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Priority Network Access Pricing for Electric Power

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Abstract

We propose a priority-pricing scheme for zonal access to the electric power grid that is uniform across all buses in each zone. The Independent System Operator (ISO) charges bulk power traders a per unit ex-ante transmission access fee based on the expected option value of the generated power with respect to the random zonal spot prices. The zonal access fee depends on the injection zone and a self-selected strike price determining the scheduling priority of the transaction. Inter zonal transactions are charged (or credited) with an additional ex-post congestion fee that equals the zonal spot price difference. The unit access fee entitles a bulk power trader to either physical injection of one unit of energy or a compensation payment that equals to the difference between the realized zonal spot price and the selected strike price. The ISO manages congestion so as to minimize net compensation payments and thus, curtailment probabilities corresponding to a particular strike price may vary by bus. We calculate the rational expectation equilibia for a three and four node system and demonstrate that the efficiency losses of the proposed second best scheme relative to the efficient dispatch solutions are modest.

1 Introduction

Transmission pricing and congestion management protocols are basic ingredients of any restructuring scheme aimed at promoting open access and competition in electricity markets. The Federal Energy Regulatory Commission (FERC) has recognized the crucial role of open access to transmission networks in Orders 888 and 889, which provide general principles for the pricing and utilization of scarce transmission capacity. One of the basic trade-offs involved in implementing FERC's open access ruling is choosing between economic efficiency and the simplicity of pricing and congestion management protocols. While it is generally agreed upon that transmission pricing should provide economic signals that will induce efficient use of the transmission grid it is not clear how precise such signals must be in order to capture most of the economic benefits from efficient congestion management. It is important to design a mechanism for regulating network access that is simple to implement, facilitates energy trading and will promote efficient network utilization.

Two extreme approaches on that spectrum are the Contract Network/Nodal Pricing approach (Hogan [5]) on one hand and the so called "postage stamp" approach on the other hand. In the nodal pricing approach, congestion management is performed through a central optimal dispatch, while transmission charges are determined ex-post and set to the nodal spot price differences (i.e. the market opportunity cost associated with using a particular transmission line). Under the assumption of perfect information (regarding generation cost) and abstraction of intertemporal aspects of the production costs and constraints this approach is "first-best" i.e. it produces the economic dispatch solution. It has been argued, however, that the claimed efficiency of the nodal pricing approach is based on unrealistic assumptions, the implementation of the idealized nodal pricing paradigm is overly complex and it relies on a highly centralized market structure that inhibits competition and customer choice. Furthermore, the ex-post determination of the transmission prices is a severe obstacle to efficient bilateral energy trading. (see Wu, Varaiya, Spiller and Oren [10]). The postage stamp approach, on the other hand, imposes a uniform charge on each unit of electricity shipped regardless of anything else (zonal differentiation has also been proposed). The simplicity of the postage stamp approach is compelling and it makes it easy for energy traders to incorporate transmission costs into their trading decisions. Unfortunately even with zonal differentiation this approach does not provide correct economic signals for transmission network usage and for congestion management. Neither does it provide locational economic signals for generation investments.

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An alternative to the nodal pricing, which in equilibrium can also achieve the first best outcome was proposed by Chao and Peck [2]. It is based on parallel markets for link based transmission capacity rights and energy trading under a set of trading rules imposed by the ISO. The trading rules specify the transmission capacity rights required to support bilateral energy trades between any two buses and are adjusted continuously to reflect changing system conditions. The decentralization in this approach and its reliance on market forces rather than on a central planning paradigm is attractive. However, its implementation would require a highly sophisticated level of electronic markets and information technology. Wilson [9] has demonstrated yet another way to achieve the first best solution by implementing a priority insurance scheme where the insurance premium varies for each pair of nodes. Neither of the above alternatives to nodal pricing offers a compelling improvement in terms of simplicity which is the primary objective of this paper.

We propose a priority insurance framework for assigning access privileges to the electricity transmission network where the premium or access fee is only differentiated according to the self-selected level of coverage but does not vary across buses within a set defined as a congestion zone. Instead, the probability of curtailment associated with each coverage level varies across buses and is endogenously derived from the congestion management protocol employed by the ISO, seeking to minimize net compensation to curtailed transactions. The reduced degrees of freedom in the premium design constrain the resulting equilibrium to produce a second best solution. However, the general direction of the market signals facilitate efficient use of scarce network resources by inducing transactions that have higher opportunity values or that impact more congestion prone segments of the grid to seek higher levels of insurance in order to obtain higher scheduling priorities at their respective buses. Furthermore, the opportunity to under-insure at injection nodes that do not impact congestion allows higher profit margins at such nodes thus providing the correct locational signals for generation investment.

The rest of the paper is organized as following: We present the formulation for both cases of a single spot market and multiple zonal spot markets in section two; in section three, we demonstrate how this scheme is implemented through numerical examples and evaluate the efficiency losses; finally, we conclude with some observations and remarks.

2 A priority insurance mechanism

We consider a market design patterned after the California restructuring plan where the network is partitioned into a few congestion zones and consumers in each zone face a uniform zonal spot price for electricity. The transmission system is operated by an Independent System Operator (ISO) that collects transmission service fees and is charged with efficient congestion management. However, our proposed transmission pricing scheme and congestion management protocol are new. For the purpose of this paper we formally define a zone as a subset of nodes sharing a common spot market (See Figure 1) All zones are mutually exclusive and collectively exhaustive. In our model, we assume that the transmission network has a fixed transmission capacity configuration and there is no uncertainty as to the availability of the transmission capacity. In each zone i, there exists a single zonal spot price $S_i \equiv S(\omega_i)$ contingent upon a random variable, ω_i, which is given exogenously. The fluctuation of S_i reflects the randomness in the supply and demand conditions. An unexpected hot summer day would cause a surge in demand for electricity, which naturally results in a high value of S_i and increased usage of the transmission network, possibly causing congestion. In such cases, the ISO needs to have an effective and efficient mechanism to allocate the limited transmission capacity to network users.

