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Generalization of Horadam's Sequence

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Abstract In this paper a new class of Fibonacci like sequence is introduced. Here we consider non-homogeneous recurrence relation to obtain generalization of Horadam's Sequence. Some identities concerning this new sequence are obtained and proved. Some examples are given in support of the results.

Keywords: *pseudo fibonacci numbers, non-homogeneous recurrence relation*

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1. Introduction

Fibonacci sequence is generalized in many ways. One of the most widely used extension is the one given by A.F. Horadam [2]. He defined the generalized sequence of numbers as follows. Let $\{W_n\}$ be a sequence defined by

$$W_n = W_n(a,b;p,q) = pW_{n-1} - qW_{n-2}, \qquad (1.1)$$

for $n \geq 2$ where p, q, a and b are integers with $W_0 = a$ and $W_1 = b$.

One can see that $\{W_n\}$ reduces to generalized Fibonacci sequence $\{U_n\}$ when $a=0$ and $b=1$, and subsequently to the Fibonacci sequence $\{F_n\}$ when $p=1$, $q=-1$. That is $U_n = W_n(0,1;p,q)$ and $F_n = W_n(0,1;1,-1)$. In a series of papers [1,3,4,5,8] various properties of the generalized sequence $\{W_n\}$ have been developed. Using Horadam Sequence and the concept of Pseudo Fibonacci sequence [A224508] defined earlier in [6] and studied further in [7], we now define a new extension of the sequence $\{W_n\}$.

Definition 1. *The generalized Pseudo Fibonacci (GPF) Sequence $\{G_n\}$ is defined as the sequence satisfying the following non-homogeneous recurrence relation*

$$G_{n+2} = pG_{n+1} - qG_n + At^n \qquad (1.2)$$

for $n \geq 0$, $A \neq 0$ and $t \neq 0$, α, β with $G_0 = a$ and $G_1 = b$.

Here a,b,p,q are integers and α,β are distinct roots of characteristic equation $x^2 - px + q = 0$ of the corresponding homogeneous equation.

First few GPF numbers are given below:

$$G_0 = a, G_1 = b,$$
$$G_2 = (pb - qa) + A,$$
$$G_3 = (p^2b - pqa - qb) + pA + At,$$
$$G_4 = (p^3b - p^2qa - 2pqb + q^2a) + (p^2 - q)A + pAt + At^2,$$
$$G_5 = (p^4b - p^3qa - 3p^2qb + 2q^2a + q^2b) + (p^3 - 2pq)A + (p^2 - q)pAt + pAt^2 + At^3.$$

Observe that each GPF number G_n, $n \geq 2$ consist of two parts. The first part is an expression in p, q, a and b, while the second is a polynomial in t whose coefficients are A times terms in p and q. This is shown in the following tables:

Table 1. First part of G_n, $n \geq 2$

n	Expression in p,q,a,b
2	$pb - qa$
3	$p^2b - pqa - qb$
4	$p^3b - 2pqb - p^2qa + q^2a$
5	$p^4b - 3p^2qb + q^2b - p^3qa + 2q^2a$
6	$p^5b - 4p^3qb - p^4qa + 3q^2a + 2q^2$

Table 2. Second part of G_n, $n \geq 2$

n	A	At	At^2	At^3
2	1			
3	p	1		
4	$p^2 - q$	p	1	
5	$p^3 - 2pq$	$p^2 - q$	p	1

From the above tables, we have the following relation between GPF and Horadam numbers W_n.

Theorem 2. *For $m \geq 2$, the term*

$$G_m = W_m(a,b;p,q) + A\sum_{k=1}^{m-1} W_k(0,1;p,q)t^{m-k-1}$$

of the sequence $\{G_m\}$ *satisfy the non-homogeneous recurrence relation*

$$G_{m+2} = pG_{m+1} - qG_m + At^m.$$

Proof. Consider

$$\begin{aligned}
&\sum_{k=1}^{m+1} W_k(0,1;p,q)t^{m-k+1} \\
&= W_1t^m + W_2t^{m-1} + W_3t^{m-2} + \ldots + W_{m+1} \\
&= W_1t^m + (pW_1 - qW_0)t^{m-1} + (pW_2 - qW_1)t^{m-2} \\
&+ \ldots + (pW_m - qW_{m-1}) \\
&= W_1t^m + p(W_1t^{m-1} + W_2t^{m-2} + \ldots + W_m) \\
&-q(W_0t^{m-1} + W_1t^{m-2} + \ldots + W_{m-1}) \\
&= W_1t^m + p\sum_{k=1}^{m} W_k(0,1;p,q)t^{m-k} \\
&-q\sum_{k=0}^{m-1} W_k(0,1;p,q)t^{m-k-1}.
\end{aligned}$$

