipe for designing collectors with maximal concentration (no radiation emitted by the absorber must be allowed to leave the concentrator outside its acceptance angle). We propose some new concentrators, including the use of compound parabolic concentrators as second stage concentrators for conventional parabolic or Fresnel mirrors. Such a combination approaches the performance of an ideal concentrator without demanding a large reflector; it may offer significant advantages for high temperature solar systems.

## I. INTRODUCTION

Many excellent articles on solar concentrators have been written[1], but despite the remark "the engineering literature indicates that most of the possible combinations of insulation, concentration and orientation of solar-heat collectors have been tried at one time or other" made by Hottel and Woertz[2] over 30 years ago, an important class of concentrators[3, 4] have been completely overlooked until very recently[5]. The new type of concentrator has been called the ideal concentrator by Winston[4] to describe its optical properties. Such a name is somewhat unfortunate in the present context, and we emphasize that this name is not to imply any value judgement whatsoever about the usefulness of this device for solar energy applications[6]. Compared to the flat mirrors, focussing parabolas and lenses that have been used until now, ideal concentrators achieve significantly higher concentration values but require a rather large reflector area; furthermore, their reflection losses are more difficult to assess than those of a simple parabola. We feel that a review is called for at the present time which compares ideal concentrators with conventional ones such as simple parabolas, *V*-troughs and Fresnel mirrors. This should help provide a rational basis for deciding which concentrator type is best suited for a particular application.

For an exhaustive answer to such a question, a detailed systems analysis may be needed, an undertaking which is beyond the scope of the present investigation. To keep this paper reasonably self-contained, we have restricted our attention to the most important general characteristics of solar concentrators: concentration, acceptance angle, sensitivity to mirror and alignment errors, size of reflector area and average number of reflections. In a companion paper[7] we have evaluated the thermal performance[8, 9] to be expected from a solar collector consisting of ideal concentrators.

Concentration of solar radiation becomes necessary when high temperatures are desired, or when, as in the case of photovoltaic cells, the cost of the absorber itself is much higher than the cost of mirrors. The heat losses from a collector are proportional to the absorber area $A_{abs}$ (to a good approximation), and hence inversely proportional to the concentration

$$C = \frac{A}{A_{abs}} = \frac{\text{aperature area}}{\text{absorber area}}. \qquad (1\text{-}1)$$

Of course, elevated temperatures can also be reached by nonconcentrating (flat plate) collectors using selective absorber coatings[10, 11]. But at very high temperatures (above 1000°K) spectral selectivity becomes less useful because of both material degradation and spectral properties. On the other hand, the radiative losses with their steep $T^4$ increase impose the most severe limitations at very high temperature, and they can only be reduced by higher concentration. For all these reasons, the concentration $C$ is one of the most important parameters of a solar concentrator.

Intimately related to the concentration is the acceptance angle, that is the angular range over which radiation is accepted without moving all or part of the collectors. Anyone who has worked with concentrators understands more or less intuitively that high concentrations entail small acceptance angles. The precise connection between these two quantities seems to be less well known, however. As we show in Section II, the second law of thermodynamics implies that the maximum possible concentration for a given acceptance half angle $\theta_c$ is $1/\sin\theta_c$ for two-dimensional (trough-like) concentrators, and $1/\sin^2\theta_c$ for three-dimensional ones.† Stated in terms of the $f$ number (= ratio of focal length/aperture diameter) this means that no optical system can have an $f$ number less than 1/2. This limit plays the same role for radiation concentrators as the Carnot efficiency $(T_1 - T_2)/T_1$ does for heat engines; we suggest therefore the use of the maximum or ideal concentration

$$C_{ideal\ 2\ dim} = \frac{1}{\sin\theta_c} \tag{I-2}$$

and

$$C_{ideal\ 3\ dim} = \frac{1}{\sin^2\theta_c} \tag{I-3}$$

as standard to which any real concentrators can be compared. While most conventional concentrators fall short of this value by a factor of at least two, the above mentioned ideal concentrators actually reach this limit.

Errors in mirror surface or alignment can be characterised by an angle $\Delta$, defined as one-sided average deviation from the perfect value. A ray undergoing $n$ reflections may deviate from the correct direction by as much as $2n\Delta$. If no radiation is to miss the target, the nominal acceptance half angle $\theta_c$ of the collector must be increased by this amount, with a corresponding loss of concentration. Obviously, the effect of mirror errors depends on the relative magnitude of $\Delta$ and the angular width of the source. In practice, the effect of the mirror errors may be somewhat smaller since the errors are likely to be random, resulting in partial cancellations. Detailed analyses of mirror errors, assuming for example a Gaussian distribution, can be found elsewhere[1], but for a first order estimate of the sensitivity to mirror errors it suffices to specify the acceptance angle and the average number of reflections $\langle n\rangle$.

The average number of reflections $\langle n\rangle$ is also needed to assess reflection losses. The fraction of the radiation incident on the aperture which is transmitted to the absorber can be approximated[12] by

$$\tau = \rho^{\langle n\rangle} \tag{I-4}$$

where $\rho$ is the reflectivity of the mirror, typically 0.75–0.95 (this formula is exact for special cases, for example if each ray undergoes one and only one reflection; in general it is exact only to lowest order in $\epsilon = 1-\rho$). Of course, $\tau$ must be multiplied by additional factors as appropriate in case there are transmission losses (e.g. absorption in a glass cover), or if a certain fraction of the radiation misses the target (either by design or by mirror errors).

For a cost analysis, at least one further parameter is needed, the ratio $R$ of reflector area $A_R$ over aperature area $A$,

$$R = \frac{A_R}{A}, \tag{I-5}$$

and we shall evaluate this quantity for various concentrator types. Other aspects, such as ease of fabrication, mechanical stability and magnitude of convective heat losses, also have a bearing on cost and performance. But they are less amenable to simple quantitative evaluation, and we shall touch upon them only when they appear to be particularly important.

As for the organization of this paper, we begin with the relationship between operating temperature, concentration and acceptance angle in Section II. In Section III we consider the aspects of the solar geometry which are relevant for solar energy collection. Section IV describes ideal two-dimensional concentrators, including some configurations not heretofore reported. In Section V we discuss the extension of ideal concentrators to three dimensions and show why there are certain failures. Section VI lists and evaluates the most important conventional concentrator types. In the final section we suggest the use of ideal concentrators as second stage concentrators for conventional optical systems, an arrangement which boosts the concentration to the highest possible values without requiring an unfavorable reflector to aperture ratio.

## II. THE SECOND LAW OF THERMODYNAMICS AND RADIATION CONCENTRATORS

The fundamental problem of radiation concentration can be stated as follows: How can radiation which is uniformly distributed over a range of angles $|\theta| \le |\theta_c|$ and incident on an aperture of area $A$, be concentrated on a smaller absorber area $A_{abs}$ and what is the highest possible concentration

$$C = \frac{A}{A_{abs}}. \tag{II-1}$$

The problem as stated covers not only direct solar radiation which is uniformly distributed over the solar disc with half angle $\delta_s = 47$ mrad $\simeq 1/4°$, but also radiation incident onto a stationary aperture from a moving point source. In the latter case the radiation becomes partially diffuse when averaged over time. This consideration is important for the design of solar concentrators which do not require any tracking during a given period, for example during the course of an entire day.

The relevance of the second law to this problem is obvious since we are dealing with an example of radiative heat transfer between two surfaces, the radiation source and the absorber. Let us first analyze the simple arrangement shown in Fig. 1.

The source is an isotropically radiating sphere of radius $r$, and the aperture of the concentrator has an area $A$ normal to the line from source to aperture and is a distance $R$ away from the center of the source. If we consider the limit $A/R^2 \to 0$ at fixed $\sin\theta_c = r/R$, then the radiation incident on $A$ is precisely of the kind specified

†This result was first derived by Winston[4] using phase space conservation. The proof presented here uses the language of radiation heat transfer which is familiar to engineers; it is more intuitive and rests on the simple fact that exchange factors cannot exceed the value one.

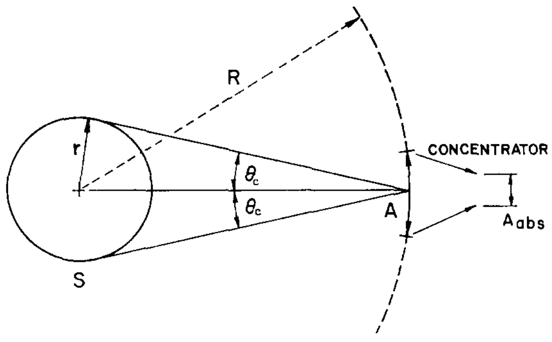



Fig. 1. Radiation transfer from source $S$ through aperture $A$ of concentrator to absorber $A_{abs}$.

above; in other words, it is uniformly distributed over all angles $|\theta| \le |\theta_c|$. For further simplification, we assume the system to be in infinite empty space, or equivalently, enclosed by black walls at absolute zero temperature. If both source and absorber are black bodies at temperatures $T_S$ and $T_{abs}$, respectively, the heat transfer between the two is easy to calculate. The source emits an amount of radiation

$$Q_S = 4\pi r^2 \sigma T_S^4 \tag{II-2}$$

of which a fraction

$$F_{S\to A} = \frac{A}{4\pi R^2}$$

hits the aperture (N.B.: the aperture is flat in the limit $A/R^2 \to 0$). With perfect concentrator optics, no radiation is lost between aperture and absorber, and thus the heat radiated from the source to the absorber is

$$Q_{S\to abs} = Q_S F_{S\to A} = A\frac{r^2}{R^2}\sigma T_S^4. \tag{II-3}$$

The absorber, in turn, radiates an amount

$$Q_{abs} = A_{abs}\sigma T_{abs}^4, \tag{II-4}$$

and the fraction $E_{abs\to S}$ of this radiation which reaches the source cannot exceed unity ($E_{abs\to S}$ is essentially an exchange factor as defined by Sparrow and Cess[13]). Hence the radiative transfer from absorber to source is

$$Q_{abs\to S} = E_{abs\to S} A_{abs}\sigma T_{abs}^4 \tag{II-5}$$

with

$$E_{abs\to S} \le 1. \tag{II-6}$$

By the second law of thermodynamics, there cannot be any net heat transfer between two bodies of equal temperatures; for the present situation this implies

$$Q_{S\to abs} - Q_{abs\to S} = 0 \text{ if } T_{abs} = T_S. \tag{II-7}$$

Combining eqns (II-3), (II-5) and (II-7), we obtain the relation

$$A\frac{r^2}{R^2} = E_{abs\to S}A_{abs}, \tag{II-8}$$

from which we can read off the concentration as

$$C = \frac{A}{A_{abs}} = \frac{R^2}{r^2}E_{abs\to S} = \frac{E_{abs\to S}}{\sin^2\theta_c}. \tag{II-9}$$

In view of the obvious constraint $E_{abs\to S} \le 1$, eqn (II-6), we conclude that the concentration must satisfy

$$C \le \frac{1}{\sin^2\theta_c}. \tag{II-10}$$

Even though we have derived this result for a particular geometry, it is completely general. For, suppose a different arrangement were used to produce the specified radiation, and an optical system were found with a concentration greater than allowed by eqn (II-10). Since aperture and angular width are the only relevant parameters, such a concentrator could also be used in the geometry of Fig. 1 and hence the second law of thermodynamics would be violated.

So far we have assumed the absorber to be surrounded by vacuum. Now, suppose the absorber is covered by a parallel slab of a transparent medium with index of refraction $n$. If the radiation incident on the slab is completely diffuse, then inside the slab it will be restricted to angles $|\theta| \le |\theta_n|$, where $\sin\theta_n = 1/n$ by Snell's law. Thus, further concentration by a factor $1/(\sin^2\theta_n) = n^2$ is allowed by eqn (II-10), and the total concentration is bounded only by

$$C \le \frac{n^2}{\sin^2\theta_c}. \tag{II-11}$$

There is no conflict, however, between this value and the second law because an emitter in a medium of index $n$ radiates $n^2$ as much energy as an emitter in vacuum, a fact evidenced by the formula for the Stephan–Boltzmann constant[14]

$$\sigma = \frac{2\pi^5}{15}\frac{n^2k^4}{c_0^2h^3}, \tag{II-12}$$

with $k$ = Boltzmann constant, $h$ = Planck's constant, and $c_0$ = velocity of light in vacuum. As an immediate corollary we learn that the increase in concentration brought about by a medium of index $n > 1$ does not reduce radiative losses (unless the medium has low thermal conductivity and is opaque to infrared). In general, the use of a medium with $n > 1$ for the purpose of increasing concentration is advisable only when high cost demands that the absorber area be as small as possible. This is relevant for photo cells, and in Section IV we shall discuss a design suitable for that application.

