

Application of Continues Wavelet Transform to Investigate the Resolution Effect of Digital Elevation Model on Topographic Index Distribution

Vahid Nourani

Abstract— Current progress in computer, geographic information system (GIS) and remote sensing technology has led to the LIDAR (light detection and ranging) data, as a new generation of high-resolution digital elevation model (DEM). Although such a high resolution DEM can be a useful tool for terrain analysis, high amount of noise is also expected to be existent in a mega-data set. To deal with this problem in this study, continues wavelet transform (CWT), as a mathematical filter, was employed to extract slope, specific area and topographic index which are important parameters in hydro-environmental studies at different resolutions. In this way the proposed methodology was applied to 1-m resolution LIDAR DEM of the Elder Creek River watershed where is a sub-basin of the South Fork Eel River basin at California. The results were also compared favorably with the results of a classic method. The result indicates that smoothing ability of the wavelet can be a promising DEM processing alternative to cope with the noise and pits of a high resolution DEM.

Index Terms— DEM, Elder Creek River, GIS, LIDAR Topography, Wavelet transform.

I. INTRODUCTION

To apply some hydro-environmental models, the spatial distribution of the so-called topographic index (TI) is calculated for the catchment from a topographic data set, such as a digital elevation model (DEM). The basic and most common form of the topographic index, $\ln(a/\tan \beta)$, is the natural logarithm of the ratio of the cumulative upslope area per unit contour length (specific catchment area), a to the local slope of the ground surface, $\tan \beta$.

The evolution of affordable, robust computers and commonplaceness of geographical information system (GIS) are encouraging the use of different GIS-based terrain algorithms to calculate the primary topographic characteristics of ' $\tan \beta$ ' and ' a ' which are then combined to form the secondary topographic characteristic of the TI. O'Callaghan and Mark [1] introduced the D8 (eight flow directions) method as the earliest and simplest algorithm for specifying flow direction and accumulation area. Thereafter,

some other methods such as multiple flow direction (MFD, [23]), modified MFD [3], infinite possible flow directions (D_∞) [4] and hybrid MFD- D_∞ [5] have also been suggested as attempts to overcome the limitation of D8 method. On the other side, finite difference, linear regression and quadratic equation methods have been employed for calculating local ground surface slope ($\tan \beta$) from DEMs [6]. There are also a number of studies which compare the patterns of TI computed in different ways (e.g., [7]). However, a recurring scale issue regarding TI is that the distribution of TI is markedly related to the topographic information content of the DEM, which can be impacted by both discretization (or aggregation) and smoothing effects [8]. Several studies have discussed the effects of scale and DEM resolution on the TI distribution (e.g., [3], [8]–[14]). Also, the recent emergence of airborne altimetry technology (e.g., light detection and ranging, LIDAR) as a means of acquiring high resolution DEMs, allows the examination of this scale issue using much finer grid cell size (see, [15]).

Beyond the shadow of a doubt, the scale dependence of TI distribution effects on the model's parameters and performance. In the case of using coarser DEMs, it has been postulated that a positive shift accompanied, sometimes by a deformation may be appeared in the TI distribution and although the calibration of parameters can often compensate for the lack of resolution, the physical interpretation of the parameters and the merit of the model in distributed predictions may be lost [10], [16]. On the other hand, some other problems may have arisen if very high resolutions DEMs are employed. Zhang and Montgomery [14] stated that increasing the resolution to 2 or 4 m would not provide important additional information and often 10 m cell size is sufficient for hydrologic simulation to have a compromise between accuracy and computation cost in large area. Cai and Wang [17] found that a 90 m DEM can derive TI distribution as well as a 30 m DEM for landscapes with significant variation in elevation; but Sorensen and Seibert [15] utilized high resolution LIDAR data to show the large influence of the DEM's information content meanwhile, simply going from a 5 to 10-m resolution can considerably affect the calculated TI. Beven *et al.* [18] describe required resolution as needing to be fine enough to reflect the effect of topography; too coarse a DEM may fail to represent convergent slope features and results in parameter inconsistency across scales, while too fine a DEM may impose perturbation to the flow directions and slope angles. Furthermore, as the resolution of topographic data is fined,

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like any other signal, there is a greater possibility that the DEM includes artificial depressions and pits which result from random noise [19]. Aforementioned studies bear out that even the implementation of high-resolution DEMs which are becoming more readily available nowadays, does not warrant the improvement of the modeling if a modification or/and DEM pre-processing is not taken into consideration. In this way, the wavelet transform as such a DEM pre-processing tool may be incorporated into a GIS framework for the de-noising and compression purposes [20].