That is,

$$\begin{aligned}
&\sum_{k=1}^{m+1} W_k(0,1;p,q)t^{m-k+1} = W_1(0,1;p,q)t^m \\
&+p\sum_{k=1}^{m} W_k(0,1;p,q)t^{m-k} - q\sum_{k=0}^{m-1} W_k(0,1;p,q)t^{m-k-1}
\end{aligned} \tag{1.3}$$

Now

$$G_{m+2} = W_{m+2}(a,b;p,q) + A\sum_{k=1}^{m+1} W_k(0,1;p,q)t^{m-k+1}.$$

Using equations (1.1) and (1.3), we write

$$\begin{aligned}
&G_{m+2} = pW_{m+1}(a,b;p,q) - qW_m(a,b;p,q) \\
&+Ap\sum_{k=1}^{m} W_k(0,1;p,q)t^{m-k} \\
&-qA\sum_{k=0}^{m-1} W_k(0,1;p,q)t^{m-k-1} + AW_1(0,1;p,q)t^m \\
&= p[W_{m+1}(a,b;p,q) + A\sum_{k=1}^{m} W_kt^{m-k}] \\
&-q[W(a,b;p,q) + A\sum_{k=0}^{m-1} W_k(0,1;p,q)t^{m-k-1}] + At^m \\
&= pG_{m+1} - qG_m + At^m.
\end{aligned}$$

Hence the theorem.

2. Some Identities for G_n

In this section, we obtain some fundamental identities for GPF sequence $\{G_n\}$.

- **Binet type Formula:**

Let $z = z(t) = At^2 - pt + q$.

Then the Binet form of G_n is given by

$$G_n = c_1\alpha^n + c_2\beta^n + zt^n, \tag{2.1}$$

where

$$\begin{cases} c_1 = \dfrac{(b - a\beta) - z(t - \beta)}{\alpha - \beta}, \\ c_2 = \dfrac{(a\alpha - b) - z(\alpha - t)}{\alpha - \beta}, \end{cases} \tag{2.2}$$

α, β are the distinct roots of the equation $x^2 - px + q = 0$ given by

$$\alpha = \frac{p+d}{2}, \beta = \frac{p-d}{2}, \tag{2.3}$$

writing $d = \sqrt{p^2 - 4p}$.

Note that

$$\alpha + \beta = p, \alpha\beta = q, \alpha - \beta = d. \tag{2.4}$$

We can deduce from (2.2) that

$$\begin{cases} c_1 + c_2 = a - z, \\ c_1 - c_2 = ((2b - ap) - z(2t - p))d^{-1}, \\ c_1c_2 = ed^{-2}, \end{cases} \tag{2.5}$$

where

$$c = abp - b^2 - a^2q - z\{bp - 2bt - 2aq + atp + A\}. \tag{2.6}$$

- **Generating function**:

Generating function $G^*(x)$ for G_n is given by

$$G^*(x) = \frac{1}{\left(1 - px + qx^2\right)}\left\{\frac{Ax^2}{1 - tx} + \left(a + bx - apx\right)\right\}.$$

We have the following result for sum of first n GPF numbers. Proofs follow from recurrence relation and Binet formula.

Proposition 3. *For* $p - q \neq \pm 1$

i) $\displaystyle\sum_{k=0}^{n-1} G_k = 1(p - q - 1)\left\{G_{n+1} - b + (p-1)(a - G_n) - A\sum_{k=0}^{n-1} t^k\right\}.$

ii) $\displaystyle\sum_{k=0}^{n-1} (-1)^k G_k = 1(p - q + 1)[(-1)^{n+1}G_{n+1}$

$\displaystyle+b - (p+1)(a + (-1)^{n+1}G_n) + A(1-t)\sum_{k=0}^{n-1} (-1)^k t^{2k}].$

Using the recurrence relation (1.2) , we have the following result. The same can also be obtained by induction.