It seems appropriate to define *ideal concentration* as the maximum concentration permitted by the second law of thermodynamics, and to use this as a standard of comparison for real concentrators. This is strictly analogous to the use of the Carnot efficiency

$$\eta_c = \frac{T_1 - T_2}{T_1} \tag{II-13}$$

in discussing heat engines. The concentration provided by any three-dimensional optical system must satisfy

$$C_{3_{dim}} \leq C_{3_{dim\ ideal}} = \frac{n^2}{\sin^2 \theta_c}, \quad \text{(II-14)}$$

where $n$ is the index of refraction of the medium surrounding the absorber. Had we considered two-dimensional systems (e.g. parabolic troughs), we would have found

$$C_{2_{dim}} \leq C_{2_{dim\ ideal}} = \frac{n}{\sin \theta_c}. \quad \text{(II-15)}$$

For imaging instruments, such as lenses, the light concentration properties are sometimes stated in terms of the $f$-number, defined as ratio of focal length $F$ and aperture diameter $D$,

$$f\text{-number} = \frac{F}{D}. \quad \text{(II-16)}$$

The image diameter $a = 2F \sin \theta_c$ determines the size of the absorber area, giving a concentration

$$C = \left(\frac{D}{a}\right)^2 = \left(\frac{D}{2F \sin \theta_c}\right)^2 = \left(\frac{1}{2f\text{-number}}\right)^2 C_{ideal}. \quad \text{(II-17)}$$

We now see that the second law of thermodynamics implies a lower limit

$$f\text{-number} \geq \frac{1}{2} \quad \text{(II-18)}$$

for the $f$-number of any imaging system.

Solar collectors which are to require little or no tracking must have a fairly large acceptance angle (see Section III), and thus they can collect a significant amount of diffuse radiation. The fraction of totally diffuse radiation accepted can easily be calculated by considering the radiation balance between absorber and aperture. The net radiation transfer between absorber and aperture is

$$Q_{A\to abs} - Q_{abs\to A} = A_A E_{A\to abs} \sigma T_A^4 - A_{abs} E_{abs\to A} \sigma T_{abs}^4 \quad \text{(II-19)}$$

where the subscripts $A$ and $abs$ refer to aperture and absorber, respectively. $E_{1\to 2}$, (essentially an exchange factor[13]), is defined as the fraction of diffuse radiation emitted by surface 1 which reaches surface 2. The relation (reciprocity relation)

$$A_A E_{A\to abs} = A_{abs} E_{abs\to A} \quad \text{(II-20)}$$

follows because no heat can be transferred at equal temperatures. Since (for any reasonable concentrator) $E_{abs\to A} = 1$ apart from absorptive losses, we conclude that the acceptance for totally diffuse radiation is

$$E_{A\to abs} = 1/C, \quad \text{(II-21)}$$

independent of any details of the concentrator (this result can also be derived from the principle of phase-space conservation).

To conclude this section, we calculate the temperature at which a solar collector can operate as a function of concentration. Since we want to illustrate only certain gross features, we make some simplifying assumptions. To avoid the complication of multiple reflections, we assume the sun and the rest of the universe to be black bodies, the sun at a temperature $T_s$, the rest of the universe (other than the sun and the absorber), collectively called ambient, at $T_{amb} = 0$. The radiation emitted by the sun and absorbed by the absorber of the collector is, by eqn (II-3) with $\sin \theta_c = \sin \delta_s = r/R$,

$$Q_{s\to abs} = \tau \alpha_{abs,\,sol} A \sin^2 \delta_s \sigma T_s^4, \quad \text{(II-22)}$$

where $\tau = 1 -$ losses due to absorption, reflection, etc. and $\alpha_{abs,\,sol} =$ absorptivity of absorber for solar radiation and $\delta_s =$ angular half width of sun $\approx 1/4°$. The radiation losses from the absorber are

$$Q_{abs,\,rad} = \epsilon_{abs,\,IR} A_{abs} \sigma T_{abs}^4, \quad \text{(II-23)}$$

where $\epsilon_{abs,\,IR}$ is the emissivity of the absorber in the IR region corresponding to $T_{abs}$. If a fraction $\eta$ of the incoming solar radiation $Q_{s\to abs}$ is extracted as useful heat and/or lost by convection or conduction, then the energy balance of the absorber reads

$$Q_{s\to abs} = Q_{abs,\,rad} + \eta Q_{s\to abs}, \quad \text{(II-24)}$$

or

$$(1-\eta)\tau \alpha_{abs,\,sol} A \sin^2 \delta_s T_s^4 = \epsilon_{abs,\,IR} A_{abs} T_{abs}^4 \quad \text{(II-25)}$$

(If $T_{amb} \neq 0$, an additional term $\tau(1 - C/C_{ideal})\epsilon_{abs,\,IR} A_{abs} T_{amb}^4$ will appear on the left-hand side of this equation).

Inserting the actual concentration $C = A/A_{abs}$ and the ideal concentration $C_{ideal} = 1/\sin^2 \delta_s$, we find

$$T_{abs} = T_s[(1-\eta)\tau(\alpha_{abs,\,sol}/\epsilon_{abs,\,IR}) C/C_{ideal}]^{1/4}. \quad \text{(II-26)}$$

(Of course, $\alpha_{abs,\,sol}$ and $\epsilon_{abs,\,IR}$ became equal as $T_{abs}$ approaches $T_s$). As expected, the highest possible absorber temperature is $T_{abs\,max} = T_s = 6000°K$, and can only be reached if no heat or radiation is lost by absorption, convection, etc. and if the concentration equals $C_{ideal\,sun} = 1/\sin^2 \delta_s \simeq 45{,}000$. For two-dimensional concentrators $C_{ideal\,2\,dim\,sun} = 1/\sin \delta_s \simeq 213$ and the absorber temperature is limited by $T_s 213^{-1/4} \simeq 1570°K$ (assuming $\alpha_{abs,\,sol} = \epsilon_{abs,\,IR}$).

## III. SOLAR GEOMETRY AND TRACKING REQUIREMENTS

To understand the tracking requirements for solar collectors, let us study the geometry shown in Fig. 2. The solar inclination $\alpha$ from the equatorial plane is given by

$$\sin \alpha = -\sin E \cos(\Omega D) \quad \text{(III-1)}$$

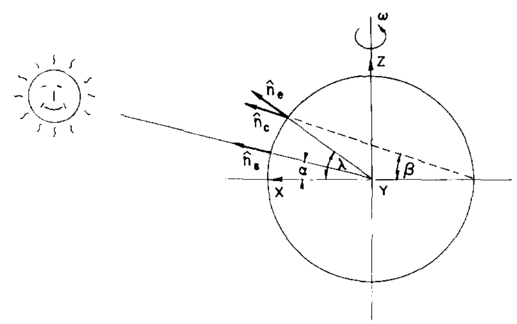



Fig. 2. Solar geometry. $x$-axis points towards solar noon; $y$-axis points east; $z$-axis = axis of rotation; $\alpha$ = solar inclination; $\lambda$ = latitude; $\beta$ = collector tilt from equatorial plane; $\hat{n}_c$ = unit vector normal to collector (assume collector runs east–west); $\hat{n}_e$ = unit vector normal to earth's surface; $\hat{n}_s$ = unit vector in direction of sun.

with

$$E = 23°27',$$

$$\Omega = 2\pi/(365.25\text{ days})$$

$D$ = time after winter solstice, in days

(assuming circular orbit).

Let $\hat{n}_s$ be the unit vector from earth to sun and $\hat{n}_c$ the unit vector normal to the collector aperture, the collector tilt $\beta$ being measured with respect to the equatorial plane. The collector is assumed to be east–west symmetric. One would like to know the angle of incidence $\theta$ of solar radiation on the collector, as well as the change in solar elevation $\Delta\theta_v$ ("vertical solar swing") during the day and during the year. Of course these angles are independent of the earth's radius (since the sun can be considered to be infinitely far away, for this discussion), and thus it is easy to see that they must also be independent of the latitude at which the collector is located. The $(x, y, z)$ coordinate system of Fig. 2 is fixed in the earth, with the $z$-axis as axis of rotation and the $x$-axis pointing towards the sun at noon. In these coordinates the unit vectors $\hat{n}_c$ and $\hat{n}_s$ take the form

$$\hat{n}_c = (\cos\beta, 0, \sin\beta) \qquad \text{(III-2)}$$

and

$$\hat{n}_s = (\cos\alpha\cos\omega t, -\cos\alpha\sin\omega t, \sin\alpha) \qquad \text{(III-3)}$$

with

$$\omega = 2\pi/(24\text{ hr})$$

and

$t$ = time after noon, in hr.

This yields the angle of incidence $\theta$ as

$$\cos\theta = \hat{n}_s \cdot \hat{n}_c = \cos\beta\cos\alpha\cos\omega t + \sin\beta\sin\alpha. \qquad \text{(III-4)}$$

In order to find the solar elevation, we consider the projection

$$\mathbf{n}_{sv} = (\cos\alpha\cos\omega t, 0, \sin\alpha) \qquad \text{(III-5)}$$

of $\hat{n}_s$ on the $y = 0$ plane, i.e. the plane spanned by the sun at noon and by the axis of rotation. The solar elevation $\theta_v$ from the equatorial plane is found by dotting the unit vector

$$\hat{n}_{sv} = (\cos\alpha\cos\omega t, 0, \sin\alpha)(1 - \cos^2\alpha\sin^2\omega t)^{1/2} \qquad \text{(III-6)}$$

into the unit vector (1, 0, 0) along the $x$-axis. The result

$$\cos\theta_v = \cos\alpha\cos\omega t(1 - \cos^2\alpha\sin^2\omega t)^{1/2}$$

can be written in the form[15]

$$\tan\theta_v = \frac{\tan\alpha}{\cos\omega t}. \qquad \text{(III-7)}$$

The extreme values of $\theta_v$ occur at solstice, and they are plotted vs time of day in Fig. 3.

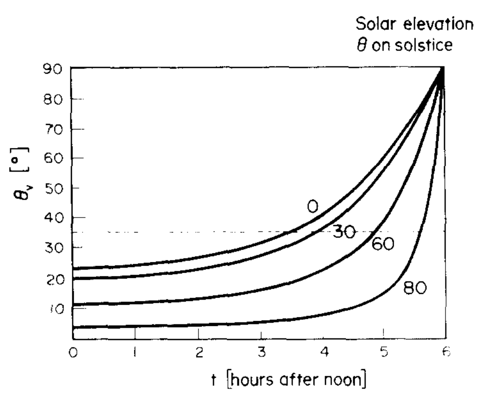



Fig. 3. Solar elevation $\theta_v$ (relative to equatorial plane) 0 days, 30 days, 60 days and 80 days from solstice, as labeled by the number next to the curves. To illustrate the use of this graph the dotted line has been added corresponding to the acceptance half angle of a stationary collector (aligned east–west) which can collect direct sunlight for at least 7 hr a day.

In order to find the vertical solar swing $\Delta\theta_v$ during the central $t$ hours of the day, one calculates $\theta_v$ from eqn (III-7) and subtracts the noon elevation $\alpha$. The largest daily swing occurs at solstice, both summer and winter, and hence we can obtain from Fig. 3 the acceptance angle which a collector must have in order to catch the sun for a specified minimum number of hours per day without any diurnal tracking. The acceptance should, of course, be even larger to accommodate the finite size of the sun (angular radius $\delta_s = 4.7$ mrad) as well as mirror inaccuracies. The latter can be characterized by an angle $\delta_m$, the amount by which the acceptance half angle will be smeared out; neglecting position errors, $\delta_m$ is twice the maximum slope error if one insists that no radiation miss the absorber and if $n = 1$.

Hence, the maximum concentration for a two-dimensional solar concentrator (aligned along the East–West direction) is

$$C_{2\,\text{dim, ideal}} = \frac{1}{\sin(1/2\Delta\theta_v + \delta_s + \delta_m)}, \qquad \text{(III-8)}$$

and this is actually achieved by the ideal concentrators described in the next section. Figure 4 shows the highest possible concentration vs collection time at solstice for a

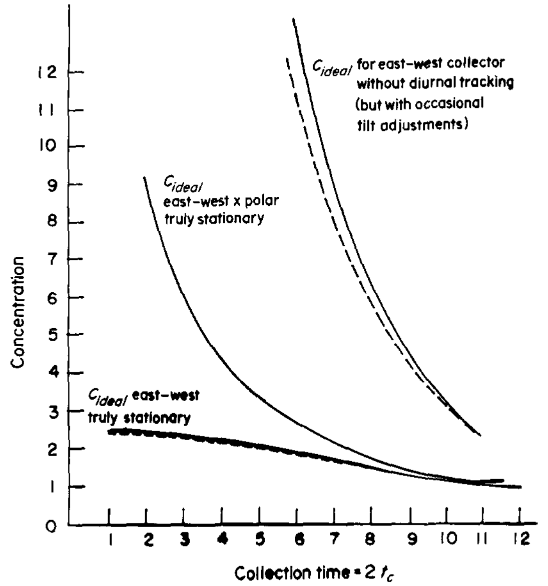



Fig. 4. Maximum possible concentration for non-tracking solar collectors, as function of minimum collection time $2t_c$. The finite size of the sun is included, with $\delta_s \simeq 1/4°$. The solid lines correspond to perfect mirrors, and the dashed lines to realistic mirrors with $\delta_s + \delta_m = 1°$. The values of $C_{ideal}$ for a combined east-west = polar concentrator can probably not be reached in practice.

collector without diurnal tracking and for a truly stationary collector. Both perfect mirrors, $\delta_m = 0$, and realistic mirrors, $\delta_s + \delta_m = 1°$, are considered; values for larger errors can be extrapolated from these curves.