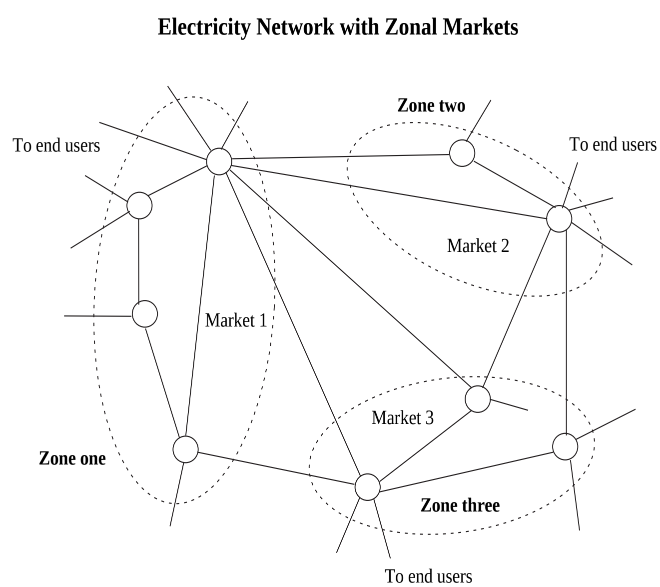

Figure 1: An electricity network with several spot markets

Our scheme offers bulk power traders wishing to engage in physical bilateral transactions a priority differentiated transmission network access tariff specific to the zone in which power is injected. In addition bilateral transactions across different zones are subject to an ex-post congestion charge (or credit) that equals to the spot price difference between the corresponding zones. It is assumed that curtailed transactions are settled either financially or through the purchase of replacement power and that the settlement price equals to the spot price at the buyer's zone.

Under the above framework, physical access to the transmission network by a generator producing power at marginal cost c per MWh can be valued as a financial "Call" option with strike price c in the zonal spot market corresponding to the injection node. Such an option is exercised only when the zonal spot price S_i exceeds the strike price and it yields the difference $S_i \Leftrightarrow c$. Hence, the actuarial value of the option is $E_{S_i}[Max(0, S_i \Leftrightarrow c)]$ with expectation taken over the random zonal spot price. Motivated by this observation our transmission pricing scheme proposes to impose a per MWh ex-ante transmission access charge in the form of an option insurance premium. The premium $X_i(c)$ in zone i equals the option value corresponding to the zonal spot price forecast and a self-selected strike price c determining the curtailment compensation. This payment would entitle a generator (or trader) to either physical access to the grid or a compensation payment that equals to the difference between the realized zonal spot price and the self-selected strike price. The ISO would then relieve congestion so as to minimize compensation payments to curtailed transactions net of the ex-post interzonal congestion payments.

If each transmission user were to select a strike price that reveals its true marginal generation cost then the above scheme would result in economic dispatch or least cost displacement. Furthermore, network users would be indifferent between physical access or compensation and would accrue zero profit whereas all the gains from producing at a cost below the spot price would go to the ISO (and ultimately to the transmission assets/rights owners). The simplicity of this approach comes from the fact that we use a single transmission access tariff that depends only on the strike price irrespective on the injection node within a zone. However, because of that simplification, users have an incentive to underinsure their transactions by selecting strike prices that are higher than their true marginal costs. In doing so they would estimate the probability of being curtailed and choose a strike price that will maximize their expected profits. Self-selected strike prices will depend on the true marginal cost and the probability of being curtailed at the particular injection node. In general, low marginal cost and high probability of curtailment will induce the selection of a lower strike price i.e., higher insurance level and higher service priority. Thus, the economic signal for congestion management is in the right direction although not exact.

The proposed mechanism can be described as a three-stage process (Figure 2).

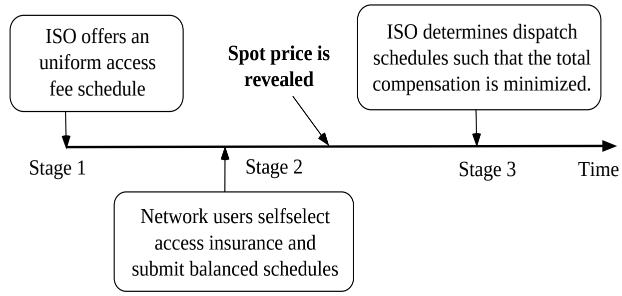

Figure 2: Timeline of the priority insurance scheme

Stage one The ISO posts a single insurance schedule $\{c, X_i(c)\}$ in each zone i, where $X_i(c)$ is the premium paid for insurance level c, allowing network users to insure network access rights for their transaction units (multiple unit can be insured at different levels)

It is assumed to be common knowledge that, when the spot price, S_i, is revealed, the ISO will manage the network congestion based on the criterion of minimizing total compensation payments net of inter-zonal congestion rent receipts. The implication of this assumption is that network users will form rational expectation about the locational service quality associated with a particular level of insurance at each node. The locational service quality is characterized in terms of the set of spot price contingencies under which transmission access at a specific bus is granted to a transaction unit insured at level c.

Stage two Before the random zonal spot prices are revealed, network users self-select an insurance level on each contracted unit in their schedules so as to maximize their expected profits. They do not need to specify the specific transaction nodes when purchasing their insurance. However, in a multi-zonal case the injection zones need to be revealed at this stage.

The spot price revelation in this time line may be interpreted as an accurate short-term spot price forecast employed by the network users to form their preferred schedule. This would be a more realistic interpretation when the reference settlement prices are the real time spot prices for imbalances.

Stage three At the third and final stage, network users submit their preferred schedules specifying injection nodes and selected insurance level for each transaction unit to the ISO. The ISO then grants transmission access or curtails submitted schedules so as to minimize total compensation payments net of ex-post congestion revenues for interzonal transaction. The curtailed transactions are paid the differences between their revealed opportunity costs and the zonal spot price corresponding to the injection node.

We next layout the formulation in both the single spot market case and the multiple zonal spot markets case. In the following formulations and the reminder of this paper we use a lossless DC-flow model to approximate the transmission constraints. The formulation can be generalized, however, to account for losses and reactive power and voltage constraints.

2.1 *Single spot market*

When there exists only one spot market in a network, the ISO simply imposes one insurance premium schedule $X(c)$ (**stage one**), which is a decreasing function of the strike price c, for the entire network.

2.1.1 A network user's self-selection problem (stage two) Given the ISO's insurance premium function $X(c)$, suppose a network user at node i subscribes to insurance level c for a transaction injecting at node i with true generation cost v. By purchasing the insurance, the user expects the transaction unit with insured cost c to obtain network access when the spot price S falls in the region $\Omega_i(c)$ (e.g. $\Omega_i(c) = [v, S_i(c)]$ where $S_i(c) = k_{i1} \Leftrightarrow k_{i2}c$) and be curtailed when the spot price S falls in $\overline{\Omega}_i(c)$, the complement region of $\Omega_i(c)$. With the rational expectation $\Omega_i(c)$, the network user chooses the optimal c so as to maximize expected profit. Namely, the network user at node i would solve the following problem to get the optimal insurance level for a type i transaction unit with true opportunity cost v

$$(NU1) \quad c_i^*(v) = \arg\max_c \int_{\Omega_i(c)} (s \Leftrightarrow v)dG(s) + \int_{\overline{\Omega}_i(c)} (s \Leftrightarrow c)dG(s) \Leftrightarrow X(c) \tag{1}$$

where $G(\cdot)$ is the cumulative distribution function of the random variable S.