Proposition 4.

$$\sum_{k=0}^{n} G_kt^{k-1} = \frac{1}{(1 - pt + qt^2)}\left[\begin{array}{l} a/t - ap + b \\ -t^n(G_{n+1} - qtG_n) \\ +A\displaystyle\sum_{k=0}^{n-1} t^{2k+1} \end{array}\right].$$

Proof. Using the recurrence relation (1.2), we have

$$G_k t^{k-1} = [pG_{k-1} - qG_{k-2} + At^{k-2}]t^{k-1}$$
$$G_{k-1} t^{k-2} = [pG_{k-2} - qG_{k-3} + At^{k-3}]t^{k-2}$$
.
.
.
$$G_2 t^1 = [pG_1 - qG_0 + At^0]t^1.$$

On summing both sides of these $(n-1)$ equations, we get,

$$\sum_{k=2}^{n} G_k t^{k-1} = p\sum_{k=1}^{n-1} G_k t^k - q\sum_{k=0}^{n-2} G_k t^{k+1} + A\sum_{k=0}^{n-2} t^{2k+1}.$$

i.e.

$$\sum_{k=0}^{n} G_k t^{k-1} - G_0 t^{-1} - G_1 = p\sum_{k=0}^{n} G_k t^k - p(G_0 + G_n t^n)$$
$$-q\sum_{k=0}^{n} G_k t^{k+1} + q(G_{n-1}t^n + G_n t^{n+1}) + A\sum_{k=0}^{n-2} t^{2k+1}.$$

i.e.

$$\sum_{k=0}^{n} G_k t^{k-1}(1 - pt + qt^2) = G_0 / t + G_1 - pG_0$$
$$-(pG_n - q(G_{n-1})t^n + qG_n t^{n+1}) + A\sum_{k=0}^{n-1} t^{2k+1}.$$

Hence

$$\sum_{k=0}^{n} G_k t^{k-1} = 1(1 - pt + qt^2)$$
$$\left\{at - ap + b - t^n(G_{n+1} - qtG_n) + A\sum_{k=0}^{n-1} t^{2k+1}\right\}.$$

We now obtain the sum of the squares of GPFs and sum of the product of two consecutive GPFs.

For simplicity, let $X = \sum_{k=0}^{n} G_k^2$, $Y = \sum_{k=0}^{n} G_k G_{k+1}$, $S_1 = \sum_{k=0}^{n} G_k t^{k-1}$ and $S_2 = \sum_{k=0}^{n} t^{2k}$.

Let $v_1 = 1 - p^2 - q^2$ and $v_2 = 1 + p^2 - q^2$.

Further let

$$P_1 = (1 - p^2)[G_0^2 - G_{n+1}^2] + (G_1^2 - G_{n+2}^2)$$
$$+2A(G_0 t^{-2} + G_1 t^{-1} - G_{n+1}t^{n-1} - G_{n+2}t^n),$$

$$P_2 = (1 + p^2)[G_0^2 - G_{n+1}^2] + (G_1^2 - G_{n+2}^2)$$
$$-2p(G_{n+1}G_{n+2} - G_0 G_1).$$

We have the following results:

Proposition 5. *For* $p \neq 0, v_1 + qv_2 \neq 0$

i) $$X = \sum_{k=0}^{n} G_k^2 = \frac{\begin{pmatrix} P_1 + qv_2 + 2(t^{-1} - q^2 t)AS_1 \\ +(q-1)A^2 S_2 + qP_2 \end{pmatrix}}{v_1},$$

ii) $$Y = \sum_{k=0}^{n} G_k G_{k+1} = v_2 P_1 - v_1 P_2$$
$$+2(t^{-1}v_2 + tqv_1)AS_1 - (v_1 + v_2)A^2 S_2 2p(v_1 + qv_2).$$

Proof. Consider

$$G_{k+2}^2 - p^2 G_{k+1}^2$$
$$= (G_{k+2} - pG_{k+1})(G_{k+2} + pG_{k+1})$$
$$= 2AG_{k+2}^2 - 2pqG_k G_{k+1} + q^2 G_k^2 - A^2 t^{2k}.$$