Some additional concentration may be gained because the azimuthal swing of the sun is less than 180° during the useful collection time. If the cutoff time is $t_c$ hours after noon, then a second concentrator, oriented along the polar axis, could in principle boost the concentration by an additional factor

$$C_{polar} = \frac{1}{\sin(\omega t_c + \delta_s + \delta_m)}. \tag{III-9}$$

Unfortunately, this may not be possible to achieve in practice. For example, a set of two crossed compound parabolic concentrators (one east-west, one polar) will reject certain rays, even though they arrive within the solid angle defined by $\theta_{east-west}$ and $\theta_{polar}$. This is due to differences between two-dimensional and three-dimensional reflectors, as discussed in Section V.

We see from Fig. 3 that a collector with an acceptance half-angle below approximately 40° needs occasional tilt adjustments. To find out when adjustments are necessary, we calculate the solar elevation $\theta_{vc}$ relative to the collector. More precisely, $\theta_{vc}$ is measured from the plane spanned by the collector normal and the east-west direction, and hence it is given by $\theta_v$ of eqn (III-7), apart from a shift by the collector tilt $\beta$,

$$\theta_{vc} = \theta_v - \beta. \tag{III-10}$$

This fact allows us to rewrite eqn (III-7) as

$$\tan(\theta_{vc} + \beta) = \frac{\tan\alpha}{\cos\omega t} \tag{III-11}$$

(Of course, this relation can also be obtained from $\cos\theta_{vc} = \hat{n}_c \cdot \hat{n}_{sv}$).

In Table 1 we list the number of tilt changes necessary for a collector with acceptance half-angle $\theta_c$, based on a point like sun. For a real collector the concentration can be obtained from $\theta_c$ by the formula

$$C = \frac{C_0}{\sin(\theta_c + \delta_s + \delta_m)} \tag{III-12}$$

where $C_0 \leq 1$ is a number [1/(2$f$-number)] which depends on the concentrator type and states by how much the concentration falls short of the ideal limit. The entries in Table 1 are computed according to the following procedure: on summer solstice the collector normal is pointed at an angle $\theta_c$ above the solar noon elevation of 23°27′. It is left in this position until the day when the collection time falls below the specified minimum, at which time the collector normal is again adjusted to an angle $\theta_c$ above solar noon elevation, etc. For a truly

Table 1. Adjustments of collector tilt. Require minimum collection time 7 hr/day (except for $\theta_c = 5.5°$ min. collect. time = 6.78 hr/day)

| Accept. Half Angle $\theta_c$ (ideal concentration for perfect mirrors and point sun) | Collection Time Aver. Over Year (hours/day) | Number of Adjust./Year | Shortest Period w/o Adjustment | Average Collection Time if Tilt Adjusted Every Day (hours/day) |
|---|---|---|---|---|
| 19.5° (3.0) | 9.22 | 2 | 180 days | 10.72 |
| 14° (4.13) | 8.76 | 4 | 35 days | 10.04 |
| 11° (5.24) | 8.60 | 6 | 35 days | 9.52 |
| 9° (6.39) | 8.38 | 10 | 24 days | 9.08 |
| 8° (7.19) | 8.22 | 14 | 16 days | 8.82 |
| 7° (8.21) | 8.04 | 20 | 13 days | 8.54 |
| 6.5° (8.83) | 7.96 | 26 | 9 days | 8.36 |
| 6° (9.57) | 7.78 | 80 | 1 day | 8.18 |
| 5.5° (10.43) | 7.60 | 84 | 1 day | 8.00 |

stationary collector, the limit of useful concentration is about two; for photovoltaic applications, this can be increased to about four by means of a dielectric medium with $n = 1.5$.

### IV. IDEAL CONCENTRATORS IN TWO DIMENSIONS

Following Winston[4], we call those concentrators which actually reach the ideal limit ($1/\sin\theta_c$ in two, $1/\sin^2\theta_c$ in three dimensions) ideal concentrators. (Of course, this name refers only to their optical properties, without any regards for practical matters such as economics). In this section, we consider only two-dimensional concentrators, also called cylindrical or trough-like.

In 1965 Winston and Hinterberger[3] discovered an example of this class, called compound parabolic concentrator (CPC) and shown in Fig. 5. It consists of parabolic reflectors which funnel the radiation from aperture to absorber. The right and the left half belong to different parabolas, as expressed by the name CPC. The axis of the right branch, for instance, makes an angle $\theta_c$ with the collector midplane, and its focus is at $A$. At the end points $C$ and $D$, the slope is parallel to the collector midplane.

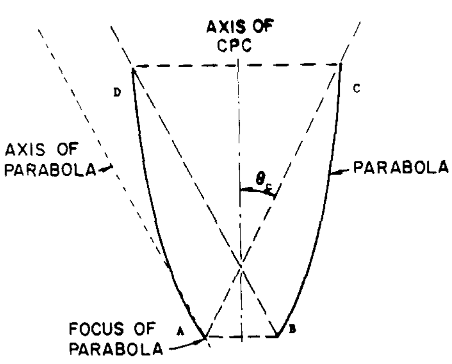



Fig. 5. Compound parabolic concentrator.

It is easy to show that this instrument has a concentration of

$$C_{ideal,2\,dim} = \frac{1}{\sin\theta_c}$$

Recalling eqn (II-9), we see that a concentrator is ideal if and only if the exchange factor $E_{abs\to source}$ for radiation going from absorber to the source is

$$E_{abs\to source}\big|_{ideal} = 1. \qquad \text{(IV-1)}$$

In other words, all radiation emitted by the absorber must get to the source, which was specified to cover an angular region $|\theta| < \theta_c$. This is equivalent to the requirement that all rays incident on the aperture inside the acceptance angle and none of the rays outside the acceptance angle pass to the absorber (this property is plotted in Fig. 6). By tracing rays emanating from the absorber, in particular from its end points $A$ and $B$, one learns that indeed no radiation from the absorber can leave the CPC outside its acceptance angle.

The CPC is not an imaging instrument, by contrast to a simple parabola. The flux distribution at the absorber can easily be found for certain special cases: rays incident at $\theta = +\theta_c(-\theta_c)$ will be brought to focus at $B(A)$, while radiation which is uniformly spread over all angles $|\theta| < \theta_c$ will be totally diffuse when it gets to the absorber. For other cases, the radiation pattern at the absorber is more complicated and has to be determined by detailed ray tracing.

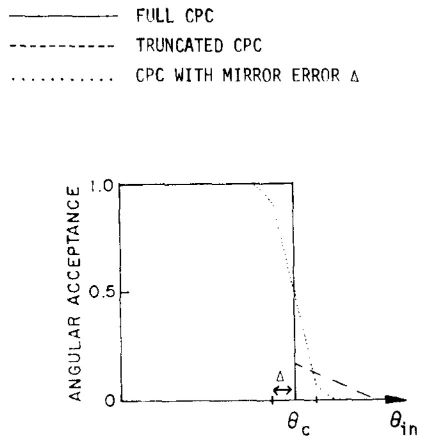



Fig. 6. Fraction of the radiation incident on aperture at angle $\theta_{in}$ which reaches absorber, for ideal concentrator in two dimensions, with acceptance half angle $\theta_c$, assuming reflectivity $\rho = 1$. ———, untruncated ideal concentrator with perfect reflectors; ------, truncated ideal concentrator with perfect reflectors; ········, untruncated ideal concentrator with surface errors $\Delta$.

Conventional imaging instruments such as Fresnel lenses or mirrors tend to fall short of the ideal concentration by a factor of two to four. However, a price must be paid for the high performance of ideal concentrators: their mirror area is rather large. Fortunately, this disadvantage can be alleviated by *truncation*. The top portion of the reflectors in Fig. 5 does not intercept much radiation, and it can be cut off without much loss in concentration. We have studied this question in another paper, and present only some results in Figs. 7 and 8. Figure 7 is a graph of reflector/aperture ratio $A_R/A$ vs concentration for various acceptance angles, both for full and for truncated CPC's. For example, a full CPC with an acceptance half-angle of 6° concentrates by a factor of 9.6 and requires a total of 10.6 m² of reflector area for each m² of aperture. When the $A_R/A$ ratio is reduced to 5, the concentration is still equal to 8.2.

The number of reflections varies both with angle of incidence $\theta_{in}$ and with point of incidence on the aperture. For solar applications one needs $\langle n(\theta_{in})\rangle$, the average over all incidence points at angle $\theta_{in}$, as well as $\langle n\rangle$, the average of $\langle n(\theta_{in})\rangle$ over all $\theta_{in}$ within the acceptance angle. Figure 8 shows $\langle n\rangle$ along with the high and low values of $\langle n(\theta_{in})\rangle$ for several acceptance angles (reflector profiles) as a function of concentration (truncation). The variation of $\langle n(\theta_{in})\rangle$ with $\theta_{in}$ decreases with truncation. This feature is important because small variation is desirable for the sake of uniform collector output.

If a CPC is truncated, some rays outside the acceptance angle ($|\theta_{in}| > \theta_c$) can reach the absorber, while of course no rays with $|\theta_{in}| > \theta_c$ are rejected. The resulting increase in angular acceptance is, however, insignificant in most

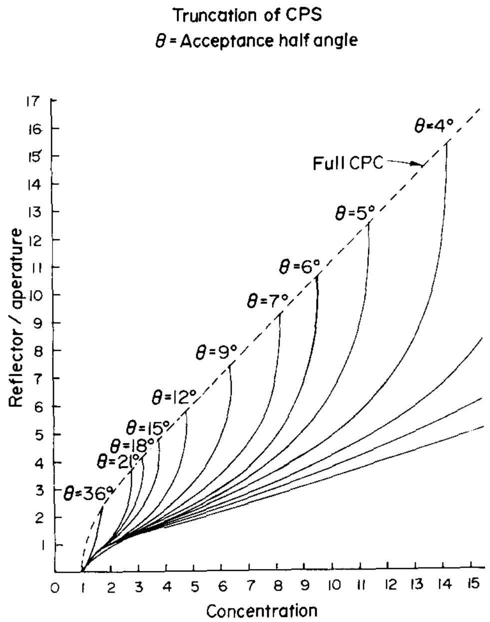



Fig. 7. Reflector/aperture ratio as function of concentration for full and for truncated CPC's.

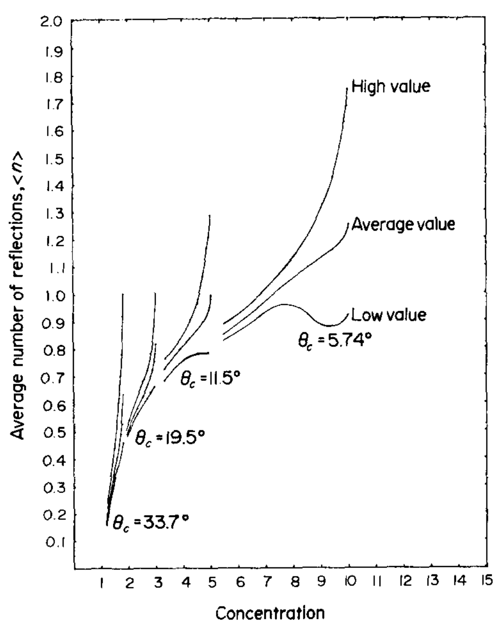



Fig. 8. Number of reflections for full and for truncated CPC, computed by ray tracing. The average over all points of impact was taken at each angle of incidence $\theta_{in}$ in order to find $\langle n(\theta_{in})\rangle$. For each of the acceptance half angle $\theta_c$ in this graph, the high and low values of $\langle n(\theta_{in})\rangle$ are shown in addition to the average $\langle n\rangle$ over all $|\theta_{in}| < \theta_c$. For example, if a CPC with $\theta = 11.5°$ is truncated to a concentration of 4, the average number of reflections ranges from a low of 0.76 to a high of 0.86 with a mean of 0.82.

practical applications, as shown in Fig. 6. For example, if direct sunlight enters a truncated CPC of concentration $\bar{C}$, with $\theta_{in} > \theta_c$, the fraction of radiation reaching the absorber is less than $1/\bar{C}$; under these conditions the collector is useless for thermal, marginal for photovoltaic applications. The fraction of diffuse radiation which is accepted is of course $1/\bar{C}$, independent of any details of the concentrator, as shown in Section II.