2.1.2 The ISO problem (stage three) After the random spot price is revealed (or accurately predicted), all network users submit their usage requests as well as their insured cost (insurance level) c for each request. By aggregating the requested transactions according to their injection node and insurance levels the ISO ends up with insured cost curves $\widetilde{D}_i(c)$ for each i. We implicitly assume in this formulation an unlimited supply of displacement power (part of which can be curtailed demand) at the zonal spot price. When the network is congested, the ISO relieves the congestion by curtailing transactions such that the total insurance compensation payments is minimized. That is, for a revealed spot price S, the ISO solves the following minimization problem subject to transmission constraints:

$$(ISO1)$$

$$\begin{array}{ll} \min\limits_{\{q_i\}} & \sum\limits_{i \in N} \int_{q_i}^{\widetilde{D}_i(s)} [s \Leftrightarrow \widetilde{v}_i(q)]dq \\ s.t. & \sum\limits_{i=1}^{n} q_i = 0 \\ & q_i = \sum\limits_{j \neq i} q_{ij} \\ & \mid q_{ij}(q_1, q_2, \cdots, q_{n-1}) \mid \leq C_{ij}, 1 \leq i < j \leq n \end{array} \tag{2}$$

where $\widetilde{v}_i(\cdot)$ is the inverse function of $\widetilde{D}_i(\cdot)$; q_i is the net amount of power injected or ejected at node i; $q_{ij}(q_1, q_2, \cdots, q_{n-1})$ is the power flow function on line (i, j); and C_{ij} is the available capacity of line (i, j). Therefore, the ISO has a compensation-minimizing dispatch schedule $(q_1^*(s), q_2^*(s), \cdots, q_n^*(s))$. And for every realized spot price $S = s$, there exists a corresponding $c_i(s)$ being the insurance level granted transmission access (i.e., allowed to inject power) at node i.

Definition 1 *The above priority insurance mechanism is coherent in an electricity network if there exists an insurance premium function $X(c)$ and rational expectation of a set of dispatch contingencies $\{\Omega_i(c)$ for all transactions injecting at node $i\}$ such that* a) *$\{\widetilde{D}_i(c)$ for all $i\}$ are the distribution curves of the insured costs of all transaction units resulting from the network users' self-selection problem $(NU1)$;* b) *$(q_1^*(s), q_2^*(s), \cdots, q_n^*(s))$ is a solution to $(ISO1)$ given $\{\widetilde{D}_i(c)$ for all $i\}$ for every revealed spot price s;* c) *$q_i^*(s) = \widetilde{D}_i(c_i(s))$ for all i, s.*

We will later show in a more general setup of multiple spot markets that if every network user reveals the truth by purchasing insurance which is equal to the true cost then our priority insurance scheme results in the economic dispatch (first best) solutions. However, network users in general have incentives to underinsure their access rights with the aforementioned choice of the insurance premium function. Our objective is to identify the coherent priority insurance schemes attempt to characterize the scheme with the smallest possible deadweight efficiency loss due to imperfect contracting and estimate those losses.

2.2 *Multiple spot markets: zonal pricing*

When there exists multiple zonal spot markets and the network is partitioned into several zones, the formulation is some what different. In this case, the ISO offers one insurance premium schedule $X_m(c)$ in each zone m. The ISO charges no ex-post fee for transactions within one zone but imposes an **additional** ex-post congestion fee (or counterflow credit) of $S_m \Leftrightarrow S_n$ per unit for transactions going from zone n to zone m, where S_m denotes the random spot price in zone m.

2.2.1 The network user self-selection problem

Like in the single spot market case, a network user choosing to purchase insurance level c for one unit injected at node i of zone m, expects physical access when the zonal spot prices $S_1, S_2, \cdots, S_k$ fall in the spot price contingency set $\Omega_i(c)$. Thus a network user chooses the optimal c such that the expected profit is maximized. The optimal c for a transaction unit injected at node i belonging to zone $m(i)$ with true cost v is determined by solving the following problem:

$$
\begin{aligned}
&(NU2)\\
c_i^*(v) = \quad & \arg\max_{c} \int_{\Omega_i(c)} (s_{m(i)} \Leftrightarrow v) dG(s_1, \cdots, s_k) \\
& + \int_{\overline{\Omega}_i(c)} (s_{m(i)} \Leftrightarrow c) dG(s_1, \cdots, s_k) \\
& \Leftrightarrow X_{m(i)}(c)
\end{aligned}
\tag{3}
$$

where $\Omega_i(c)$ is the region of spot price contingencies under which the insurance level c would guarantee access to the network for a transaction unit injected at node i; $\overline{\Omega}_i(c)$ is the complement of $\Omega_i(c)$; and $G(\cdot)$ is the joint cumulative distribution function of the random variables $\{S_1, S_2, \cdots, S_k\}$.

In practice, we may offer a set of discrete insurance levels $\{c_1, c_2, \cdots, c_k\}$ and the corresponding set of premia $\{x_1, x_2, \cdots, x_k\}$. If the number of discrete levels is small we may wish to customize them to each zone. We will illustrate the merits of such an approach in an example.

2.2.2 The ISO problem

By aggregating all submitted insurance levels c, the ISO ends up with curtailment supply curves $\widetilde{D}_i(c)$ at each node i. When the network is congested, the ISO relieves the congestion by curtailing transactions so as to minimize the total compensation payments. Namely, the ISO solves the following minimization problem subject to transmission constraints:

$$
\begin{aligned}
&(ISO2)\\
\min_{\{q_i\}} \quad & \sum_{i \in N_S} \int_{q_i}^{\widetilde{D}_i(S_{m(i)})} [S_{m(i)} \Leftrightarrow \widetilde{v}_i(q)] dq \\
& \Leftrightarrow \frac{1}{k} \sum_{1 \le m < n \le k} (S_m \Leftrightarrow S_n)\left(\sum_{j \in Z_m} q_j \Leftrightarrow \sum_{j' \in Z_n} q_{j'}\right) \\
s.t. \quad & \sum_{i=1}^{n} q_i = 0 \\
& q_i = \sum_{j \neq i} q_{ij} \quad i = 1, 2, \cdots, n. \\
& | q_{ij}(q_1, \cdots, q_{n-1}) | \le C_{ij} \quad 1 \le i < j \le n
\end{aligned}
\tag{4}
$$

where N_S denotes the set of supply nodes; Z_m denotes the node set of zone m; $m(i)$ denotes the zone to which node i belongs; and q_i is the net amount of power injected or ejected at node i. Hence, the ISO has a compensation-minimizing dispatch schedule $(q_1^*(s), q_2^*(s), \cdots, q_n^*(s))$ for every realized zonal spot price vector $(s_1, s_2, \cdots, s_m)$. There exists again a corresponding $c_i(s_1, s_2, \cdots, s_m)$ at node i, which is the insurance level purchased by the marginal transaction unit granted network access at node i for a revealed zonal spot price vector $(s_1, s_2, \cdots, s_m)$. If all network users reveal the truth by purchasing insurance $c^*(v) = v$, then the ISO's compensation-minimizing schedule is indeed the social welfare (gain from trade) maximizing schedule which is defined as follows.