Hence, summing up to n+1 terms on both sides, we get,

$$\sum_{k=0}^{n} G_{k+2}^2 - p^2 \sum_{k=0}^{n} G_k^2$$
$$= 2A\sum_{k=0}^{n} G_{k+2}^2 + q^2 \sum_{k=0}^{n} G_k^2 - 2pq\sum_{k=0}^{n} G_k G_{k+1} - \sum_{k=0}^{n} A^2 t^{2k}.$$

Adjusting the variables of summation and simplifying, we get

$$v_1 X + 2pqY = P_1 + At^{-1}S_1 - A^2 S_2.$$

Similarly, starting with

$$G_{k+2}^2 - q^2 G_k^2$$
$$= 2pG_{n+1}G_{n+2} - 2AqG_n t^n - p^2 G_{n+1}^2 + A^2 t^{2n}$$

and simplifying as above, we get

$$v_2 X - 2pY = P_2 - 2qAtS_1 + A^2 S_2.$$

Solving these two equations for X and Y, we get the required results.

Next result deals with sum of even and odd terms of GPF sequence. Again for simplicity, let $E = \sum_{i=1}^{n} G_{2i}$ and $O = \sum_{i=1}^{n} G_{2i-1}$, $O_A = A\sum_{i=1}^{n} t^{2i-1}$, $E_A = A\sum_{i=0}^{n} t^{2i}$ so that $E_A + O_A = A\sum_{i=0}^{2n} t^i$.

Proposition 6. *The sum of the even (odd) indexed terms of* $\{G_n\}$ *is given by*

(i) $$\sum_{k=1}^{n} G_{2k} = \frac{1}{\{p^2 - (1+q^2)\}}[p(G_{2n+1} - G_1)$$
$$+(1+q)(qG_0 + G_{2n+2} - pG_{2n+1})$$
$$-pA\sum_{k=1}^{n} t^{2k-1} - (1+q)\sum_{k=0}^{n} t^{2k}]$$

and

(ii) $$\sum_{k=1}^{n} G_{2k-1} = \frac{1}{\left\{p^2 - \left(1 + q^2\right)\right\}}$$
$$[pqG_0 + pG_{2n+2} - p^2 G_{2n+1} + (1+q)(G_{2n+1} - G_1)$$
$$-(1+q)A\sum_{k=1}^{n} t^{2k-1} - Ap\sum_{k=0}^{n} t^{2k}]$$

Provided $p^2 \neq 1+q^2$.

Proof. From the recurrence relation (1.1), we have,

$$pG_{2k} = G_{2k+1} + qG_{2k-1} - At^{2k-1}.$$

Summing up to n terms, we get

$$p\sum_{k=1}^{n} G_{2k} = \sum_{k=1}^{n} G_{2k+1} + q\sum_{k=1}^{n} G_{2k-1} - A\sum_{k=1}^{n} t^{2k-1},$$

which on simplifying yields,

$$pE = (1+q)O + G_{2n+1} - G_1 - O_A. \quad (2.7)$$

Similarly using the relation

$$pG_{2k+1} = G_{2k+2} + qG_{2k} - At^{2k},$$

and summing up to n terms, we get,

$$pO = (1+q)E + G_{2n+2} - pG_{2n+1} + qG_0 - E_A. \quad (2.8)$$

Solving (2.7) and (2.8), we get the required results.

We have following identity.

Proposition 7. *For* $m,n > 0$,

$$G_mG_n - qG_{m-1}G_{n-1} = (b-zt)G_{m+n-1}$$
$$+(z-a)qG_{m+n-2} + (aq-bt)zt^{m+n-2}$$
$$+z[G_mt^n + G_nt^m - G_{n-1}qt^{m-1} - G_{m-1}qt^{n-1}].$$

Proof. Using Binets formula (2.1),

$$\text{L.H.S.} = (c_1\alpha^m + c_2\beta^m + zt^m)(c_1\alpha^n + c_2\beta^n + zt^n)$$
$$-q(c_1\alpha^{m-1} + c_2\beta^{m-1} + zt^{m-1})(c_1\alpha^{n-1} + c_2\beta^{n-1} + zt^{n-1}).$$

On simplification, we get,

$$\text{L.H.S.} = [(b-a\beta) - z(t-\beta)]c_1\alpha^{m+n-1}$$
$$-[(a\alpha-b) - z(\alpha-t)]c_2\beta^{m+n-1} - z^2t^{m+n}$$
$$+zt^nG_m + zt^mG_n - qz\{t^{m-1}G_{n-1} + t^{n-1}G_{m-1} - zt^{m+n-2}\}$$