The suitability of the CPC concept for solar applications is under investigation at Argonne National Laboratory[6]. Several different design variations are being constructed and tested, including single large CPC units as well as panels containing many small CPC's (to reduce edge losses).

Before proceeding to different concentrator types, we mention some reflector configurations which transport radiation from one place to another, with no change in concentration. Consider, for instance, the CPC (in Fig. 9a) which, for solar applications in midlatitudes, will be tilted at an angle between 10° and 80°. Calculations[7] indicate that in CPC solar collectors convective and radiative heat losses are comparable. Radiative losses can be reduced by selective absorber coatings, but this will not improve the performance greatly unless convection losses are suppressed at the same time. The latter can be accomplished either by evacuation (impractical unless absorber is placed inside evacuated cylindrical glass tube) or by placing the absorber horizontally, facing downward, as shown in Fig. 9.

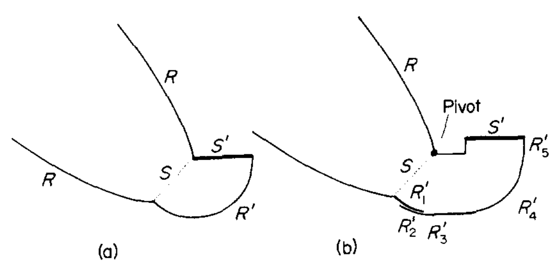



Fig. 9. Convection suppressing cavities for CPC.

The radiation impinging upon surface $S$ in Fig. 9a is piped around the corner to the horizontal surface $S'$ by a cylindrical reflector $R'$. Obviously, there is no change in concentration, and no radiation is lost apart from absorption by imperfect mirror surfaces. The losses due to absorption are easy to estimate because diffuse radiation passing from $S$ to $S'$ will undergo on the average

$$\langle n \rangle_{cylinder} = \frac{\phi}{2} \tag{IV-2}$$

reflections[12] where $\phi$ is the angle between $S$ and $S'$ in radians. Therefore, the fraction of light transmitted from $S$ to $S'$ is

$$\rho^{\langle n \rangle} = \rho^{\phi/2} \tag{IV-3}$$

where $\rho$ is the reflectivity of the mirror $R'$.

Figure 9b includes some ramifications of this idea. It may be desirable to create a stagnant air layer below the absorber, and this can be accomplished by the parallel reflector section $R'_5$. An additional straight section $R'_3$ may be necessary for mechanical reasons. The average number of reflections for diffuse radiation passing between parallel plates of length $l$ and separation $h$ is given by the formula[12]

$$\langle n \rangle_{parallel\ plate} = \frac{l}{h}. \qquad \text{(IV-4)}$$

and the fraction of radiation transmitted is again approximated by $\rho^{\langle n \rangle}$. Inserting realistic numbers, one finds that the convection suppressing cavity[16] of Fig. 9(b) will cause approximately 1–1.5 extra reflections, corresponding to absorption losses of 10–15 per cent if good reflector materials ($\rho \sim 0.9$) are used.

Further work is needed to decide to what extent this type of cavity is practical. Even if wind is kept out by a cover glass, uneven heating may create small convection cells inside the cavity thus impairing its insulating value. This kind of question has to be settled by experiment.

Since the tilt of the CPC may have to be adjusted periodically, a venetian blind arrangement suggests itself. In Fig. 9(b), the CPC reflector $R$ and the cylindrical section $R_1'$ can pivot around point $P$, with sections $R_1'$ and $R_2'$ sliding past each other.

In 1974 Winston and Hinterberger[17] discovered that the absorber of a two-dimensional ideal concentrator need not be flat and parallel to the aperture. They proved that radiation incident with $|\theta| < \theta_c$ on an aperture of width $l$ can be concentrated onto any convex absorber of circumference $l \sin \theta_c$ (see Fig. 10). Sections $AD$ and $AF$ of the concentrator are convolutes of sections $AC$ and $AB$ of the absorber. For the rest of the concentrator, one demands that at any point $P$ of the reflector the reflector normal bisect the angle between the line $PT$ (= tangent to absorber at $T$) and the ray incident on $P$ at angle $\theta_c$ (with respect to collector axis). With starting point $D(F)$ and slope at each point specified, the entire concentrator section $DE(FG)$ is uniquely determined. Mathematically speaking, there is one boundary condition and a first order differential equation. If the absorber is straight, the reflector consists of parabolic sections, possibly combined with circular sections. For example, in Fig. 11(a), the portions $DE$ and $FG$ are parabolic, while $AD$ and $AF$ are circular. Note that in Fig. 11(b), sections $DD'$ and $D'E$ belong to different parabolas. The reflector shape corresponding to a circular absorber has been calculated in Ref. [18].

The concentrators shown in Fig. 11 may be very attractive for solar energy collection; not only is the absorber material used more efficiently than in other designs, but there are no losses through the back. This may be quite an important advantage because it may be too costly to reduce the effective $U$-value of the back of a collector much below $0.5 \times 10^{-4}$ w/cm$^2$ °K. Compared to frontal $U$-values for CPC type solar collectors[7], approximately $3 \times 10^{-4}$ w/cm$^2$ °K for three-fold and $1.4 \times 10^{-4}$ w/cm$^2$ °K for ten-fold concentration, the losses through the back are indeed significant.

As for the $U$-values of the collectors in Fig. 11, they can be calculated with the formulas derived in [7] and [8]. Differences from the CPC arise (i) from differences (about ±30 per cent) in the free convective heat transfer coefficient, and (ii) from the fact that the shape factor for radiation from absorber to aperture is smaller than for a CPC. The latter point is relevant for mirrors with high absorptivities for low temperature infrared (i.e. second surface mirrors) because then the reflectors can act as a radiation shield, provided of course their backs are thermally insulated. As far as heat losses are concerned, configuration 11(b) appears to be particularly favorable.

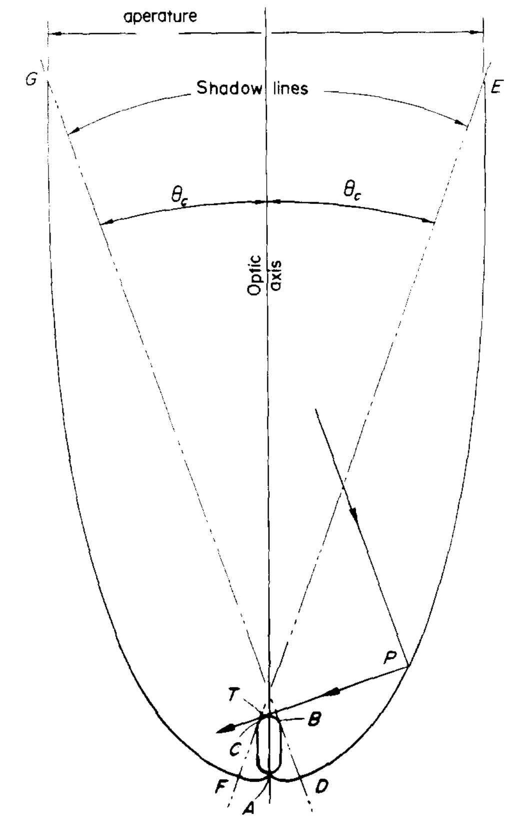



Fig. 10. Ideal cylindrical concentrator with arbitrary absorber shape.

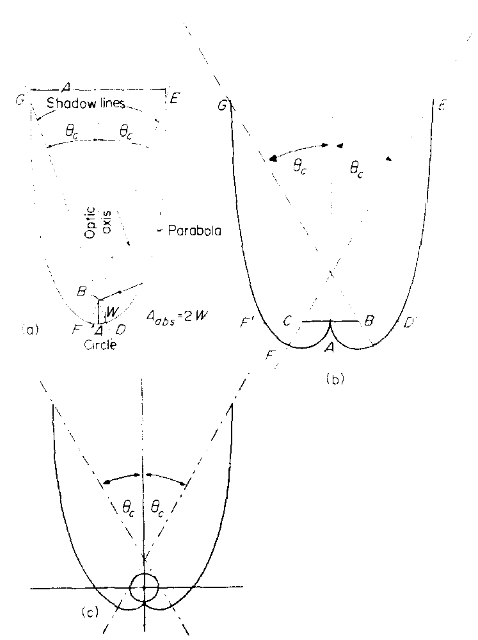



Fig. 11. Examples of ideal cylindrical concentrators.

However, the optical losses have to be considered too, and these are somewhat higher than for a CPC. The average number of reflections can be calculated analytically [12]; for example, for the configuration of Fig. 11(a) it is about 50 per cent higher than for a CPC of comparable concentration.

The concentrators mentioned so far possess uniform concentration for all angles of incidence $|\theta| < \theta_c$ (apart from the usual $\cos\theta$ factor for non-normal incidence). For heating and cooling applications, however, the load varies with the seasons, and a collector with variable output might be more appropriate. The concentrators shown in Fig. 12 do indeed have this property. They consist of a single parabola $CD$ whose axis is parallel to one of the extreme rays and whose focus is at the edge $B$ of the absorber; the parabola concentrates all radiation incident on the aperture $BD$ with $|\theta| < \theta_c$ onto the surface $BC$.

The examples shown here have an acceptance half-angle $\theta_c = 36°$, and thus they are truly stationary with a collection time of at least 7 hr per day. Their concentration is $1.7 = 1/\sin\theta_c$ at normal incidence, but varies from zero (winter for a, summer for b) to 3.4 (summer for a, winter for b), including the $\cos\theta$ factor. This "sea shell" collector is of course constructed according to the same principle as the other ideal concentrators; no radiation emitted by the absorber is allowed to escape from the collector outside its acceptance angle, as is readily verified. The change in concentration can be varied by truncation to fit the demand curve for a particular user. For example, with the truncation point at $T$ in Fig. 12(a) the concentration ranges from 0.7 to 1.7 (again including the cosine factor) with a mean value of 1.5 at $\theta = 0$.

The "sea shell" can be combined with the convection suppressing cavities described earlier, and we have included this feature in Fig. 12. With a selective absorber coating, the collector in Fig. 12(a) may well be suited to drive absorption air conditioners which need temperatures of 100°C or more, a range in which ordinary flat-plate collectors are considered to be inadequate.

The ordinary CPC (Fig. 5) and the "sea shell" (Fig. 12) are special limiting cases of a general class of asymmetric concentrators sketched in Fig. 13. The axis of the left (right) parabola subtends an angle $\phi_l(\phi_r)$ with the absorber normal; $F_l(F_r)$ is the focus of the left (right) parabola. The acceptance angle is $2\theta_c = \phi_l + \phi_r$ and the geometric concentration is $C = 1/\sin\theta_c$. The effective concentration varies from a minimum at $\theta_{in} = \phi_r$ to a maximum at or near $\theta_{in} = \phi_l$, due to the change in projected aperture area (normal to $\theta_{in}$). The ordinary CPC corresponds to $\theta_c = \phi_l = \phi_r$ and the "sea shell" to $2\theta_c = \phi_l$, $\phi_r = 0$.