Definition 2 *For a set of zonal spot prices $(S_1, S_2, \cdots, S_k)$, a dispatch schedule $(q_1, q_2, \cdots, q_n)$ is a social welfare maximizing (or, economic dispatch/first best) schedule if it is a solution to the (ED) problem.*

$$
\begin{aligned}
&(ED)\\
\max_{\{q_i\}} \quad & \sum_{i \in N_D} q_i \cdot S_{m(i)} \Leftrightarrow \sum_{i \in N_S} \int_0^{q_i} D_i^{-1}(q) dq \\
s.t. \quad & \sum_{i=1}^{n} q_i = 0 \\
& q_i = \sum_{j \neq i} q_{ij} \quad i = 1, 2, \cdots, n. \\
& | q_{ij}(q_1, q_2, \cdots, q_{n-1}) | \le C_{ij} \quad 1 \le i < j \le n
\end{aligned}
\tag{5}
$$

where N_D and N_S denote the demand node set and the supply node set, respectively; $D_i^{-1}(q)$ is the true inverse supply cost curve at supply node i.

We summarize the above as a proposition and provide the proof in the appendix.

Proposition 1 *Suppose all network users purchase insurance with strike price revealing their true costs, i.e. $c_i^*(v) = v$. Then we have $\widetilde{D}_i(c^*(v)) = D_i(v)$ where $D_i(v)$ is the true cost curve at node i, and the solutions $(q_1^*(s), q_2^*(s), \cdots, q_n^*(s))$ of the ISO problems ($ISO1$ & $ISO2$) are also the corresponding social welfare maximizing solutions.*

Proof. See the Appendix. ∎

The concept of coherent insurance scheme in the multiple spot market case is similarly defined as in **Definition 1** with $(NU2)$ replacing $(NU1)$ and $(ISO1)$ replacing $(ISO2)$.

2.3 *The choice of premium function X(c)*

It is important to note that a proper choice of the insurance premium function $X(c)$ by the ISO is key in our scheme. The choices of premium functions provide the self-selection incentives and lead to different insurance purchase distributions with different social welfare implications. In this paper we focus on the special case where the ISO chooses $X(c)$ as the expected benefit accrued to a transaction unit, with true cost $v = c$, from physical access to the grid. In the following proposition we show that under this premium function no transaction unit would have any incentive to overinsure its access to the network, i.e., the optimal solution to the self-selection problem $c^*(v)$ is always no less than the true cost v. One of the implications of this result is that there is no adverse selection of revealed injection node at **stage three** where users submit their preferred schedules. If a user were to overinsure, there might be an incentive to reveal a false injection node in order to obtain compensation when the user would have curtailed supply voluntarily due to low spot price realization. But with underinsurance compensation is never paid when the users' true cost exceeds the spot price and hence there is no incentive for misrepresenting the injection node.

Proposition 2 *If the ISO chooses*

$$X_m(c) = E_{S_m}(Max(S_m \Leftrightarrow c, 0))$$

as the insurance premium function in each zone m, then $c^*(v) \geq v$ where $c^*(v)$ is the optimal solution to the self-selection problem $(NU1\&2)$ of a transaction unit with true cost v.

Proof. Consider a transaction unit with true cost v. If the unit is overinsured, i.e. $c < v$, then the following is true,

$$\begin{aligned}
0 &= E_{S_m}(Max(S_m - c, 0)) - X_m(c) \\
&\qquad\qquad\qquad\qquad \text{(By the definition of } X_m(c)) \\
&= \int_{\Omega_i(c)} \max(s_{m(i)} - c, 0) dG(s_1, s_2, \cdots, s_k) \\
&\quad + \int_{\overline{\Omega}_i(c)} \max(s_{m(i)} - c, 0) dG(s_1, \cdots, s_k) - X_{m(i)}(c) \\
&\geq \int_{\Omega_i(c)} \max(s_{m(i)} - v, 0) dG(s_1, s_2, \cdots, s_k) \\
&\quad + \int_{\overline{\Omega}_i(c)} \max(s_{m(i)} - c, 0) dG(s_1, \cdots, s_k) - X_{m(i)}(c)
\end{aligned}$$

The expression to the right of the above inequality is the objective function in the self-selection problem (NU2). Since this objective can achieve value zero under true insurance $c = v$, it follows that $c^*(v) \geq v$. ∎

3 Numerical examples

In this section, we take a classical three-node network (Figure 3) with one spot market to show how a coherent priority insurance scheme is obtained. We then compute the efficiency loss of the particular scheme with respect to the economic dispatch solution. As previously mentioned, we use a DC-flow approximation and assume no losses in all our examples. In the specific three-node network, each transaction is uniquely characterized by its injection node since we only consider one net demand node. As for an example in the case of multiple spot markets, we present a four-node network with two spot markets (two zones) and explore the efficiency properties when user's choices are restricted to one and two discrete levels of insurance in each zone.

3.1 *Single spot market: three-node network*

Consider a three node network with transmission line capacity $(C_{12}, C_{13}, C_{23}) = (136MW, 300MW, 254MW)$ and equal admittance of 1. Node 3 is the location of

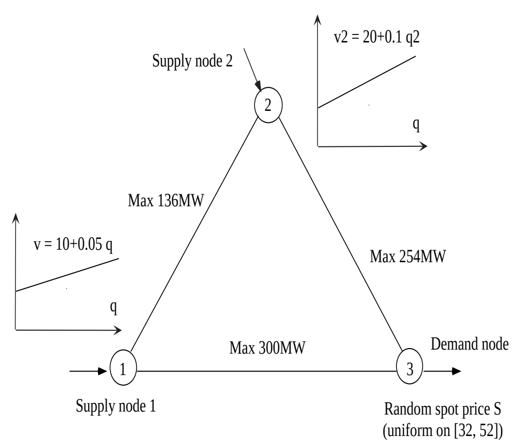

Figure 3: A three node network

the spot market with uniformly distributed random spot price $S \sim U(32, 52)$ and the cumulative distribution function of S is:

$$G(s) \quad = \quad \begin{cases} 0 & , \quad s \leq 32 \\ \dfrac{s \Leftrightarrow 32}{20} & , \quad 32 < s \leq 52 \\ 1 & , \quad s > 52 \end{cases}$$

We first compute the economic dispatch (first best) solution for each realization of the spot price S and the expected social welfare (gain from trade) of the first best solution. A social planner's objective of maximizing social welfare is equivalent to minimizing the shaded areas representing the displacement costs, as depicted in Figure 4. Therefore, a social welfare maximizing ISO solves the following problem to obtain the economic dispatch