Since, $\alpha-\beta = d, c_1d = (b-a\beta) - z(t-\beta)$ and $c_2d = (a\alpha-b) - z(\alpha-t)$,

$$G_mG_n - qG_{m-1}G_{n-1} = (b-zt)G_{m+n-1} + (z-a)qG_{m+n-2}$$
$$+(aq-bt)zt^{m+n-2} + z[G_mt^n + G_nt^m - G_{n-1}qt^{m-1} - G_{m-1}qt^{n-1}].$$

By letting $m = n$ we have the following result.

Corollary 8.

$$G_n^2 - qG_{n-1}^2 = (b-z)G_{2n-1} + (z-a)qG_{2n-2}$$
$$+(aq-bt)zt^{2n-2} + 2z[t^nG_n - qt^{n-1}G_{n-1}].$$

Note that the above corollary along with Proposition 3.(i) can be used to find $\sum_{k=0}^{n} G_k^2$ obtained in Proposition 5(i).

Next we prove a version of Catalan's Identity for GPF numbers.

Proposition 9.

$$G_{n+r}G_{n-r} - G_n^2 = eq^{n-r}u_{r-1}^2$$
$$+zt^n[t^rG_{n-r} + t^{-r}G_{n+r} - 2G_n]$$

where $u_r = W_r(1,p;p,q)$ *and* e *is as defined by* (2.6).

Proof. Using (2.3)

$$\text{L.H.S.}$$
$$= (c_1\alpha^{n+r} + c_2\beta^{n+r} + zt^{n+r})(c_1\alpha^{n-r} + c_2\beta^{n-r} + zt^{n-r})$$
$$-(c_1\alpha^n + c_2\beta^n + zt^n)^2$$
$$= c_1^2\alpha^{2n} + c_2^2\beta^{2n} + z^2t^{2n} + c_1c_2(\alpha^{n+r}\beta^{n-r} + \alpha^{n-r}\beta^{n+r}$$
$$+zt^{n+r}(c_1\alpha^{n-r} + c_2\beta^{n-r}) + zt^{n-r}(c_1\alpha^{n+r} + c_2\beta^{n+r})$$
$$-\{c_1^2\alpha^{2n} + c_2\beta^{2n} + z^2t^{2n}$$
$$+2(c_1c_2\alpha^n\beta^n + zc_2\underline{e}ta^nt^n + zc_1t^n\alpha^n)\}$$
$$= c_1c_2\alpha^n\beta^n(\alpha^r\beta^{-r} + \alpha^{-r}\beta^r - 2) + zt^{n+r}G_{n-r}$$
$$-z^2t^{2n} + zt^{n-r}G_{n+r} - z^2t^{2n} + zt^nG_n + 2z^2t^{2n}$$
$$= c_1c_2(\alpha\beta)^{n-r}(\alpha^r - \beta^r)^2 + zt^n[(t^rG_{n-r} + t^{-r}G_{n+r} + G_n]$$
$$= eq^{n-r}\frac{(\alpha^r-\beta^r)^2}{(\alpha-\beta)^2} + zt^n[(t^rG_{n-r} + t^{-r}G_{n+r} + G_n]$$
$$= eq^{n-r}u_{r-1}^2 + zt^n[(t^rG_{n-r} + t^{-r}G_{n+r} + G_n].$$

From this result we immediately have a version of Cassini's identity for GPF numbers.

Corollary 10.

$$G_{n+1}G_{n-1} - G_n^2 = eq^{n-1} + zt^n[tG_{n-1} + t^{-1}G_{n+1} - 2G_n].$$

Next we have an expression for G_{2n} in terms of binomial coefficients.