In this study, the capability of the wavelet transform which has recently found wide application in geophysics [21], topographic feature extraction [19], [22], [23] and hydrological simulation [24]–[26] is examined as an alternative for DEM and TI processing computed from a high-resolution LIDAR DEM.

II. STUDY AREA

The study area which is the subject of this research is the Elder Creek sub-basin of the South Fork Eel River in northern California, USA (Fig. 1). This 16.84 km² basin drains a landscape of steep hill slopes and narrow canyons. The length of its main channel is about 8 km with an average stream gradient of 80 m/km. The texturally-filtered DEM of the Elder Creek Basin was extracted from 1.0 m LIDAR DEM of the South Fork Eel River watershed which is a part of University of California at Berkley's NCALM (National Center for Airborne Laser Mapping) project and can be retrieved via (www.ncalm.org) (Fig. 2).

III. WAVELET TRANSFORM

The one dimensional (1-D) wavelet transform of a continuous signal, $f(x)$, is defined as [27]:

$$d_w(a_w, b_w) = \frac{1}{\sqrt{a_w}} \int_{-\infty}^{+\infty} \psi^* \left(\frac{x-b_w}{a_w} \right) f(x) dx \quad (1)$$

Where ψ^* corresponds to the complex conjugate and $\psi(x)$ is

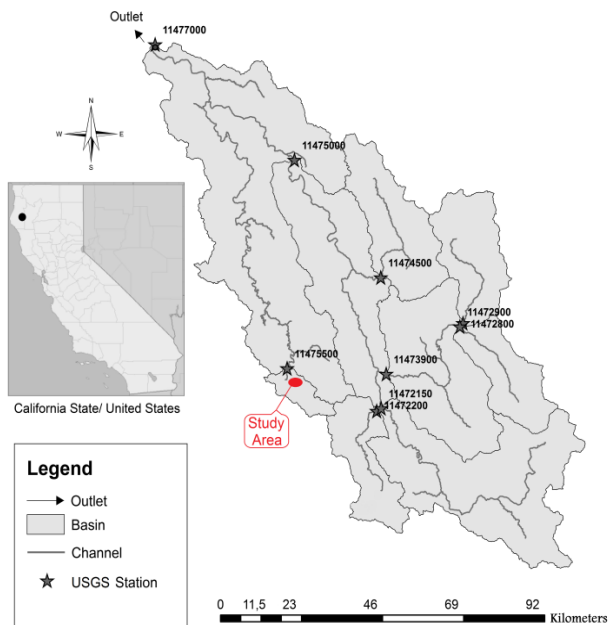


Fig. 1 Eel River map and study area location

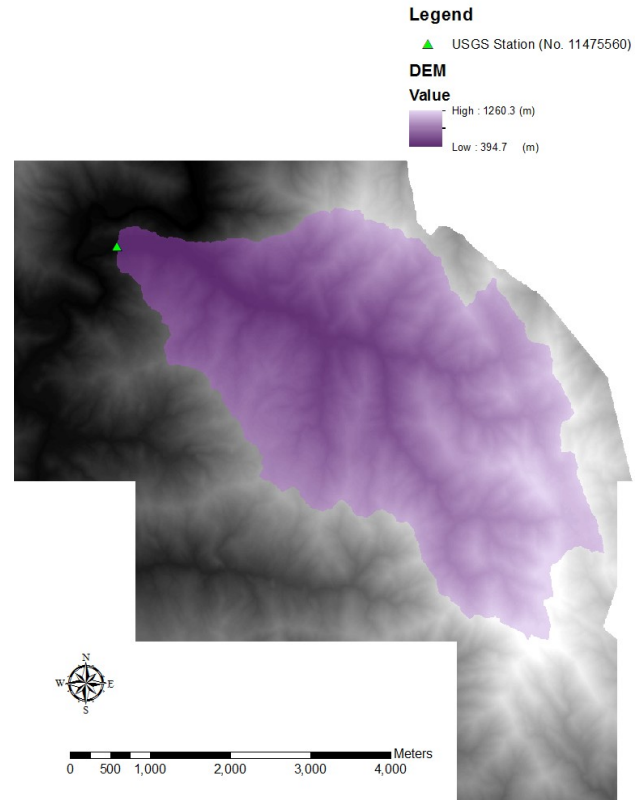


Fig. 2 Study area DEM

called wavelet function or mother wavelet. The parameter a_w acts as a dilation factor, while b_w corresponds to a temporal or spatial translation of the function $\psi(x)$, which allows the study of the signal around b_w . In order to be classified as a wavelet, a function must have finite energy, and it must satisfy the following "admissibility conditions" [27]:

$$\int_{-\infty}^{+\infty} \psi(x) dx = 0, \quad C_\psi = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(w)|^2}{|w|} dw < \infty \quad (2)$$

where, $\hat{\psi}(w)$ is Fourier transform of $\psi(x)$; i.e., the wavelet must have no zero frequency component. In order to obtain a reconstruction formula for the studied signal, it is necessary to add "regularity condition" to the previous ones [27]:

$$\int_{-\infty}^{+\infty} x^l \psi(x) dx = 0 \quad \text{Where } l = 1, 2, \dots, k-1 \quad (3)$$

So the original signal may be reconstructed using the inverse wavelet transform as [27]:

$$f(x) = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_0^\infty \frac{1}{\sqrt{a_w}} \psi \left(\frac{x-b_w}{a_w} \right) d_w(a_w, b_w) \frac{da_w db_w}{a_w^2} \quad (4)$$

Almost all of the algorithms for specifying flow direction and specific area including D8 and D ∞ , use first order finite difference method to compute local slope. This method magnifies the DEM's noise since it considers a sudden jump and discontinuity, as a step function, between the elevations of two adjacent cells. Thus, a smoothing filter may be utilized to reduce the noise effect when high-resolution DEM is used to compute the local slopes. In the context of channel network extraction, Lashermes *et al.* [22] applied convolution product with the kernel 'g' on the elevation height (h) as:

$$\frac{\partial(h * g)}{\partial x} = \frac{\partial h}{\partial x} * g = h * \frac{\partial g}{\partial x} \quad (5)$$

$$\text{in which } (h * g)_{(x)} = \int_0^x h_{(t)} g_{(x-t)} dt$$

to compute the smoothed slope via the continues wavelet transform (CWT).

Comparison of (5) with (1) implies that slope in x-direction smoothed by the kernel 'g', i.e. $m_x = \frac{\partial h}{\partial x} * g$, is equal to the wavelet coefficient of DEM with mother wavelet of $\frac{\partial g}{\partial x}$. Similarly, m_y and then magnitude and direction of steepest slope are computed for each cell as:

$$m = \sqrt{m_x^2 + m_y^2} \quad (6)$$

and,

$$\tan \beta = \gamma \pi + \arctan \left(\frac{m_y}{m_x} \right) \quad (7)$$

Where, $\gamma=0$ if $m_x > 0$; $\gamma=1$ if $m_x < 0$ and $m_y > 0$; $\beta = -1$ if $m_x < 0$ and $m_y < 0$. Thereafter, computed slope can be used to calculate specific upslope area and TI distribution. In this study, the first derivative of the Gaussian function as the Gaussian wavelet with unit energy, in the form of (8) was applied to the 1-m resolution DEM in both x and y directions to compute magnitude and direction of slope and then spatial distribution of TI were extracted.

$$\frac{\partial g}{\partial x} = \left(\frac{32}{\pi} \right)^{\frac{1}{4}} x e^{-x^2} \quad (8)$$

The Gaussian wavelet is popular in geophysical applications, particularly in terrain analysis because its shape, as shown in Fig. 3, is similar to the topography form. It is worthy of note that the wavelet transform should be applied to an extended area started from outside of the study's region in order to cope with the edge effect [28].

IV. RESULTS AND DISCUSSION

The NCLAM's 1-m resolution LIDAR DEM was re-sampled into 2, 4, 8, 16, 32 and 60-m resolution DEMs. Instead of using the localized mean which may lead to an overly smoothed topography, the aggregation process was performed by passing the aggregation window of target size across the original 1-m resolution DEM and assigning the elevation of the central pixel of window to the corresponding pixel in the coarser DEM. For the re-sampled DEMs the classic sink removal method [29] was applied and TI distributions were computed using the most common flow pathway algorithm (i.e., D8) and also CWT methods. The results have been presented in Figs. (4a) and (4b) for D8 and CWT methods, respectively which indicate that by increasing the grid cell size, not only a positive shift in mean of TI distribution is appeared but also higher order