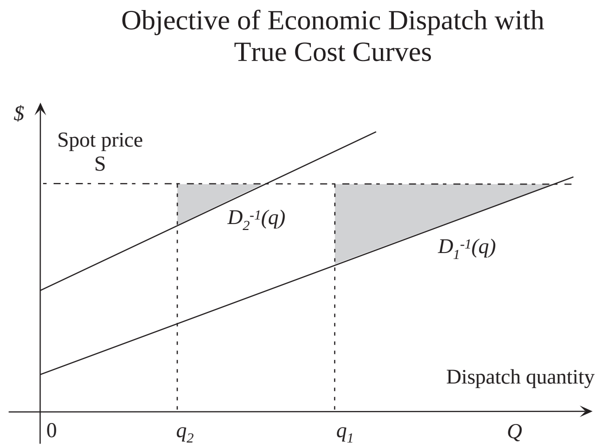

Figure 4: Objective of economic dispatch

for a realized $S(\omega) = s$:

$$\begin{aligned} SW(s) &\equiv \min_{(q_1,q_2,q_3)} \sum_{i=1}^{2} \int_{q_i}^{D_i(s)} [s \Leftrightarrow v_i(q)]dq \\ s.t.\ &\sum_{i=1}^{3} q_i = 0 \\ &\begin{pmatrix} \Leftrightarrow 136 \\ \Leftrightarrow 254 \\ \Leftrightarrow 300 \end{pmatrix} \le \begin{pmatrix} \frac{1}{3} & \Leftrightarrow\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \\ \Leftrightarrow\frac{2}{3} & \Leftrightarrow\frac{1}{3} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \le \begin{pmatrix} 136 \\ 254 \\ 300 \end{pmatrix} \end{aligned} \tag{6}$$

The solution is given by

$$\begin{cases} \widehat{q}_1 = \frac{20}{9}(210 \Leftrightarrow s) \\ \widehat{q}_2 = \frac{20}{9}(2s \Leftrightarrow 15) \end{cases}, \quad 32 \le s \le 52 \tag{7}$$

And the expected social welfare is $E[SW] = 10697$.

We now turn to the computation of a coherent priority insurance scheme where the ISO posts the insurance premium function $X(c)$ given in (3.9). The economic interpretation of $X(c)$ is that it equals the expected benefit accrued to a transaction with true unit cost c receiving physical access to the network and hence avoiding a settlement cost at the spot market price. This premium can also be interpreted as the actuarial value of a financial "call option" with strike price c with respect to the underlying spot market.

$$\begin{aligned} X(c) &= E_S[\max(S \Leftrightarrow c, 0)] \\ &= \begin{cases} 42 \Leftrightarrow c & , \quad 0 \le c \le 32 \\ \frac{1}{40}(52 \Leftrightarrow c)^2 & , \quad 32 < c \le 52 \\ 0 & , \quad c > 52 \end{cases} \end{aligned} \tag{8}$$

We conjecture that a network user who selected insurance level (or, insured cost) c for a transaction injected at node 1 expects the transaction unit to get access for $S \in [\max(32, v), S_1(c)]$ where $S_1(c) = k_1 \Leftrightarrow k_2 c$. The degrees of freedom in computing the rational expectation equilibrium allow us to parameterize the contingency set under which access is provided in terms of a two parameter linear function defining $S_1(c)$. Then the optimal c chosen for a transaction unit with true cost v injected at node 1 is given by the solution of the following problem

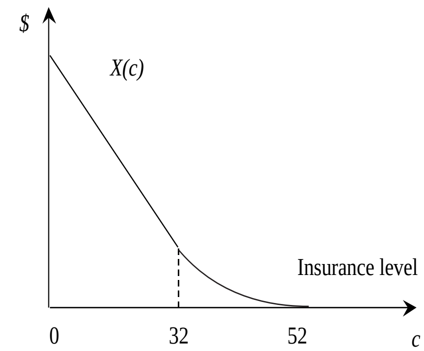

Figure 5: Insurance premium function

$$\begin{aligned} c_1^*(v) &= \arg\max_c \int_{\max(v,32)}^{S_1(c)} (s \Leftrightarrow v) dG(s) \\ &\quad + \int_{S_1(c)}^{52} (s \Leftrightarrow c) dG(s) \Leftrightarrow X(c) \\ &= \begin{cases} \underline{c}_1 & , \quad v \le v_1' \\ \frac{(k_1-32)+k_2 v}{2k_2} & , \quad v_1' < v \le v_1'' \\ \overline{c}_1 & , \quad v > v_1'' \end{cases} \end{aligned} \tag{9}$$

Similarly, we conjecture the spot price interval for which a unit transaction with insurance level c injecting at node 2 gets access to be $[S_2(c), 52]$ where $S_2(c) = k_3 + k_4 c$. Then the optimal insurance level for a transaction unit with true cost v injected at node 2 is determined by the self-selection problem:

$$\begin{aligned} c_2^*(v) &= \arg\max_c \int_{\max(c,32)}^{S_2(c)} (s \Leftrightarrow c) dG(s) \\ &\quad + \int_{S_2(c)}^{52} (s \Leftrightarrow v) dG(s) \Leftrightarrow X(c) \\ &= \begin{cases} \underline{c}_2 & , \quad v \le v_2' \\ \frac{(52-k_3)+k_4 v}{2k_4} & , \quad v_2' < v \le v_2'' \\ \overline{c}_2 & , \quad v > v_2'' \end{cases} \end{aligned} \tag{10}$$

For each S, we have the marginal insurance levels c_1 and c_2 at node 1 and 2, respectively, such that $S = k_1 \Leftrightarrow k_2 c_1(S) = k_3 + k_4 c_2(S)$. The shapes of the resulting inverse insurance distribution curves $\widetilde{D}_i^{-1}(q)$ $(i = 1, 2)$ are illustrated in Figure (6).

When the random spot price is revealed, network users submit their usage requests along with their insurance levels. Therefore, the above insurance distribution curves $\widetilde{D}_i(c)(i = 1, 2)$ are revealed to the ISO. In case of network congestion, the ISO determines dispatch schedules based on the criterion of minimizing total curtail-

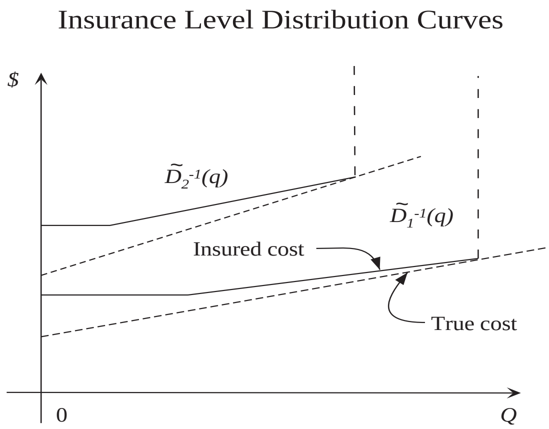

Figure 6: Insured cost distribution curves

ment compensation payments.