Proposition 11.

$$G_{2n} = (-q)^n\sum_{i=0}^{n}\binom{n}{i}(-p/q)^{n-i}G_{n-i} - z[(pt-q)^n - t^{2n}].$$

Proof. We have

$$\text{R.H.S.} = (-q)^n\sum_{i=0}^{n}\binom{n}{i}(-p/q)^{n-i}G_{n-i} - z[(pt-q)^n - t^{2n}]$$
$$= (-q)^n\sum_{i=0}^{n}\binom{n}{i}(-p/q)^{n-i}(c_1\alpha^{n-i} + c_2\beta^{n-i} + zt^{n-i})$$
$$-z[(pt-q)^n - t^{2n}]$$
$$= c_1\sum_{i=0}^{n}\binom{n}{n-i}(p\alpha)^{n-i}(-q)^i + c_2\sum_{i=0}^{n}\binom{n}{n-i}(p\beta)^{n-i}(-q)^i$$
$$+z\sum_{i=0}^{n}\binom{n}{n-i}(pt)^{n-i}(-q)^i - z[(pt-q)^n - t^{2n}]$$
$$= c_1(p\alpha-q)^n + c_2(p\beta-q)^n + z(pt-q)^n$$
$$-z[(pt-q)^n - t^{2n}].$$

Since, $\alpha^2 = p\alpha - q$ and $\beta^2 = p\beta - q$, we get,

$$\text{R.H.S.} = c_1\alpha^{2n} + c_2\beta^{2n} + zt^{2n} = G_{2n}.$$

Hence,

$$G_{2n} = (-q)^n\sum_{i=0}^{n}\binom{n}{i}(-p/q)^{n-i}G_{n-i} - z[(pt-q)^n - t^{2n}]$$

which is the required result.

3. Examples

In this section we present some examples in support of some results obtained in Section 2.

Examples: Consider $G_{n+2} = G_{n+1} - 2G_n + (-1)^n$, with $G_0 = 0, G_1 = 1$.

Here, $p = 1, q = 2, t = -1, A = 1$.

First few terms of G_n are $G_0 = 0,$ $G_1 = 1,$ $G_2 = 2,$ $G_3 = -1,$ $G_4 = -4,$ $G_5 = -3,$ $G_6 = 6,$ $G_7 = 11,$ $G_8 = 0,$ $G_9 = -23,$ $G_{10} = -22.$

(1) Verification of Proposition 5(i).

When $n = 5$, we have $S_1 = -1,$ $S_2 = 6,$ $P_1 = -128,$ $P_2 = -6,$ $v_1 + qV_2 = -8$. Then

$$\text{L.H.S.} = \sum_{k=0}^{5} G_k^2 = G_0^2 + G_1^2 + \ldots + G_5^2 = 31.$$

$$\text{R.H.S.} = -128 - 6 + 6 - 120 - 8 = -248 - 8 = 31.$$

Result is verified.

(2) Verification of Proposition 5 (ii).

Here let $n = 6$. We have, $P_1 = -19,$ $P_2 = -241,$ $S_1 = -7,$ $S_2 = 7.$ Then

$$\text{L.H.S} = \sum_{k=0}^{6} G_k G_{k+1} = G_0 G_1 + G_1 G_2 + \ldots + G_6 G_7 = 64.$$

$$\text{R.H.S.} = 38 - 964 - 140 + 42 - 16 = 64.$$

Result is verified.

(3) Verification of Proposition 7.

Let $m = 2$ and $n = 3$. Then $z = 14$.

$$\text{L.H.S.} = G_2 G_3 - 2G_1 G_2 = 2(-1) - 2(2) = -6.$$

$$\text{R.H.S.} = -5 - \tfrac{1}{2} - \tfrac{1}{4} - \tfrac{3}{4} + \tfrac{1}{2} = -6.$$

Result is verified.

(4) Verification of Proposition 9.

Let $n = 6$ and $r = 2$. $z = \dfrac{1}{4}, e = -2, u_1 = 1$

$$\text{L.H.S} = G_8 G_4 - G_6^2 = -36.$$

$$\text{R.H.S} = -32 - 14X(4 + 12) = -32 - 4 = -36.$$

Result is verified.

(5) Verification of Proposition 11.

Let $n = 5$.

$$\text{L.H.S.} = G_{2n} = -22$$

$$\text{R.H.S.} = (-q)^n \sum_{i=0}^{n} \binom{n}{i} (-p/q)^{n-i} G_{n-i} - z[(pt - q)^n - t^{2n}]$$

$$= -32(83/32) - 1/4(-244) = -83 + 61 = -22.$$

Result is verified.

4. Conclusion

The well known Horadam sequence is generalized via non homogeneous recurrence relation to obtain a Fibonacci like sequence. All the usual identities and properties of Fibonacci like sequences are obtained for the new generalization of Horadam sequence.

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