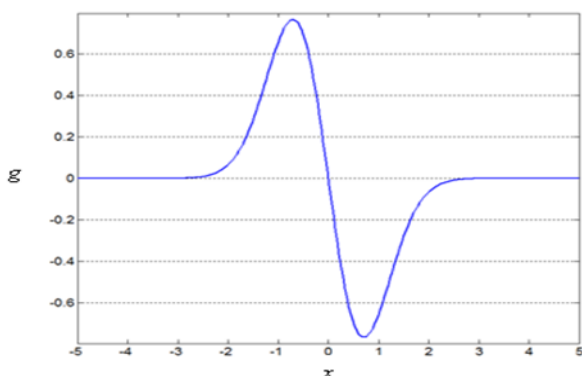


Fig. 3 Gaussian wavelet

momentums of the distribution may also be changed. Finer grid sizes typically lead to larger values of $\tan \beta$ and smaller values of a since steeper hill slopes and smaller drainage area are resolved and as a result provide smaller TI values. Fig. 5 compares calculated TI distributions by two algorithms using 1- and 32-m resolution DEMs. In CWT method, the calculated local slope is smaller than computed slope by D8. Therefore, the discrepancy between two distributions is more marked especially in the beginning of the distribution which grid sizes typically lead to larger values of $\tan \beta$ and smaller values of a since steeper hill slopes and smaller drainage area are resolved and as a result provide smaller TI values. Fig. 5 compares calculated TI distributions by two algorithms using 1- and 32-m resolution DEMs. In CWT method, the calculated local slope is smaller than computed slope by D8. Therefore, the discrepancy between two distributions is more marked especially in the beginning of the distribution which his related to steep headwaters and more sensitive to the cell size. However in the coarser grid size, the whole trend of the flow pathway is almost identical in both methods. Of course, the magnitude of the discrepancy depends on the catchment's terrain and as an evidence for the importance of slope; the TI distribution is more sensitive to the grid size and flow direction algorithm for the steeper catchments such as Elder Creek Basin.

Also, according to Figs.(6a) and (6b) which show respectively the spatial distributions of TI within the basin extracted from 1 and 8-m resolution DEMs, for the finer DEM the structure of the TI distribution is much complex and irregular. In addition to the slope, the specific area may also cause this complexity. The smallest value of the specific area equals to the grid cell size and in a low resolution DEM, a cell in a down slope valley position will have a large specific area value. However, the cells in the similar positions of the corresponding high resolution DEM may have either a large specific area if they are located along the channel or a small specific area if they have slightly higher elevation than the surrounding cells and located outside of the channel.

A large number of TIs with low and even negative values can be seen in the computed distribution by 1-m resolution DEM (see Fig.6a). According to the fractal property of the topography there is an exponential relationship between the elevation difference (Δh) and distance (ϵ) of two adjacent cells within the basin's DEM as [30], [6]:

$$\Delta h = \alpha \epsilon^{2-D} \rightarrow \tan \beta = \frac{\Delta h}{\epsilon} = \alpha \epsilon^{1-D} \quad (9)$$

Since the fractal dimension (D) and proportionality coefficient (α) are constant for the topography, by decreasing the grid cell size (ϵ) the local slope is increased. Furthermore, it has been already shown that the uncertainty in slope has a grid spacing dependence and as grid is fined, there is a greater probability of error in slope [31]. Consequently, it is clear that in a high-resolution DEM, the high slopes result in very low (even negative) values of TI on steep slopes. The complexity of the local slope ($\tan \beta$) distribution, extracted from 1-m resolution DEM, at different parts of the basin may lead to this outcome (Fig. 7). Similar result and argument were reported by Quinn *et al.* [3] and Lane *et al.* [32] using 5 and 2-m resolution DEMs, respectively. Quinn *et al.* [3] reasoned that this is because fining DEM resolution tends to

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increase the numbers of cells with a low specific area value whilst leaving the overall slope gradient the same. Also as it was previously mentioned, the fractal theory which implies the scale effect on the local slope value and also the noise and pits are other important reasons for the phenomenon. In spite of DEM and slope smoothing potential of applied CWT, some negative TI values (about 0.7%) can be still seen in Fig. (6a). Two arguments are conceivable for such performance of the CWT. The first is related to the basis geometry, so that at very steep slope or cliff having a shape like the step function, the filtering may even yield to much larger slope values. The second is that the applied CWT just acts on x and y directions without any filtering across diagonal direction wherever may need to be smoothed.

The effect of change in TI distribution may be imposed to the hydro-environmental model. Although the change in the TI distribution is offset by calibration of the model's parameter(s) to match the simulated and observed hydrographs, such re-optimized parameters may lead to unreasonable results when tested against internal field data [3].