$$IP(s) \equiv \min_{(q_1,q_2,q_3)} \sum_{i=1}^{2} \int_{q_i}^{\widetilde{D}_i(s)} [s \Leftrightarrow \widetilde{D}_i^{-1}(q)]dq$$

$$s.t. \sum_{i=1}^{3} q_i = 0$$

$$\begin{pmatrix} \Leftrightarrow 136 \\ \Leftrightarrow 254 \\ \Leftrightarrow 300 \end{pmatrix} \leq \begin{pmatrix} \frac{1}{3} & \Leftrightarrow\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \\ \Leftrightarrow\frac{2}{3} & \Leftrightarrow\frac{1}{3} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \leq \begin{pmatrix} 136 \\ 254 \\ 300 \end{pmatrix} \tag{11}$$

The solution of (11) is

$$\begin{cases} q_1^* = \frac{20}{k_2}(30 + k_1 \Leftrightarrow 10k_2 \Leftrightarrow 2s) \\ q_2^* = \frac{10}{k}(2s \Leftrightarrow k_3 \Leftrightarrow 20k_4 \Leftrightarrow 29) \end{cases}, \quad 32 \leq s \leq 52 \tag{12}$$

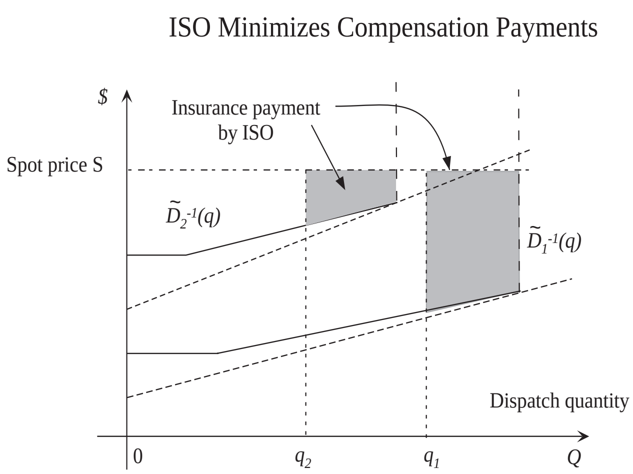

Figure 7: Objective of the ISO's minimization problem

Figure (7) gives a graphic representation of the ISO's objective of minimizing total insurance payments based on the revealed insured cost distribution curves.

Invoking the equilibrium condition c) in **Definition 1**, we can solve for the free parameters of the rational expectation equilibrium to obtain $\{k_1 = 317.78, k_2 = 9, k_3 = \Leftrightarrow 39.72, k_4 = 2.25\}$. The resulting network access contingencies corresponding to the rational expectation equilibrium are characterized by the boundary functions $\{S_i(c), i = 1, 2\}$, given by:

$$\begin{cases} S_1(c) = \begin{cases} \Leftrightarrow 9c + 317.78 & \text{for } c \in [29.53, 31.75] \\ 0 & o.w. \end{cases} \\ S_2(c) = \begin{cases} 2.25c \Leftrightarrow 39.72 & \text{for } c \in [31.88, 40.77] \\ \infty & o.w. \end{cases} \end{cases} \tag{13}$$

The spot-price contingency sets under which network access is granted to each insurance level at the two supply nodes are illustrated in Figure (8).

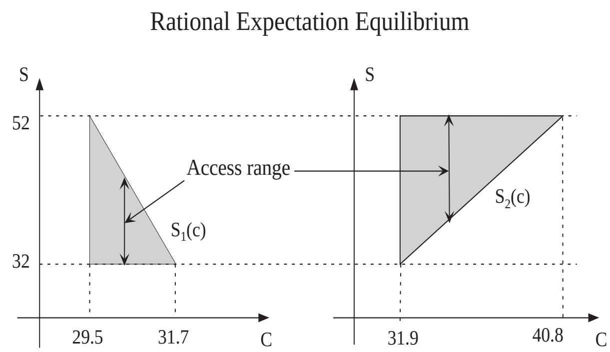

Figure 8: Rational expectation of network access price interval

We substitute the solution $\{k_i, i = 1, 2, 3, 4\}$ into (12) and get the induced dispatch schedules $\{(q_1^*(s), q_2^*(s)), 32 \leq s \leq 52\}$ under the above priority insurance scheme . The expected social welfare of the induced schedules is $E[SW^*] = 10593$. This amounts to only 0.974% efficiency loss. For this simple example, our calculation shows that the efficiency losses associated with the minimum compensation dispatch solution under the priority insurance scheme is rather small as compared to the first best solution. Figure 9 illustrates a comparison between the economic dispatch (first best) solution and the minimum compensation (second best) for every realization of the spot price S.

To check the robustness of the above result we performed a modest sensitivity analysis calculating the efficiency loss for slightly varied different sets of parameters. Basically we vary the fixed costs of the true cost distribution curves so as to change the difference between true supply functions at the different supply nodes. The computation indicates that the efficiency losses are still of similar magnitudes.

Parameter set	$\begin{cases} v_1 = 9 + 0.05q_1 \\ v_2 = 22.5 + 0.1q_2 \end{cases}$	$\begin{cases} v_1 = 10 + 0.05q_1 \\ v_2 = 20 + 0.1q_2 \end{cases}$	$\begin{cases} v_1 = 10 + 0.05q_1 \\ v_2 = 23 + 0.1q_2 \end{cases}$
Efficiency loss	0.972 %	0.974 %	1.01 %

Table 1: Sensitivity of Efficiency Loss

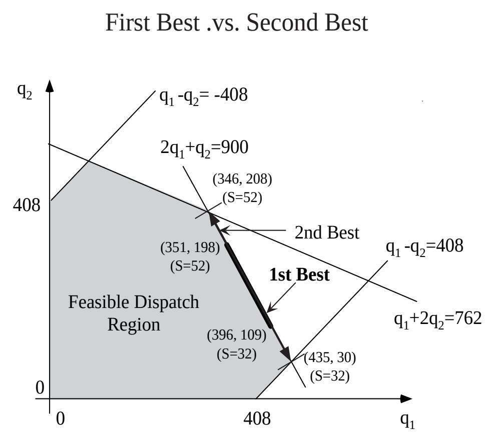

Figure 9: The comparison between 1st best and 2nd best solutions

3.2 *Multiple spot markets: 4-node network*

We next turn to the multiple zonal spot markets case. Consider a 4-node network with two spot markets and two supply nodes as shown in Figure 10. Node 1 and 4 belong to zone one while node 2 and 3 belong to zone two. The link 3-4 connecting the two zones is the only congested link with line-flow capacity of 80MW. All lines are of equal impedance of one.