For photovoltaic applications, the two-stage CPC design [19] of Fig. 14 takes advantage of the extra

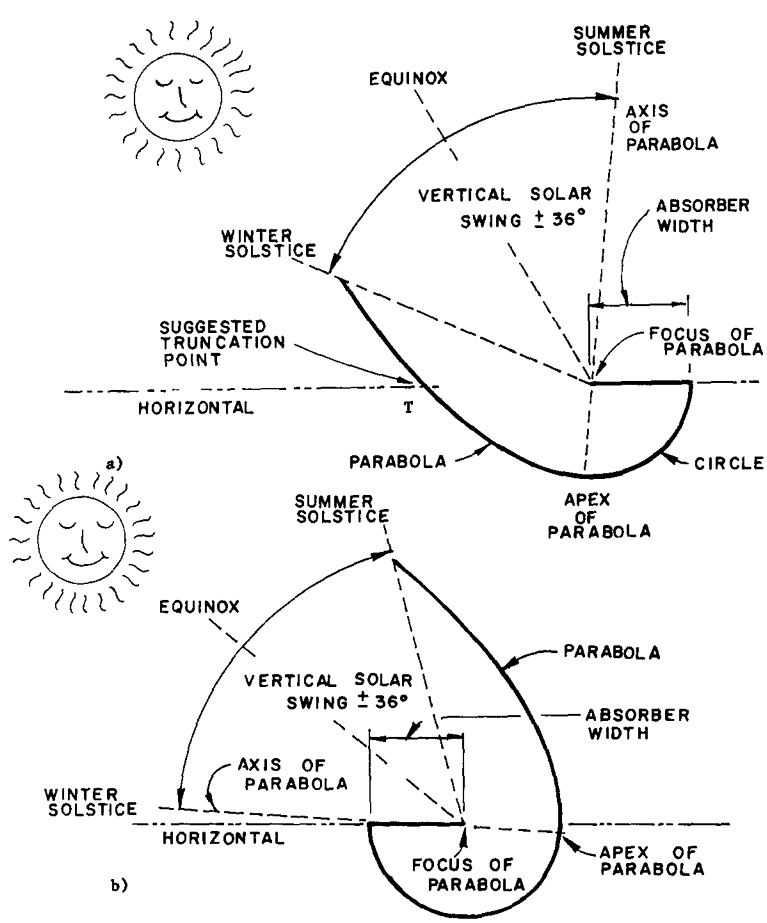



Fig. 12. Stationary "sea shell" collector with variable concentration, with maximal output in summer (a) and winter (b).

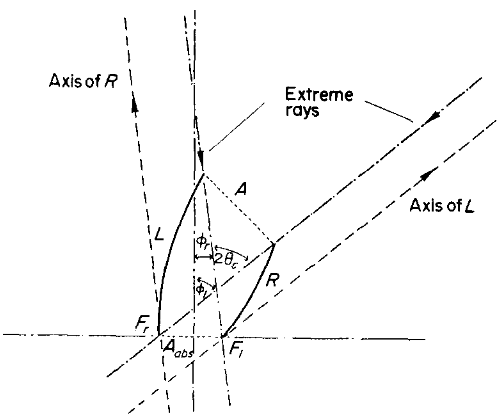



Fig. 13. Asymmetric ideal concentrator with acceptance angle $2\theta_c = \phi_l + \phi_r$ and geometric concentration $C = 1/\sin\theta_c$. The effective concentration varies with angle of incidence. $A$ = aperture, $A_{abs}$ = absorber, $R$ = right parabola, $L$ = left parabola, $F_r$ = focus of $R$, $F_l$ = focus of $L$.

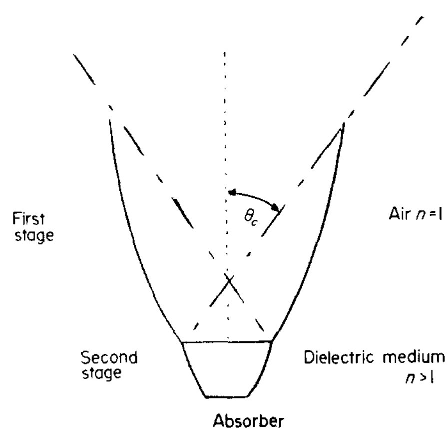



Fig. 14. Two stage CPC suitable for photovoltaic applications. The first stage concentrates by $1/\sin\theta_c$, the second by $n$.

concentration allowed by a dielectric medium with index of refraction $n > 1$ (two CPC's in tandem require less material than a single CPC filled entirely with the medium). If the medium is a liquid such as water or oil, the solar cells can be cooled while collecting useful low-grade heat. In fact, the relative proportions of electric and thermal energy match the demands of a typical residence. This approach is particularly attractive if used as terminal concentrators (see Section VII) for a conventional Fresnel mirror, because then the higher concentration necessitates efficient cooling and the extra cost of the second CPC plus liquid is only a small fraction of the total.

As for the sensitivity to mirror surface errors, the analysis is equally simple for all ideal concentrators considered here because their geometry implies that all rays incident near the cutoff angle, i.e. with $|\theta_{in}| \lessapprox \theta_c$, undergo exactly one reflection on their way to the absorber. In almost all practical applications, the acceptance half angle $\theta_c$ will be larger than 5°, and it is reasonable to assume that the mirror surface errors $\Delta$ will be fairly small compared to $\theta_c$. Therefore, all of the rays with $|\theta_{in}| < \theta_c - \Delta$ and none of the rays with $|\theta_{in}| > \theta_c + \Delta$ will reach the absorber, while in the transition region $\theta_c - \Delta < |\theta_{in}| < \theta_c + \Delta$ some rays are accepted and some are rejected. The resulting angular acceptance is shown schematically by the dotted line in Fig. 6. (The equality of the areas under the straight and dotted lines—weighted by appropriate cosine factors—follows from eqn II-21).

### V. RAY TRACING IN THREE DIMENSIONS AND FAILURE OF IDEAL CONCENTRATORS

When tracing rays in three-dimensional concentrators it is not sufficient to consider only rays which lie in a plane of symmetry. Nonplanar rays can show quite a different behavior, and they are an essential complication. As an illustration, we take the extension of the two-dimensional CPC to three dimensions. One might expect that the corresponding figure of revolution, a cone with compound parabolic profile, would act like an ideal three-dimensional concentrator with concentration $1/\sin^2\theta_c$. This turns out to be almost, but not quite, correct. Figure 15 shows how much of the radiation incident on the aperture, at angle $\theta$, is actually transmitted to the absorber even if the mirrors are perfect; this graph has been obtained by a Monte Carlo technique[4]. Some rays within the nominal acceptance half-angle $\theta_c$ are rejected while some rays with $|\theta| > \theta_c$ do get to the absorber. The transition region $\theta_t$ is small compared to $\theta_c$, a typical example being $\theta_t \approx 1°$ for $\theta_c = 16°$. Therefore, if all radiation within a specified cone of half-angle $\theta_c$ is to reach the absorber of a CPC cone, its profile must be chosen according to a nominal acceptance half-angle $\theta'_c = \theta_c + 1/2\theta_t$; this results in an actual concentration which is 5–10 per cent below the ideal limit.

Before proceeding further, it is appropriate to state the law of specular reflection in vector notation. Define the unit vectors

$\hat{\imath}$ = direction of incident ray

$\hat{n}$ = direction of normal of reflector surface

$\hat{r}$ = direction of reflected ray

all three pointing away from the surface. The law of specular reflection states that

(i) angle of incidence = angle of reflection,

$$\hat{\imath}\cdot\hat{n} = \hat{r}\cdot\hat{n} \qquad \text{(V-1)}$$

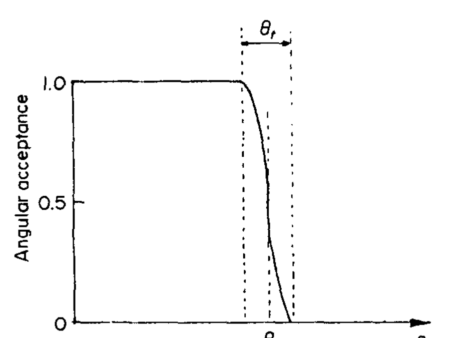



Fig. 15. Fraction of the radiation incident on aperture an angle $\theta_{in}$ which reaches absorber of compound parabolic cone (three dimensional CPC) with nominal acceptance angle $\theta_c$. Perfect reflectors with $\rho = 1$ are assumed.

and

(ii) $\hat{\imath}$, $\hat{n}$ and $\hat{r}$ lie in the same plane

$$(\hat{\imath} \times \hat{r}) \cdot \hat{n} = 0. \qquad \text{(V-2)}$$

Given any two of these vectors, the third one is uniquely determined by eqns (V-1) and (V-2), apart from trivial minus signs. If $\hat{\imath}$ and $\hat{r}$ are specified, $\hat{n}$ must be chosen orthogonal to $\hat{\imath} \times \hat{r}$ and to $\hat{\imath} - \hat{r}$; hence, it must have the direction of $(\hat{\imath} \times \hat{r}) \times (\hat{\imath} - \hat{r})$, and it turns out to be

$$\hat{n} = (\hat{\imath} + \hat{r})(2 + 2\hat{\imath} \cdot \hat{r})^{-1/2}. \qquad \text{(V-3)}$$

On the other hand, with $\hat{\imath}$ and $\hat{n}$ given, $\hat{r}$ must have the form $a\hat{\imath} + b\hat{n}$ with $a$ and $b$ fixed by $\hat{r}^2 = 1$ and $\hat{\imath} \cdot \hat{n} = \hat{r} \cdot \hat{n}$; the result is

$$\hat{r} = -\hat{\imath} + 2(\hat{\imath} \cdot \hat{n})\hat{n}. \qquad \text{(V-4)}$$

In order to see under what conditions and to what extent a three-dimensional problem can be treated as a two-dimensional one, consider the projections of $\hat{\imath}$, $\hat{n}$ and $\hat{r}$ onto a plane, say the $(x, y)$ plane. The projected angles of incidence $\alpha_{i,xy}$ and $\alpha_{r,xy}$ are given by

$$\cos \alpha_{i,xy} = i_{xy} \cdot n_{xy} = i_x n_x + i_y n_y$$
$$= \hat{\imath} \cdot \hat{n} - i_z n_z$$

and

$$\cos \alpha_{r,xy} = r_{xy} \cdot n_{xy} = r_x n_x + r_y n_y$$
$$= \hat{r} \cdot \hat{n} - r_z n_z,$$

and they are equal if, and only if, $i_z n_z = r_z n_z$. This is obviously satisfied if $\hat{n}$ lies in the projection plane. Thus, in any trough-like concentrator aligned along the $z$-axis all incident rays with the same $(x, y)$ projection (plane of the paper in Fig. 16) are represented by the same two-dimensional ray tracing diagram, no matter how large their elevation from the $(x, y)$ plane.

Therefore, rays with the same $(x, y)$ components but different $z$ components need not be traced separately. Suppose a planar ray entering with $\hat{\imath} = (i_x, i_y, 0)$ has been found to leave in the direction $\hat{s} = (s_x, s_y, 0)$. Then a ray entering with

$$\hat{\imath}' = (i_x\sqrt{1 - (i_z')^2},\, i_y\sqrt{1 - (i_z')^2},\, i_z'), \qquad \text{(V-5)}$$

$i_z'$ arbitrary, has the same $(x, y)$ projection and leaves with

$$\hat{s}' = (s_x\sqrt{1 - (i_z')^2},\, s_y\sqrt{1 - (i_z')^2},\, i_z'), \qquad \text{(V-6)}$$

no matter how many reflections have occurred. (The equality of $i_z'$ and $s_z'$ follows from eqn V-4).

For nonplanar rays the elevation from the $(x, y)$ plane changes with each reflection. Let $\hat{\imath} = (i_x, i_y, i_z)$ be an arbitrary incident ray, and write the normal $\hat{n}$ of the reflecting surface as

$$\hat{n} = (\cos \gamma, \sin \gamma, 0);$$

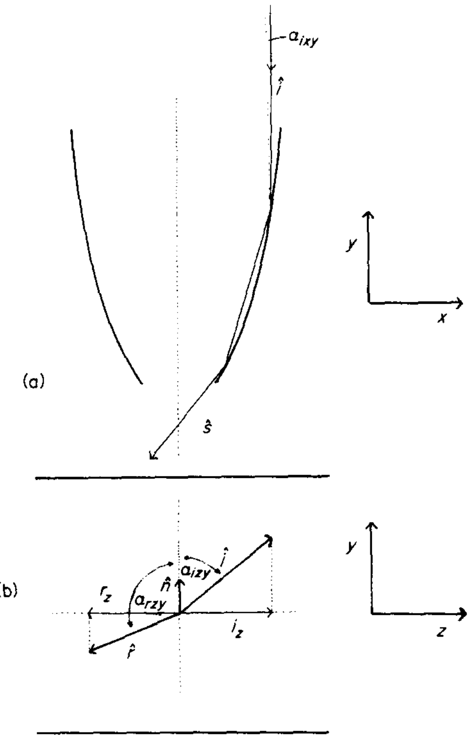



Fig. 16. Reflection in troughlike (along $z$-axis) concentrator. Projection on $(x, y)$ plane $(a)$, and on $(y, z)$ plane $(b)$. The $(x, y)$ projection is independent of the elevation from the $(x, y)$ plane.

then the reflected ray has the direction

$$\hat{r} = ([i_x \cos 2\gamma + i_y \sin 2\gamma], [i_x \sin 2\gamma - i_y \cos 2\gamma], -i_z). \qquad \text{(V-7)}$$

Let $\alpha_{i,zy}$ and $\alpha_{r,zy}$ be the projected angles on the $(y, z)$ plane, as indicated in Fig. 16; they are given by

$$\tan \alpha_{i,zy} = \frac{i_z}{i_y} \quad \text{and} \quad \tan \alpha_{r,zy} = \frac{r_z}{r_y}. \qquad \text{(V-8)}$$

Inserting the $y$ and $z$ components of $r$, we obtain

$$\tan \alpha_{r,zy} = \frac{\tan \alpha_{i,zy}}{\cos 2\gamma - \sin 2\gamma \tan \alpha_{i,xy}}, \qquad \text{(V-9)}$$

where $\alpha_{i,xy} = \tan^{-1}(i_x/i_y)$ is the angle shown in Fig. 16(a). The term $\sin 2\gamma \tan \alpha_{i,xy}$ is positive, since $\hat{\imath}$ and $\hat{n}$ lie on the same side of the reflector, and hence the denominator in eqn (V-9) is less than one. In the following, we consider only the case when the reflected ray points downward, i.e. $r_y < 0$; this implies that the denominator is positive. We can conclude, therefore, that

$$\alpha_{r,zy} > \alpha_{i,zy} \qquad \text{(V-10)}$$

except in the trivial case when $\gamma = 0$, corresponding to a reflector wall parallel to the $(y, z)$ plane.

