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New Lower Bound below the Information Rate of Phase Noise Channel Based on Kalman Carrier Recovery

Luca Barletta, Maurizio Magarini, and Arnaldo Spalvieri

Abstract

A new lower bound below the information rate transferred through the Additive White Gaussian Noise (AWGN) channel affected by discrete-time multiplicative phase noise is proposed in the paper. The proposed lower bound is based on the Kalman approach to data-aided carrier phase recovery, and is less computationally demanding than known methods based on phase quantization and trellis representation of phase memory or on particle filtering. Simulation results show that the lower bound is close to the actual channel capacity, especially at low-to-intermediate signal-to-noise ratio.

Index Terms

Coherent communication, Phase noise, Channel capacity, Information rate, Kalman filtering, Kalman carrier phase recovery.

1. Introduction

Multiplicative phase noise is a major source of impairment in coherent communication. In the context of radio transmission, phase noise is introduced by local oscillators used for up-conversion and down-conversion. The impact of phase noise on the performance of orthogonal frequency division multiplexing (OFDM) systems is studied, for instance, in [1]. Also, single carrier systems, especially recent systems based on frequency domain equalization as [2], suffer from phase noise and require specific mitigation techniques [3]. With the advent of coherent optics, the role of phase noise is becoming well recognized also in the context of optical transmission, see e.g. [4].

Much is known about phase noise with first-order memory. The basic model of phase noise with first-order memory is that of Wiener phase noise, [4], [5], where the power spectral density of the spectral line is a slope of -20 dB/decade. Several methods have been proposed in the literature to combat the detrimental effects of Wiener phase noise. Among these methods we cite the iterative demodulation and decoding techniques of [6-8], the insertion of pilot symbols [9-11], and soft differential demodulation [12]. Computation of the capacity of the multiplicative Wiener phase noise plus additive white Gaussian noise (AWGN) channel, which is a channel with memory and continuous state, is a challenging problem. A Monte Carlo approach based on phase space quantization and trellis representation of phase memory has been recently proposed in [13-15] for computing the constrained channel capacity (i.e. the capacity with a fixed source). In the recent papers [16], [17], the authors have proposed a lower bound below the capacity of Wiener phase noise channel based on demodulation aided by the past data, where the transfer function of the causal filter used for phase estimation is worked out by the Kalman approach. Compared to methods based on phase quantization and trellis representation of phase memory, such as [14], the method proposed here is less computationally demanding, since a (Kalman) filter is used in place of a trellis. At low-to-intermediate signal-to-noise ratio, it can be seen that the proposed lower bound is so close to the actual channel capacity that the new lower bound of [16], [17] gives virtually the actual channel capacity.

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Less is known about phase noise with memory of higher order. Phase noise with second-order memory can be used to model microwave oscillators characterized by a power spectral density that is a slope of -40 dB/decade at low frequency, followed by a slope of -20 dB/decade at intermediate frequency, and then a white noise floor at high frequency. In [18] it is shown that phase noise with this spectrum can be optimally tracked with a second-order phase lock-loop (PLL). A general method for working out an approximation to channel capacity is the particle method proposed in [19].

In this paper we improve over the results of [16], [17], showing that the capacity of the channel affected by multiplicative phase noise with higher-order memory can be closely lower-bounded by data-aided demodulation based on a higher-order Kalman filter. Compared to [19], the method proposed here is much less computationally demanding.

The outline of the paper is as follows. In Section II the channel model and the source model are introduced. Section III reports the general method behind the bound, while in Section IV the specific bound is presented. Section V gives simulation results, while in Section VI the conclusion is drawn.

2. Channel and Source Model

Let u_i^k indicate the column vector

$$(u_i, u_{i+1}, \cdots, u_k)^T \quad i \le k$$
$$(u_i, u_{i-1}, \cdots, u_k)^T \quad i > k,$$

where the superscript ^{T} denotes transposition and $u_i^k \in U_i^k$, and let U indicate a possibly non-stationary process, $U = (U_0, U_1, \cdots)$, whose generic realization is the sequence $(u_0, u_1, \cdots)$. When U_i^k is a continuous set, $p(u_i^k)$ is used to indicate the multivariate probability density function, while when U_i^k is a discrete set $p(u_i^k)$ indicates the multivariate mass probability and $|U_i|$ denotes the number of elements in U_i. When the process is stationary, the time index is skipped and U is used in place of U_i.

The k-th output of the channel is

$$y_k = x_k e^{j\phi_k} + w_k, \quad k = 1,2,\cdots, \tag{1}$$

where j is the imaginary unit, Y is the complex channel output process, X is the channel complex input modulation process made by i.i.d. random variables with zero mean and unit variance, W is the complex AWGN process with zero mean and variance SNR^{-1}, and Φ is the phase noise process which is assumed to be independent of X and W.

Let us introduce the z-transform

$$\Omega(z) = \frac{z^{-1}V(z)}{(1-z^{-1})\prod_{k=2}^{N_\alpha}(1-\alpha_k z^{-1})}, \tag{2}$$

where $V(z)$ is the z-transform of an i.i.d. sequence of Gaussian random variables with zero mean and variance γ^2, and

$$|\alpha_k| < 1, \quad N_\alpha \geq 1,$$

and write the z-transform of the phase sequence as

$$\Phi(z) = \Omega(z)\prod_{k=1}^{N_\beta}(1-\beta_k z^{-1}), \tag{3}$$

with

$$|\beta_k| \leq 1, \quad {}_{\beta}\geq 0.$$

Note that, since Φ is a phase, it can be reduced modulo 2π, thus allowing for the pole on the unit circle in (2). Sequences $\{\omega_k\}$ and $\{\phi_k\}$ can be obtained as

$$\omega_k = v_k + \sum_{i=1}^{N_\alpha} a_i \omega_{k-i}, \quad \phi_k = \omega_k + \sum_{i=1}^{N_\beta} b_i \omega_{k-i}, \tag{4}$$

where coefficients $\{a_i\}$ and $\{b_i\}$ are such that

$$(1-z^{-1})\prod_{k=2}^{N_\alpha}(1-\alpha_k z^{-1}) = 1 - \sum_{k=1}^{N_\alpha} a_k z^{-k},$$

$$\prod_{k=1}^{N_\beta}(1-\beta_k z^{-1}) = 1 + \sum_{k=1}^{N_\beta} b_k z^{-k}.$$