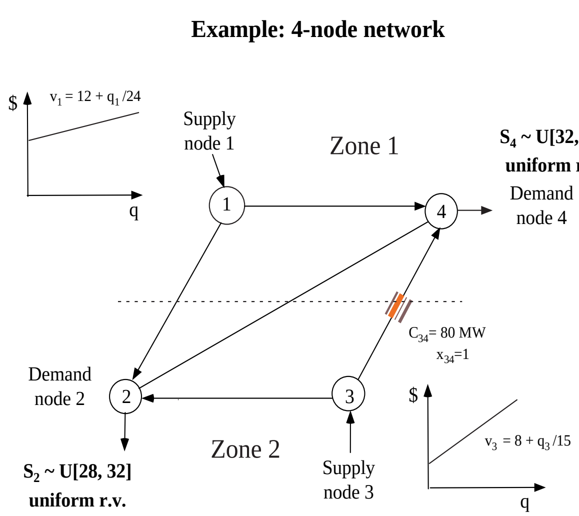

Figure 10: An example of 4 node network

We assume the spot prices in zone 1 and zone 2 are jointly uniformly distributed over interval $[28, 32] \times [32, 40]$. The marginal distributions are $S_4 \sim U[32, 40]$ and $S_2 \sim U[28, 32]$, respectively. The true inverse supply cost curves at node 1 and node 3 are:

$$\begin{cases} v_1 &= 12 + q_1/24 \\ v_3 &= 8 + q_3/15 \end{cases}$$

The economic dispatch (first best) solutions for any given spot prices (s_2, s_4) are given by

$$\begin{cases} \widehat{q}_1 &= 12(s_2 + s_4 \Leftrightarrow 24) \\ \widehat{q}_2 &= \dfrac{\Leftrightarrow 3056 + 399 s_2 \Leftrightarrow 201 s_4}{4} \\ \widehat{q}_3 &= \dfrac{15(5s_2 \Leftrightarrow 3s_4 \Leftrightarrow 16)}{2} \\ \widehat{q}_4 &= \dfrac{1424 \Leftrightarrow 201 s_2 + 159 s_4}{4} \end{cases}$$

The expected social welfare of the first best solutions is $E_S[SW] = 8652$.

We consider the simplest situation where the ISO offers a one-level insurance scheme in the two zones, i.e. network users selections are restricted to $\{c^1, x_i^1\}$ or $\{\infty, 0\}$ in zone i $(i = 1, 2)$. The premia x_i^1 are given by $X_i(c^1)$ $(i = 1, 3)$ where

$$\begin{aligned} X_1(c) &= E_{S_4}[\max(S_4 \Leftrightarrow c, 0)] \\ &= \begin{cases} 36 \Leftrightarrow c &, \ c \le 32 \\ \frac{1}{16}(40 \Leftrightarrow c)^2 &, \ 32 < c \le 40 \\ 0 &, \ c > 40 \end{cases} \end{aligned} \tag{14}$$

and

$$\begin{aligned} X_3(c) &= E_{S_2}[\max(S_2 \Leftrightarrow c, 0)] \\ &= \begin{cases} 30 \Leftrightarrow c &, \ c \le 28 \\ \frac{1}{8}(32 \Leftrightarrow c)^2 &, \ 28 < c \le 32 \\ 0 &, \ c > 32 \end{cases} \end{aligned} \tag{15}$$

The network users' self-selection problem amounts to the individual rationality condition, namely, $c_i^*(v) = c^1$ $(i = 1, 2)$ if and only if the expected benefit of purchasing c^1 is no less than 0 . The ISO minimizing insurance compensation problem is

$$\begin{aligned} \min_{\{q_i; q_i^\alpha\}} \quad & \sum_{i=1,3} \sum_{\alpha=0}^{1} p_i^\alpha (\overline{q}_i^\alpha(S) \Leftrightarrow q_i^\alpha) \Leftrightarrow (S_4 \Leftrightarrow S_2)(q_3 \Leftrightarrow q_2) \\ s.t. \quad & q_1 + q_3 \Leftrightarrow q_2 \Leftrightarrow q_4 = 0 \\ & q_i = \sum_{\alpha=0}^{1} q_i^\alpha \quad (i = 1, 3) \\ & \mid q_{ij}(q_1, q_2, q_3) \mid \le C_{ij} \\ & q_i^\alpha \le \overline{q}_i^\alpha(S) \\ & q_i \ge 0, \ q_i^\alpha \ge 0 \end{aligned} \tag{16}$$

where

q_i^1 is the number of access requests with insurance at node i $(i = 1, 3)$
q_i^0 is the number of uninsured access requests at node i $(i = 1, 3)$
$\overline{q}_i^1(S)$ is the total number of access requests with insurance at node i $(i = 1, 3)$
$\overline{q}_i^0(S)$ is the total number of uninsured access requests at node i $(i = 1, 3)$
$p_1^1 = \max(S_4 \Leftrightarrow c^1, 0)$, $p_3^1 = \max(S_2 \Leftrightarrow c^1, 0)$ are the insurance payments
$p_i^0 = 0$ $(i = 1, 3)$

By varying the insurance level c^1, we calculated several equilibrium solutions and found that the social welfare efficiency losses is not very sensitive to the choice of insurance level c^1. Taking $c^1 = 28.5$, the solutions to the ISO problem is

$$\begin{cases} q_1 = 396MW, & q_2 = 0MW \\ q_3 = 48.8MW, & q_4 = 444.8MW; \quad (5s_2 \Leftrightarrow 3s_4 \leq 57) \\ & \\ q_1 = 396MW, & q_2 = 646.75MW \\ q_3 = 307.5MW, & q_4 = 56.75MW; \quad (5s_2 \Leftrightarrow 3s_4 > 57) \end{cases}$$

which yield an expected social welfare of 7254.0. The calculation accounts for random rationing among insured access requests when transmission constraints prohibit scheduling of all such requests. The corresponding efficiency loss is equal to 16.15% which is roughly the smallest efficiency loss achievable with one insurance level.