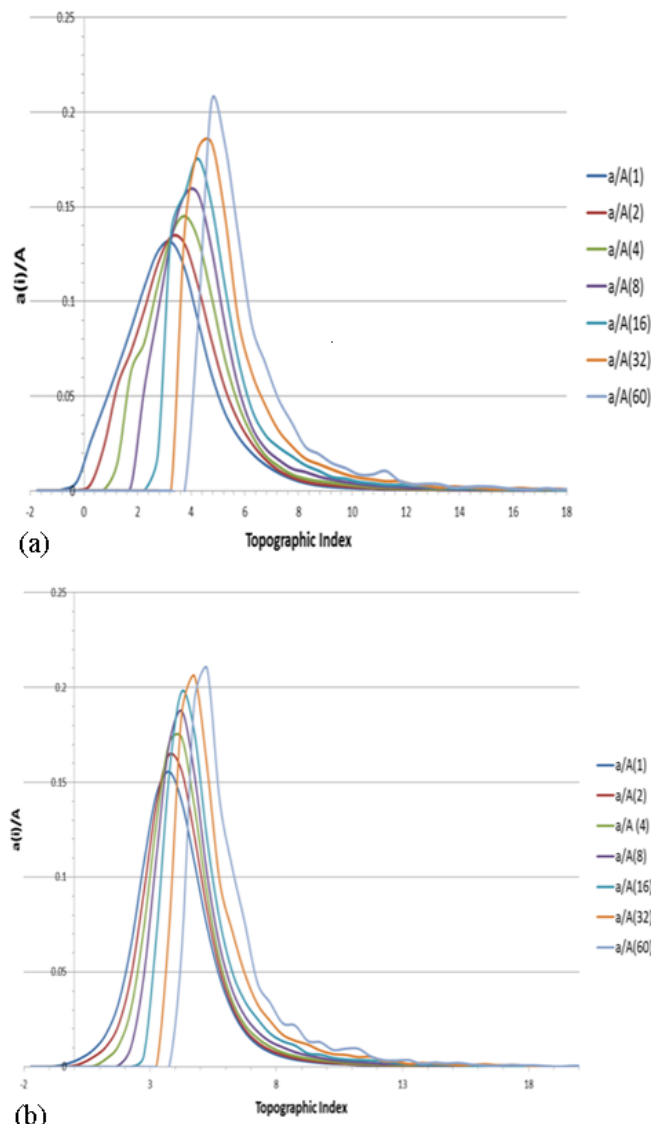


Fig. 4 Computed TI distributions by a) D8 b) CWT methods Therefore, considering the developments in new generation of high resolution DEM, it may become apparent that the application of finer grid resolution is more appropriate to the observed processes.

However as mentioned formerly, some other problems may be faced when a high-resolution DEM is used; such as corruption of the terrain analysis by error in terms of instrument noise or scale irregularities in general [31], [22], and [19].

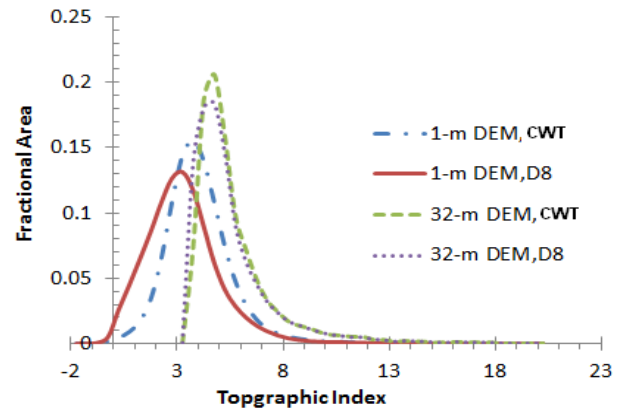


Fig. 5 Comparison of calculated TI distributions by two algorithms using 1- and 32-m resolution DEMs.