With this information, one learns at once that the crossed double CPC of Fig. 17, consisting of one CPC along the $z$-axis with acceptance half-angle $\theta_1$ followed by a second CPC along the $x$-axis with acceptance half-angle

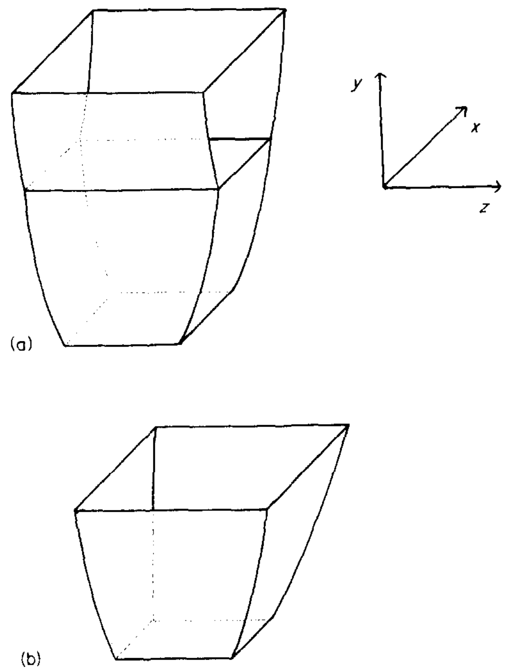



Fig. 17. Combination of two orthogonal CPC's. Two successive units ($a$), single unit ($b$). Optically, ($b$) is somewhat better than ($a$), but neither attains the ideal concentration.

$\theta_2$, is not an ideal concentrator, i.e. it fails to concentrate by the full amount $(1/\sin\theta_1)(1/\sin\theta_2)$. In particular, for rays hitting the lower edge of the first CPC, where $\gamma = \gamma_{max} = (\pi/4) - (\theta_1/2)$ and the "aberrations" are largest, the angle $\alpha_{r,zy}$ is given by

$$\tan\alpha_{r,zy} = \frac{\tan\alpha_{i,zy}}{\sin\theta_1 - \cos\theta_1 \tan\alpha_{i,xy}}. \qquad \text{(V-11)}$$

This angle $|\alpha_{r,zy}|$ must be less than $\theta_2$ in order for the ray to be accepted by the second CPC, but in fact it ranges from a minimum of $\tan^{-1}(\tan\alpha_{i,zy}/\sin\theta_1)$, which is already larger than $\alpha_{i,zy}$, all the way to $\pi/2$. Therefore, the second CPC will necessarily reject some of the rays which do not lie in the $xy$ plane.

Two perpendicular CPC's could be combined into a single groined unit, as in Fig. 17, presumably with somewhat better performance (but certainly short of the ideal limit). This configuration may be employed as stationary concentrator in front of ordinary flat-plate collectors.

## VI. SOME CONVENTIONAL CONCENTRATOR TYPES

In this section we review some of the best known concentrator types[1], and list their respective values of concentration $C$, of reflector to aperture ratio $R$ and of number of reflections $\langle n\rangle$ as a function of angular acceptance. The angular acceptance is specified by the half-angle $\delta$ where $2\delta$ is the angular range for which all rays reach the absorber. In most applications, $\delta$ will be the angular radius of the sun, $\delta_s = 4.7$ mrad, augmented by an appropriate amount $\delta_m = 2\langle n\rangle\Delta$ to account for mirror and tracking inaccuracies.

(A) *Parabola*

A two-dimensional parabola, i.e. a parabolic trough, with cylindrical absorber is shown in cross-section in Fig. 18. The placement is symmetric about the line from source to absorber, and the aperture of the parabola is determined by the rim angle $\phi = \sphericalangle OAB$. If the radius $a$ of the absorber is chosen as small as possible without losing any radiation, then the concentration is

$$C_{2\ dim\ parab.,\ cyl.\ abs} = \frac{2x_A}{2\pi a} = \frac{\sin\phi}{\pi\sin\delta} = \frac{\sin\phi}{\pi} C_{ideal\ 2\ dim}. \qquad \text{(VI-1)}$$

The corresponding statement for a three-dimensional parabola with a spherical absorber is

$$C_{3\ dim\ parab.,\ cyl.\ abs} = \frac{\sin^2\phi}{4\sin^2\delta} = \frac{\sin^2\phi}{4} C_{ideal\ 3\ dim}. \qquad \text{(VI-2)}$$

In both cases the maximum occurs at a rim angle $\phi = \pi/2$ and falls a factor $1/\pi$ or $1/4$ short of the ideal limit. (For cylindrical parabolas, concentrations slightly higher than this, but still below the values for one-sided flat absorpers, are possible if the absorber has different horizontal and vertical dimensions.) In practical designs, one may prefer different values of $a$ and $\phi$ if the attendant loss of concentration is less important than other considerations.

The concentration for a flat absorber depends on whether it is one-sided or two-sided. A two-sided absorber does not lose any radiation because of shading, but its surface area is twice as large compared to a one-sided absorber. The corresponding concentrations are related, both in two and in three dimensions, by

$$C_{1\ sided} = 2C_{2\ sided} - 1. \qquad \text{(VI-3)}$$

For a two-dimensional parabola with rim angle $\phi$, the concentration is

$$C_{2\ dim\ parab.,\ flat\ 1\ sided} = \frac{\sin\phi\cos(\phi+\delta)}{\sin\delta} - 1 \qquad \text{(VI-4)}$$

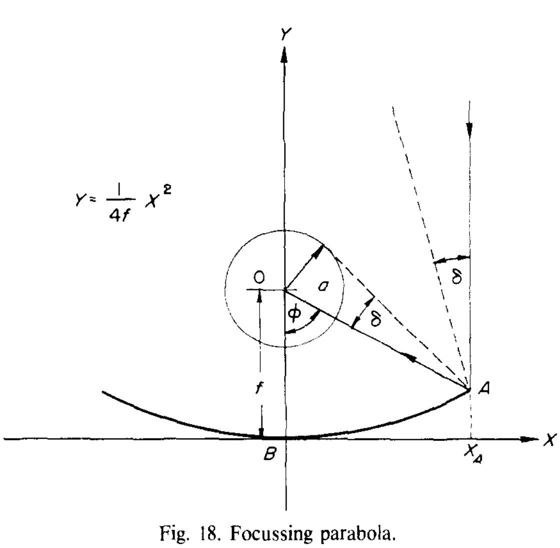



Fig. 18. Focussing parabola.

and reaches a maximum at

$$\phi_{\max} = \frac{1}{2}\left(\frac{\pi}{2} - \delta\right), \qquad \text{(VI-5)}$$

corresponding to a concentration

$$C_{2\,dim\,flat,\,1\,sided\,max} = \frac{1}{2\sin\delta} - \frac{3}{2} = \frac{1}{2}C_{2\,dim\,ideal} - \frac{3}{2}. \qquad \text{(VI-6)}$$

The analogous statements in three dimensions read

$$C_{3\,dim\,flat,\,1\,sided} = \frac{\sin^2\phi\cos^2(\phi+\delta)}{\sin^2\delta} \qquad \text{(VI-7)}$$

and

$$C_{3\,dim\,flat,\,1\,sided\,max} = \frac{1}{4\sin^2\delta} - \frac{1}{2\sin\delta} - \frac{3}{4}$$

$$= \frac{1}{4}C_{3\,dim\,ideal} - \frac{1}{2\sin\delta} - \frac{3}{4}. \qquad \text{(VI-8)}$$

For the ratio $R$ of aperture to reflector area, we find in two dimensions

$$R_{2\,dim\,parab.}(\phi) = \frac{A_R}{A} = \left[\frac{1}{\cos(\phi/2)} + \left(\cos\frac{\phi}{2}\right) \times \log\cot\left(\frac{\pi}{4} - \frac{\phi}{4}\right)\right]\Big/2 \qquad \text{(VI-9a)}$$

which becomes for $\phi = \pi/4$

$$R_{2\,dim\,parab.}\left(\phi = \frac{\pi}{4}\right) = 1.03 \qquad \text{(VI-9b)}$$

and for $\phi = \pi/2$

$$R_{2\,dim\,parab.}\left(\phi = \frac{\pi}{4}\right) = 1.15. \qquad \text{(VI-9c)}$$

In three dimensions we have

$$R_{3\,dim\,parab.}(\phi) = \frac{2(1/\cos(\phi/2) - \cos^2(\phi/2))}{3\sin^2(\phi/2)} \qquad \text{(VI-10a)}$$

with

$$R_{3\,dim\,parab.}\left(\frac{\pi}{4}\right) = 1.04 \qquad \text{(VI-10b)}$$

and

$$R_{3\,dim\,parab.}\left(\frac{\pi}{2}\right) = 1.22. \qquad \text{(VI-10c)}$$

(B) *Fresnel mirrors*

For large installations, it is often advantageous to use a field of Fresnel mirrors, in other words, to break up a single parabolic mirror into many small segments each of which can be moved separately to direct the light into a common focus. An example of this is the central receiver or "power tower", a solar power plant concept in which the absorber is on top of a tower surrounded by a field of heliostatic mirrors [20–24].

The complete analysis of a three-dimensional Fresnel mirror field is quite complex, involving a detailed description of shading and blocking for various angles of incidence. (*Shading* occurs if direct sunlight fails to reach a mirror because it is intercepted by some other mirror. *Blocking* occurs if light reflected by a mirror fails to reach the absorber because it is intercepted by some other mirror.) In the following, we assume that only a fraction $\psi$ of the ground is covered by mirrors and that the mirror spacing is chosen to minimize shading and blocking. Values of $\psi \approx 0.5$ have been suggested for practical central receiver designs [22–24]. In terms of the effective ground cover $\psi$, the concentration is easily calculated. For normal incidence (at other angles there would be an additional cosine term) and a round absorber the concentration is, in two dimensions

$$C_{2\,dim,\,Fresnel,\,cyl.\,abs.} = \psi\frac{\sin\phi}{\pi\sin\delta}$$

$$= \psi\frac{\sin\phi}{\pi}C_{2\,dim\,ideal} \qquad \text{(VI-11)}$$

and in three dimensions

$$C_{3\,dim,\,Fresnel,\,spher.\,abs.} = \psi\frac{\sin^2\phi}{4\sin^2\delta}$$

$$= \psi\frac{\sin^2\phi}{4}C_{3\,dim\,ideal} \qquad \text{(VI-12)}$$

where $\phi$ is the rim angle. For flat absorbers the results are

$$C_{2\,dim,\,Fresnel,\,flat} = \psi\frac{\cos(\phi+\delta)\sin\phi}{\sin\delta}$$

$$= \psi\cos(\phi+\delta)\sin\phi C_{2\,dim\,ideal} \qquad \text{(VI-13)}$$

and

$$C_{3\,dim,\,Fresnel,\,flat} = \psi\left[\frac{\cos(\phi+\delta)\sin\phi}{\sin\delta}\right]^2$$

$$= \psi[(\cos(\phi+\delta)\sin\phi]^2 C_{3\,dim\,ideal}. \qquad \text{(VI-14)}$$

In practical designs, the mirror spacing is chosen uniform, i.e. independent of mirror position, and hence $\psi$ is independent of rim angle. Then the maximum for a flat absorber is reached at $\phi = \pi/4$. With spherical absorber and fixed $\psi$, eqn (VI-12) does not reach a maximum, but obviously excessive rim angles require too much land area and hence values around $\phi = 60°$ are chosen in practice. If the highest possible concentration is desired, then the mirror spacing should vary with mirror position, and the maximum of eqns (IV-13) and (IV-14) will occur at rim angles somewhat different from 45°. Compared to a round absorber, a flat absorber has the advantage that it can be replaced by a cavity which combines higher absorptivity with lower convection heat losses.