We now consider a two-level insurance scheme with one in each zone, i.e. network users selections are restricted to $\{c_i^1, x_i^1\}$ or $\{\infty, 0\}$ in zone i $(i = 1, 2)$ and $c_1^1 \neq c_2^1$. The ISO minimum compensation problem becomes:

$$\begin{array}{ll} \min\limits_{\{q_i; q_i^\alpha\}} & \sum\limits_{i=1,3} \sum\limits_{\alpha=0}^{1} p_i^\alpha (\overline{q}_i^\alpha(S) \Leftrightarrow q_i^\alpha) \Leftrightarrow (S_4 \Leftrightarrow S_2)(q_3 \Leftrightarrow q_2) \\ s.t. & q_1 + q_3 \Leftrightarrow q_2 \Leftrightarrow q_4 = 0 \\ & q_i = \sum\limits_{\alpha=0}^{1} q_i^\alpha \quad (i = 1, 3) \\ & \mid q_{ij}(q_1, q_2, q_3) \mid \leq C_{ij} \\ & q_i^\alpha \leq \overline{q}_i^\alpha(S) \\ & q_i \geq 0,\ q_i^\alpha \geq 0 \end{array} \tag{17}$$

where

q_i^1 is the number of access requests with insurance at node i $(i = 1, 3)$
q_i^0 is the number of uninsured access requests at node i $(i = 1, 3)$
$\overline{q}_i^1(S)$ is the total number of access requests with insurance at node i $(i = 1, 3)$
$\overline{q}_i^0(S)$ is the total number of uninsured access requests at node i $(i = 1, 3)$
$p_1^1 = \max(S_4 \Leftrightarrow c^1, 0)$, $p_3^1 = \max(S_2 \Leftrightarrow c^1, 0)$ are the insurance payments
$p_i^0 = 0$ $(i = 1, 3)$

For the instance of $c_1^1 = 30$ and $c_2^1 = 21$, we have $\{\overline{q}_1^1(S) = 432MW, \overline{q}_3^1(S) = 195MW\}$. The solutions to the ISO compensation minimization problem are:

$$\begin{cases} q_1 = 432MW, & q_2 = 0MW \\ q_3 = 41.6MW, & q_4 = 473.6MW; \quad (5s_2 \Leftrightarrow 3s_4 \leq 42) \\ & \\ q_1 = 432MW, & q_2 = 383.5MW \\ q_3 = 195MW, & q_4 = 243.5MW; \quad (5s_2 \Leftrightarrow 3s_4 > 42) \end{cases}$$

The above dispatch schedules yield an expected social welfare of 8156.3 which amounts to an efficiency loss of 5.7% as compare to the expected social welfare of economic dispatch solutions. Note that given the rational expectation about the ISO minimum compensation dispatch over the corresponding spot price contingency region, the expected benefit for a transaction unit with true cost v purchasing c_1^1 and c_2^1 are $(c_1^1 \Leftrightarrow v)$ and $91(c_2^1 \Leftrightarrow v)/150$, respectively. Therefore the marginal insurance purchasing units at node 1 and node 2 have true costs of $v_1^* = c_1^1 = 30$ and $v_3^* = c_2^1 = 21$, respectively.

By adding one more insurance level to each zone in the previous two-level insurance example, e.g. taking $c_1^1 = 30$ $c_1^2 = 31.5$ and $c_2^1 = 21$ $c_2^2 = 27$, we reduce the efficiency loss from 5.7% to 4.4%. Note these insurance levels were not optimized to achieve the minimum efficiency loss. It is reasonable to expect that optimizing the insurance levels and adding more levels of insurance in each zone will further reduce the efficiency loss to an acceptable level.

4 Conclusion

At the intuitive level, our scheme can be viewed as a hybrid of priority insurance and a postage stamp approach. The different levels of insurance characterized by the revealed opportunity costs may be interpreted as postage stamps with different priorities. These priorities allow for network users self-selection which in turn provides economic signals for the efficient rationing of scarce transmission resources. With a single zone in a transmission network, since we constrain the admissible insurance schemes to be uniform, we cannot expect a first best solution. However, if we partition a network into more zones and allow more different insurance premium schedules to be offered, the efficiency gains can be improved. The limiting case, where the insurance scheme is node specific, is equivalent to a nodal pricing approach. In essence, our scheme takes out part of the time and locational "price variability" present in a nodal pricing scheme and allows "quantity variability" in the form of uncertain access at a given price. Stable prices with a measure of uncertainty in service quality is a prevalent practice in most service industries. What is important to realize is that the proposed pricing scheme and the corresponding congestion management protocols are quite simple. The mathematical complexity is in attempting to calculate the market equilibrium. In reality that part is performed by the market itself.

A Appendix

A.1 Proof of Proposition 1

Proof. Notice that the (ED) problem and the $(ISO2)$ have the same set of constraints. It is therefore sufficient to show that the objectives of the two problems are equivalent provided $\widetilde{D}_i(c^*(v)) = D_i(v)$. Let

$v_i(q)$ denote $D_i^{-1}(q)$.

$$\max \sum_{i \in N_D} q_i \cdot S_{m(i)} \Leftrightarrow \sum_{i \in N_S} \int_0^{q_i} v_i(q)dq$$

(objective of the (ED) problem)

$$\begin{aligned}
\Leftrightarrow \quad & \max \sum_{i \in N_D} q_i \cdot S_{m(i)} \Leftrightarrow \sum_{i \in N_S} \int_0^{D_i(S_{m(i)})} v_i(q)dq \\
& + \sum_{i \in N_S} \int_{q_i}^{D_i(S_{m(i)})} v_i(q)dq \\
\Leftrightarrow \quad & \max \sum_{i \in N_D} q_i \cdot S_{m(i)} + \sum_{i \in N_S} (D_i(S_{m(i)}) \Leftrightarrow q_i) S_{m(i)} \\
& \Leftrightarrow \sum_{i \in N_S} (D_i(S_{m(i)}) \Leftrightarrow q_i) S_{m(i)} \\
& \quad + \sum_{i \in N_S} \int_{q_i}^{D_i(S_{m(i)})} v_i(q)dq \\
\Leftrightarrow \quad & \max \sum_{i \in N_D} q_i \cdot S_{m(i)} + \sum_{i \in N_S} (D_i(S_{m(i)}) \Leftrightarrow q_i) S_{m(i)} \\
& \Leftrightarrow \sum_{i \in N_S} \int_{q_i}^{D_i(S_{m(i)})} [S_{m(i)} \Leftrightarrow v_i(q)]dq \\
\Leftrightarrow \quad & \max \sum_{i \in N_D} q_i \cdot S_{m(i)} + \sum_{i' \in N_S} q_{i'} \cdot S_{m(i')} \Leftrightarrow IP \\
\Leftrightarrow \quad & \max \sum_{j=1}^{k} Q_j \cdot S_j \Leftrightarrow IP \\
\Leftrightarrow \quad & \min IP \Leftrightarrow \sum_{j=1}^{k} Q_j \cdot S_j \\
\Leftrightarrow \quad & \min IP \Leftrightarrow \frac{1}{k} \sum_{1 \le l < j \le k} (Q_l \Leftrightarrow Q_j)(S_l \Leftrightarrow S_j)
\end{aligned}$$

(objective of the $(ISO2)$ problem)

where

$$\begin{aligned}
Q_k & \equiv \sum_{i \in Z_k} q_i \text{ with } Z_k \text{ denoting the node set of zone } k. \\
IP & \equiv \sum_{i \in N_S} \int_{q_i}^{D_i(S_{m(i)})} [S_{m(i)} \Leftrightarrow v_i(q)]dq
\end{aligned}$$

The last equivalent relationship utilizes the fact that $\sum_{i=1}^{k} Q_i = 0$. ∎

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