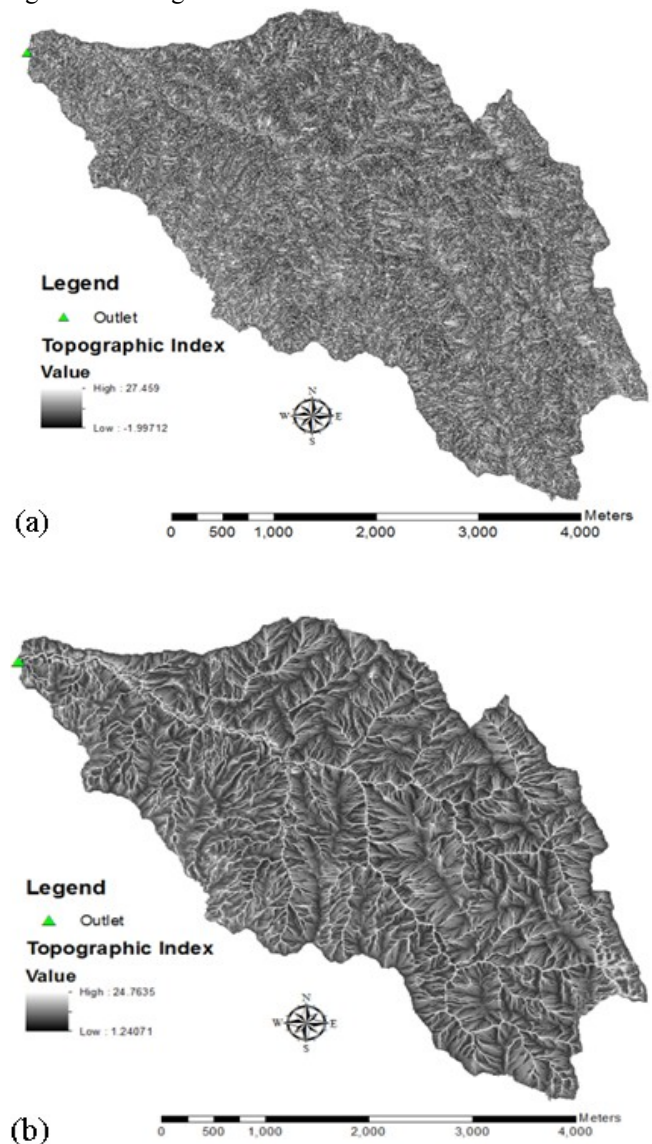


Fig. 6 Spatial distribution of TI extracted from DEM with resolution a) 1-m, b) 8-m

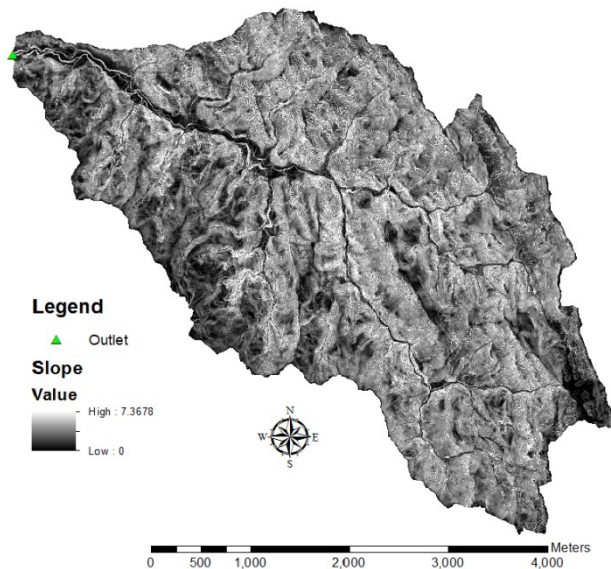


Fig. 7 Spatial distribution of the slope extracted from original 1-m resolution DEM

V. CONCLUSION

In this study, the concept of continuous wavelet transform (CWT) was used to compute local ground surface slope and then cumulative drainage area and topographic index (TI) distribution of the watershed which are important geomorphologic parameters in the hydro-environmental studies. The Gaussian CWT was applied to the DEMs at different resolutions extracted from the high-resolution LIDAR DEM. A systematic trend could be seen in the computed TI distributions at different resolutions; so that by decreasing the resolution of the DEM, the mean of the distribution is increased. Furthermore, there is a discrepancy between the mean and formation of the computed distributions by the classic D8 and proposed CWD methods. The smoothing ability of the CWT allows the method to handle the existing pits and noise in the high-resolution DEM.

It is recommended to employ the wavelet-based TI distribution in a hydro-environmental model and verify its ability in a real world problem. A few studies have been already presented to downscale the TI distribution from a coarse-resolution DEM and in most of them by neglecting the scale effect on the slope value, it has been assumed that TI distribution at different scales have similar shapes but not necessarily similar means (e.g., [33], [34]). However as a research plan for the future study, the fractal (or multi-fractal) concept and proposed wavelet analysis may be conjugated and used for downscaling and re-scaling of the TI distribution.

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