Being a parabolic mirror that has been broken up into many small segments, a Fresnel mirror is very similar to a parabola, and thus the formulas for the aperture to reflector ratio $R$ are the same, while the formulas for the concentration differ only by the effective ground cover $\psi$.

### (C) *V-Troughs*

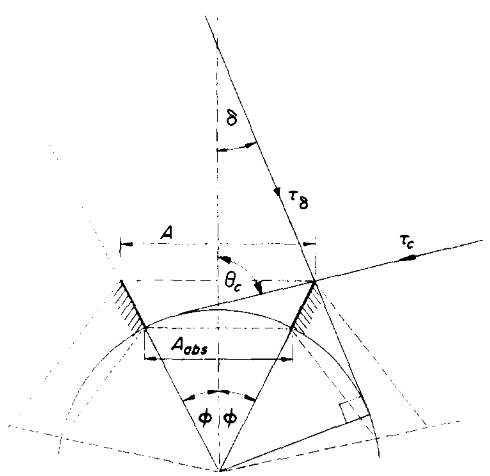
Two-dimensional, i.e. linear, $V$-trough solar concentrators have been discussed for example by Hollands[25]. Here we present a slightly different analysis in order to clarify the similarities and differences between $V$-troughs and compound parabolic concentrators. We follow the usual method of images to treat multiple reflections, shown graphically in Fig. 19(a), and we neglect, for simplicity, the difference between the polygon and the circle (called reference circle).

A $V$-trough can be specified (apart from its overall size) by two dimensionless numbers, for example, by the trough angle $\phi$ and by the concentration

$$C = \frac{A}{A_{abs}}.$$

Let $\delta$ be the largest angle of incidence at which no radiation is rejected, and $\theta_c$ the angle of incidence beyond which no ray can reach the absorber. These angles can be found, to a good approximation, by drawing the tangents $\tau_\delta$ and $\tau_c$ of the reference circle which pass through an endpoint of the aperture. It is easy to show that $\delta$ and $\theta_c$, the angles which $\tau_\delta$ and $\tau_c$ make relative to the trough axis, are related by

$$\theta_c = \delta + 2\phi, \qquad \text{(VI-15)}$$




(a)

Fig. 19(a). $V$-trough concentrator, with mirror images and reference circle. The rays $\tau_\delta$ and $\tau_c$ have angle of incidence $\delta$ and $\theta_c$, respectively; they pass through the edge of the absorber and are tangential to the reference circle.

(b)

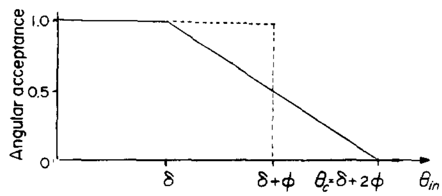



Fig. 19(b). Angular acceptance of $V$-trough (schematic, neglecting difference between polygon and circle in Fig. 19(a)).

while the concentration is

$$C_v = 1/\sin(\delta + \phi). \qquad \text{(VI-16)}$$

Of course, the conditions $\delta + \phi < \pi/2$ and $\phi < \pi/4$ must be met to avoid loss of any radiation with $|\theta_{in}| < \delta$. In order to clarify the similarities and differences between $V$-trough and CPC, Fig. 19(b) shows schematically the angular acceptance of a $V$-trough. All rays with $|\theta_{in}| < \delta$ are accepted while all rays with $|\theta_{in}| > \theta_c$ are rejected; the transition region between full acceptance and full rejection has (approximately) width $2\phi$ and is centered around $\delta + \phi = \sin^{-1}(1/C_v)$. As a $V$-trough becomes very narrow, i.e in the limit $\phi \to 0$, its concentration and angular acceptance approach that of a CPC (see Fig. 6). However, the reflector to aperture ratio

$$R_v = \frac{1 - \sin(\delta + \phi)}{\sin \phi} \qquad \text{(VI-17)}$$

and the reflection losses become very unfavorable for small trough angles $\phi$. The number of reflections can be determined by various methods (analytically[12], graphically or by ray tracing). If the trough is shallow enough so that no multiple reflections occur, $\langle n \rangle$ for nearly normal incidence is simply given by $1 - 1/C$.

A quantitative comparison between $V$-trough and CPC is difficult because of the large number of parameters that should be considered simultaneously. Even disregarding reflector cost and solar energy collection, the comparison involves additional parameters ($R, \langle n \rangle$, acceptance angle and truncation) besides the value of the concentration. We mention just two examples which are somewhat arbitrary but typical. If $\delta = 19.5°$ ($C_{ideal} = 3$), then a truncated CPC achieves $C = 2.9$ with $R = 2.8$, and a mean number of reflections of 0.75. A $V$-trough with a comparable number of reflections has an opening half-angle of $\phi = 11°$, and thus it gives only a concentration of $C = 2$ with a value of $R = 2.5$ for the reflector to aperture ratio. If the acceptance angle is $\delta = 5.7°$ ($C_{ideal} = 10$), then a truncated CPC achieves $C = 7.8$ with $R = 4.9$ and 1.0 reflections on the average. A $V$-trough with roughly the same number of reflections and acceptance angle has an opening half-angle $\phi = 10°$ and concentrates only by $C = 3.7$ with a value $R = 4.1$.

The higher the concentration the greater the relative advantage of the CPC over the $V$-trough, and above $C \approx 3$ a $V$-trough appears to be impractical. For low concentrations (and relatively large trough angles, $\phi$ around 20°) a $V$-trough becomes comparable to a CPC as far as $C$, $\langle n \rangle$ and $R$ are concerned, but the wide transition region between full acceptance ($|\theta_{in}| < \delta$) and full rejection ($|\theta_{in}| > \theta_c = \delta + 2\phi$) will put the $V$-trough at a disadvantage. Compared to a CPC with its sharp cut off (say at $t = t_c$), a $V$-trough will begin to miss some radiation early in the afternoon (at $t < t_c$) and continue to collect later in the afternoon (at $t > t_c$). However, the radiation collected at $|t| > t_c$ is of lower intensity and less valuable than the radiation which is missed at $|t| < t_c$. This is particularly critical for thermal collectors which have to overcome a constant heat loss before they can produce any useful energy.

## VII. SECOND STAGE CONCENTRATORS

With increasing concentration $C$, the reflector area of a CPC grows like $1 + aC$, with $a \approx 1/2$ to 1 depending on truncation, and the average number of reflections $\langle n \rangle$ grows like $1/2 \log C$. Therefore, a single CPC trough (cone) is likely to be impractical for concentrations above ten (one hundred). For higher concentrations, a two-stage system becomes advantageous because it reaches almost the ideal limit without excessive reflector/aperture ratio and transmission loss.

To be specific, we consider the combination of a Fresnel mirror field with a CPC as second stage or terminal concentrator.

The radiation emerging from a conventional concentrator such as a lens or a parabolic mirror has an angular divergence half angle $\phi < \pi/2$. It can thus be further concentrated by a matching compound parabolic concentrator (CPC)[26]. For example, the system shown in Fig. 20 attains an overall concentration

$$C_{2\,dim} = \psi \cos(\phi + \delta) C_{2\,dim\,ideal} \text{ in two-dimensions,} \tag{VII-1}$$

and

$$C_{3\,dim} = \psi \cos^2(\phi + \delta) C_{3\,dim\,ideal} \text{ in three-dimensions,} \tag{VII-2}$$

where $\delta$ is the angular half width of the source, a result that follows directly from eqns (VI-13 and 14). For normal angles of incidence and in the limit $\phi \to 0$ and $\delta \to 0$ shading and blocking problems disappear. Hence, the effective ground cover $\psi$ can be chosen equal to one, and the concentration approaches the ideal limit. Very small rim angles $\phi$ are, of course, impractical because then the CPC becomes too deep and requires too many reflections. There is, however, an intermediate range of values of $\phi$, around 10°–30°, for which the overall concentration is still close to the ideal limit while the average number of reflections in the CPC is below one.

In practice, rim angles larger than 30° may be desirable; for example, the cost of the tower for the central receiver can be justified only if radiation is received from a sufficiently wide mirror field. This requirement can easily be met without sacrificing concentration, if several intermediate angle CPC's are combined to form a "fly eye" terminal concentrator. Such a design is well suited for use with a cavity absorber, and Fig. 21 includes this feature.

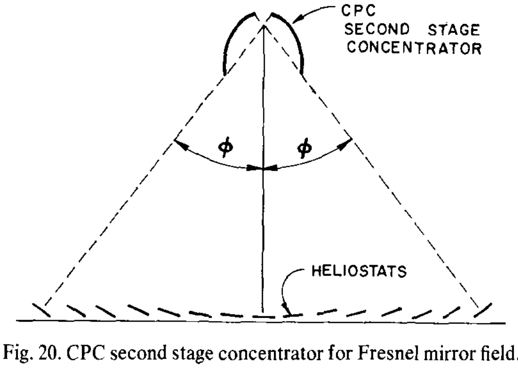



Fig. 20. CPC second stage concentrator for Fresnel mirror field.

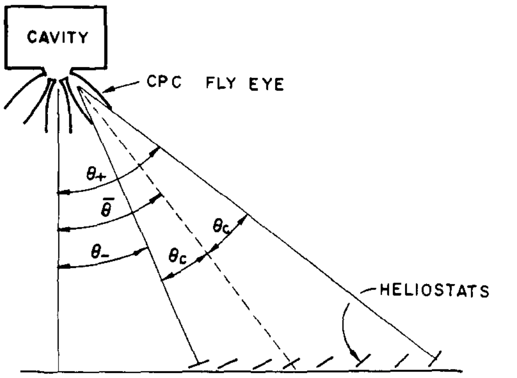



Fig. 21. Central receiver (power tower) with CPC "fly eye" second stage concentrator. The drawing is schematic and the CPC and cavity are shown much too large in proportion to the tower height.

As for the aiming strategy, the mirror field is divided into zones, one for each CPC, and in each zone the mirrors are aimed at their respective CPC. The zones are independent of each other, and the total concentration can be derived easily, if an effective ground cover $\psi(\bar{\theta})$ is assumed for each zone. Taking $\delta \ll 1$ which is appropriate for direct solar radiation, we find for the zone explicitly labeled in Fig. 21, a concentration

$$C_{2\,dim}(\bar{\theta}) = \psi(\bar{\theta}) \frac{\cos^2\theta_c}{\cos\theta_-} \frac{1}{\delta} = \psi(\bar{\theta}) \frac{\cos^2\theta_c}{\cos\theta_-} C_{2\,dim\,ideal} \tag{VII-3}$$

in two dimensions, and

$$C_{3\,dim}(\bar{\theta}) = \psi(\bar{\theta}) \frac{\cos^2\theta_c \cos\theta_+}{\cos\bar{\theta}\cos\theta_-} \frac{1}{\delta^2} = \psi(\bar{\theta}) \frac{\cos^2\theta_c \cos\theta_+}{\cos\bar{\theta}\cos\theta_-} C_{3\,dim\,ideal} \tag{VII-4}$$

in three dimensions.

Unlike the power tower with a simple flat absorber, i.e. without a terminal concentrator, the arrangement presented here does not suffer a loss of concentration if rim angles larger than 45° are chosen. For example, with $\theta_c = 10°$ and $\bar{\theta} = 50°$, corresponding to an outer rim angle of 60°, eqns (V-3 and V-4) yield concentrations of $1.265\psi C_{2\,dim\,ideal} = 74.4$ and $0.9844\psi C_{3\,dim\,ideal} = 7615$ for $\psi = 0.5$ and $\delta = 0.0076$. These numbers exceed the corresponding values for flat absorbers without terminal concentrators by a factor of 2.9 in two dimensions and by a factor of 5.1 in three dimensions.

The concentration is about twice as high as that achieved by a straight $V$-cone terminal concentrator, a design proposed by Brumleve[23]. The mirrors near the center of the field are aimed at the center of the absorber. The beams reflected by the outer mirrors are too wide to fit into the absorber, and they are folded in half, so to speak, by a $V$-cone in front of the absorber. This requires a special aiming strategy: the mirrors near the right edge of the field, for example, are aligned so that the left edge of the reflected beams hits the left edge of the absorber. Rim angle, absorber width and the proportions of the

$V$-cone are chosen to insure that no rays are rejected or lost. For large rim angles, this type of $V$-cone becomes quite wide; for example for $\phi = 60°$, the outer diameter would be about 20 m for a tower height of 100 m, while a "fly eye" would have an outer diameter of only 12 m (or as little as 6 m if the CPC's are truncated). Furthermore, a single large and deep CPC cone can be replaced by a densely packed honeycomb array of many small cones with almost no loss in optical performance. Figure 22 displays the variation of concentration $C$ with rim angle $\phi$ for a central receiver with spherical absorber, with flat absorber, with $V$-cone second stage and with "fly eye" second stage. The results are presented in a form which is independent of $\delta$ by using as abscissa the ratio $C/(\psi C_{ideal})$. For the "fly eye", we have assumed that the CPC cones achieve only 90 per cent of the ideal concentration; this is a margin of safety which guarantees that the failure zones discussed in Section V and Fig. 15 do not cause any light rays to be rejected.

Of course, concentration is only one of several factors determining the design of a solar power plant, and an analysis of the entire system is needed before the most suitable concentrator type can be chosen. At this point, we can only list some advantages and disadvantages of a CPC terminal concentrator.

The fact that for a given acceptance angle $2\delta$, the concentration reached by the CPC is about four times as high as for a system without second stage concentrator, is an obvious advantage for the design of ultrahigh temperature power plants (using power cycles such as high temperature gas turbines, magnetohydrodynamics or thermionic conversion). But it can be just as important for solar collectors of low or intermediate temperature, because the acceptance half angle $\delta = \delta_s + \delta_m$ can be doubled for a specified concentration, thus allowing a

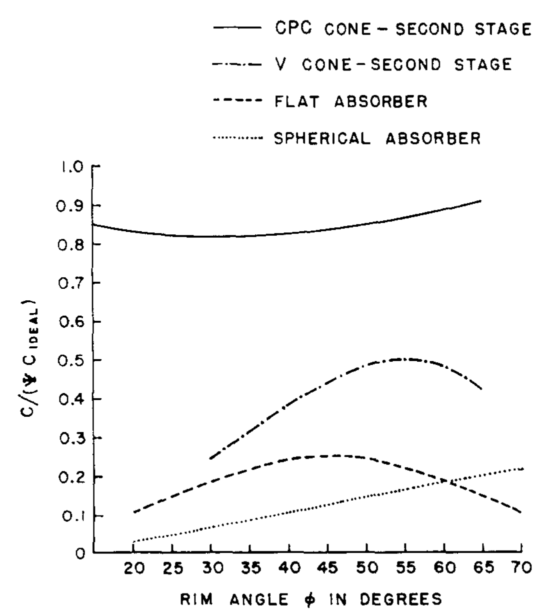



Fig. 22. Concentration $C$ of central receiver with and without second stage concentrator. The results are presented in terms of ground cover $\psi$ and $C_{ideal} = 1/(\sin\delta)^2$, and are thus independent of acceptance half-angle $\delta$. The theoretical upper limit corresponds to $C/(\psi C_{ideal}) = 1$.

very significant relaxation of the mirror accuracy. For example, a power tower with CPC and with effective mirror and tracking error $\delta_m = 2\Delta = 4.7$ mrad achieves as high a concentration as a power tower without CPC but with perfect mirrors. (This $\delta_m$ refers to the first stage: as for contour errors of the second stage CPC, they are insignificant as long as they are small compared to $\theta_c$). Since the heliostats constitute a large fraction of the total cost of a central receiver solar power plant and since this cost depends strongly on the accuracy requirement, significant savings may be possible.

Being relatively deep and narrow, a CPC can act as convection suppressor if the absorber is at the top and the aperture at the bottom. Furthermore, we note in Fig. 22 that the overall concentration of the two stage system is nearly independent of rim angle, a fact which offers great flexibility for design. For instance, in some locations a mountain slope or a building wall may provide a natural support for a Fresnel mirror field, and a matching CPC second stage would guarantee high concentration for almost any geometry.

As for disadvantages, a CPC with an acceptance half-angle around 15° requires on the average one reflection; this causes losses, around 10 per cent for good reflectors, and may necessitate cooling. The heat extracted by this cooling loop may or may not be useable, depending on the thermal conversion system and on the stability of reflector materials at elevated temperatures. As an illustration of how a CPC mirror may serve to preheat a working fluid, we mention a conventional superheated steam cycle which needs about 75 per cent of the heat at 300°C and the remaining 25 per cent at temperatures up to 500°C. The energy and temperature requirements can easily be satisfied by a two-stage cavity, the first part being a CPC, operating as a preheater at 300°C, and the second part being an ordinary radiation cavity at 500°C. The specular reflectivity of the CPC could be as low as 0.30, which is easy to maintain at 300°C.

Reflectivities around 95 per cent can be maintained in the second stage CPC if it is coated with silver and placed behind a glass window either in an inert gas atmosphere or in a vacuum. For example, in small line focus systems, the entire second stage (CPC + absorber) can be put inside an evacuated glass tube: this is advantageous because of its low thermal losses.

The radiation incident upon the top of a power tower can be reflected to the ground and collected there. If matching CPC cones are used as terminal concentrators[27], overall concentration values of several thousand are possible.

A CPC second stage is attractive in combination with many other concentrator types, for instance with the fixed mirror plus moving-receiver concept of Russell[28], or with a linear Fresnel lens. In some applications, solar collectors will be built as panels consisting of many small parallel CPC troughs. To protect the reflectors from dirt and snow, a transparent cover will have to be placed on top with the usual transmission losses. If instead of a simple cover a linear Fresnel lens of small rim angle (low concentration $C_1 \leqslant 5$, and hence small aberrations) is used, with a small second stage CPC in the focal plane, the overall concentration will be nearly ideal. Furthermore less reflector is needed, and transmission losses are much smaller.

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## VIII. NOMENCLATURE

$A$ = area of aperture
$A_{abs}$ = area of absorber
$A_R$ = area of reflector
$C = A/A_{abs}$ = concentration (with subscripts to indicate type of concentrator)
$E$ = 23°27′ = inclination of earth's axis
$E_{1\to 2}$ = fraction of the diffuse radiation emitted by surface 1 which reaches surface 2 directly or after intervening specular reflections and/or refraction
$F_{1\to 2}$ = fraction of the diffuse radiation emitted by surface 1 which reaches surface 2 directly (= shape factor)
$\hat{i}$ = unit vector in direction of incident ray
$n$ = number of reflections
$\langle n \rangle$ = average number of reflections
$\hat{n}$ = unit vector normal to reflector surface
$\hat{n}_c$ = unit vector normal to collector
$\hat{n}_s$ = unit vector from point on earth's surface to sun
$Q$ = radiation heat transfer (with appropriate subscripts, e.g. $Q_S$ = total radiation emitted by $S$, $Q_{S\to a}$ = radiation emitted by $S$ and absorbed by $a$)
$\hat{r}$ = unit vector in direction of reflected ray
$T$ = absolute temperature (with appropriate subscripts)
$\alpha$ = absorptivity
$\alpha_i$ = angle of incidence
$\alpha_r$ = angle of reflection
$\beta$ = collector tilt from equatorial plane
$\delta = \delta_s + \delta_m$ = effective angular half-width of sun (or other source) as seen through imperfect mirrors
$\delta_s$ = angular half width of sun = 4.7 mrad ≈ 1/4°
$\delta_m = 2\langle n \rangle \Delta$ with $\Delta$ = error in mirror surface and alignment (onesided deviation from perfect)
$\epsilon$ = emissivity (with appropriate subscripts)
$\eta_c$ = Carnot efficiency
$\phi$ = rim angle
$\theta_c$ = angular half-width of source (= acceptance half angle of ideal concentrator)
$\theta_{in}$ = angle of incidence
$\rho = 1 - \alpha$ = reflectivity
$\sigma$ = Stephan–Boltzmann constant
$\tau$ = fraction of the radiation incident on aperture which gets to absorber.

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16. These ideas for convection suppressing cavities evolved in discussions with Prof. B. T. Chao.
17. R. Winston and H. Hinterberger, Principles of cylindrical concentrators for solar energy. *Solar Energy* **17**, 255 (1975).
18. A. Rabl, Argonne Report SOL 75-05 (Nov. 1975). *Applied Optics*, to be published.
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22. A. F. Hildebrandt *et al.*, *EOS Trans. of American Geophysics Union*, **53**(7), 684–692 (1972); and University of Houston and McDonnel Douglas Astronautic Co. progress reports NSF/RANN/SE/GI-39456/PR.
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24. Honeywell, Inc., and Black & Veatch, Dynamic conversion of solar generated heat to electricity. NASA Report. NASA CR-134726 (1976).
25. K. G. T. Holland, A concentrator for thin-film solar cells. *Solar Energy* **13**, 149 (1971).
26. Actually, for this application a compound elliptical concentrator is better than a CPC, see A. Rabl and R. Winston, Ideal Concentrators for Finite Sources and Restricted Exit Angles (Feb. 1976), to be published. However, the difference in concentration between a compound elliptic and a compound parabolic concentrator is small, of order $\cos\phi/(C_1 - 1)$ where $\phi$ is the rim angle and $C_1$ the concentration of the first stage.
27. A. Rabl, Tower reflector for solar power plant. Argonne Report SOL 75-04 (July 1975), *Solar Energy*, to be published. T. M. Knasel *et al.*, The tower reflector as an alternative to the tower boiler concept for a central receiver solar thermal electric conversion plant. Science Applications, Inc., Report (1975).
28. J. L. Russell, Central station solar power. *Power Engng.* (Nov. 1974).

**Resumen**—A pesar de que la mayoría de las variaciones de concentradores solares ha sido estudiada o construída en algún momento u otro, se ha dejado de lado hasta hace poco tiempo una importante clase de los mismos. Estos nuevos concentradores han sido llamados "ideales" en virtud de sus propiedades ópticas, y por ejemplo, el concentrador parábolico compuesto, está siendo ensayado actualmente por la Argonne National Laboratory. Los concentradores "ideales" difieren radicalmente de los instrumentos convencionales como las parábolas de enfoque. Ellos actúan como un embudo de radiación y no poseen un foco. Para un ángulo dado de aceptación la concentración sobrepasa en dos o cuatro veces la de otros concentradores solares, pero se requiere relativamente un gran área de reflexión. El número de reflexiones varía según el ángulo de incidencia, pero en valor promedio es cercano a uno en la mayoría de los casos de interés. En función de ayudar a proveer una base radional para decidir cual tipo de concentrador es mejor para una aplicación particular, nosotros hemos comparado una variedad de concentradores en términos de sus características principales, p.ej., concentración, ángulo de aceptación, sensibilidad de los errores de los espejos, tamaño del área reflectora y el número promedio de reflexiones.

Le conexión entre la concentración, el ángulo de aceptación y la temperatura de operación se analiza simplemente en términos intuitivos, llegando directamente a recetas para el diseño de colectores de concentración máxima (no debe permitirse que ninguna radiación emitida por el absorbedor salga fuera del ángulo de aceptación). Nosetros proponemos algunos neuvos concentradores, incluyendo el uso de los parábolicos compuestos como segunda etapa para parábolicos convencionales o espejos de Fresnel. Esta combinación approxima el comportamiento al de un concentrador "ideal" sin necesitar un reflector grande; esto puede ofrecer ventajas significativas para los sistemas solares de alta temperatura.

**Résumé**—Bien que presque tous les types de concentrateurs solaires aient été étudiés ou construits à un moment ou à un autre, une importante catégorie de concentrateurs a été négligée jusqu'à très récemment. Un exemple de ces nouveaux concentrateurs appelés idéaux au plan de leurs propriétés optiques, le paraboloïde composé, est actuellement expérimenté au Laboratoire National d'Argonne. Les concentrateurs idéaux diffèrent radicalement des dispositifs traditionnels tels que la parabole focalisante. Ils fonctionnent comme des conduits de rayonnement et n'ont pas de foyer. Pour un angle d'interception donné, leur concentration surpasse celle des autres concentrateurs solaires, d'un facteur de deux à quatre, mais ils nécessitent une surface de captation assez importante. Le nombre de réflexions varie avec l'angle d'incidence, autour d'une valeur moyenne unitaire pour la plupart des cas intéressants. Dans le but de contribuer à la fourniture de bases rationnelles pour décider du type de concentrateur convenant le mieux à une application particulière, nous avons comparés divers concentrateurs solaires du point de vue de leurs caractéristiques générales les plus importantes, et en particulier: concentration, angle d'interception, sensibilité aux erreurs du miroir, dimensions de la surface réfléchissante et nombre moyen de réflexions.

La relation entre concentration, angle d'interception et température de fonctionnement d'un collecteur solaire est analysée en termes simples et intuitifs, conduisant à une recette de conception des collecteurs à concentration maximale (aucune radiation émise par l'absorbeur ne doit être consentie vers l'extérieur en dehors de l'angle d'interception). Nous proposons quelques concentrateurs nouveaux, y compris l'utilisation de paraboliques composés comme second étage de concentration pour les miroirs paraboliques ou de Fresnel traditionnels. Une telle combinaison approche des performances idéales sans nécessiter de réflecteurs de grandes dimensions; elle peut offrir des avantages intéressants pour les dispositifs solaires à haute tempéraure.