Thanks to equations (2) and (3), the phase noise channel can be cast in the general framework of state-space approach for modeling dynamic systems, where the state at time k is the $(N+1)$ column vector

$$\mathbf{s}_k = \omega_k^{k-N}, \quad k = 0, 1, \cdots, \tag{5}$$

where $N = \max(N_\alpha - 1, N_\beta)$. The state evolution is

$$\mathbf{s}_{k+1} = \mathbf{F} \cdot \mathbf{s}_k + (v_k, (\mathbf{0}_N)^T)^T, \tag{6}$$

where $\mathbf{0}_N$ is a column vector made by N zeros, and the square matrix $\mathbf{F}$ of size

$(N+1)\times(N+1)$ is

$$\mathbf{F} = \begin{bmatrix} a_1 & a_2 & \dots & a_{N_\alpha} & (\mathbf{0}_{N-N_\alpha+1})^T \\ & & \mathbf{I}_N & & \mathbf{0}_N \end{bmatrix}, \tag{7}$$

where $\mathbf{I}_N$ is the identity matrix of size N, being understood that $\mathbf{I}_0$ and $\mathbf{0}_0$ are empty matrices. The measurement is (1) with

$$\phi_k = \mathbf{b}^T \cdot \mathbf{s}_k, \tag{8}$$

where $\mathbf{b} = (1, b_1, b_2, \ldots, b_{N_\beta}, (\mathbf{0}_{N-N_\beta})^T)^T$.

The popular Wiener phase noise is obtained by putting $N_\beta = 0$, $N_\alpha = 1$, and $a_1 = 1$. In this case the power spectral density of the continuous-time complex exponential $e^{j\phi(t)}$, whose samples taken at symbol frequency generate the sequence $e^{j\phi_k}$, is the Lorentzian function

$$L(f) = \frac{4\gamma^2 T}{\gamma^4 + 16\pi^2 f^2 T^2},$$

where T is the symbol repetition interval and f is the frequency. The parameter γ^2 can be expressed as

$$\gamma^2 = 2\pi B_{FWHM} T,$$

where B_{FWHM} is the full-width half-maximum bandwidth of the spectral line.
The second-order model of [18] is obtained with

$$\beta_1 = 0.9937,\ \beta_2 = 0.7286, \alpha_2 = 0.9999. \tag{9}$$

The power spectral density of Wiener phase noise and of the second-order phase noise normalized with respect to γ^2 are reported in Fig. 1. Also, the spectrum of white phase noise, that is obtained with

$$b_1 = -1,\ a_1 = 1,$$

is reported in Fig. 1.

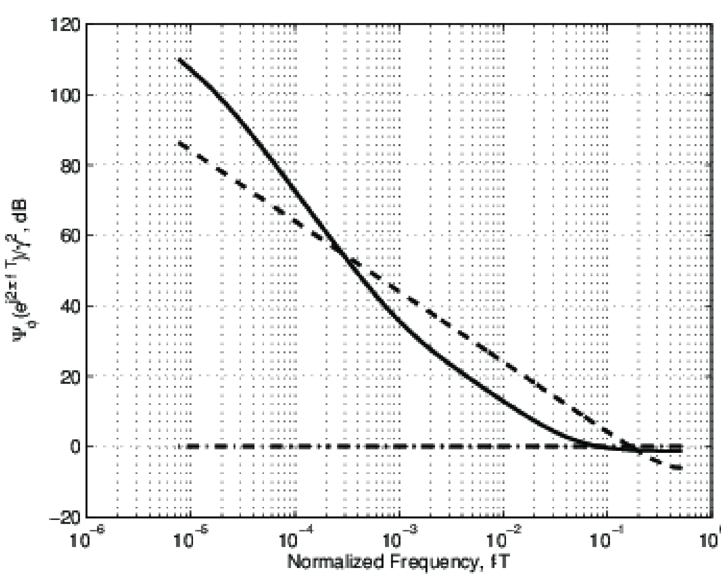

Figure 1. Dashed line: power spectral density of Wiener phase noise. Solid line: power spectral density of the second-order phase noise with (9). Dashed-dotted line: power spectral density of white phase noise.

3. The Auxiliary Probability Method

The bound that we are going to present is based on the Kullback-Leibler (KL) divergence. The normalized KL divergence between the multivariate mass probability functions $p(u_1^n)$ and $q(u_1^n)$ is

$$\lim_{n\to\infty}\frac{1}{n}E_p\left\{\log_2\frac{p(u_1^n)}{q(u_1^n)}\right\}\geq 0, \tag{10}$$

where $E_p\{\cdot\}$ denotes the expectation over $p(u_1^n)$ and $(n)^{-1}$ is the normalization factor. From the normalized KL divergence one has the following upper bound on the entropy rate of process U :

$$\overline{H}(U)=\lim_{n\to\infty}\frac{1}{n}E_p\left\{\log_2\left(\frac{1}{q(u_1^n)}\right)\right\}\geq H(U). \tag{11}$$

Let us regard the *auxiliary* multivariate mass probability $q(u_1^n)$ as an approximation to $p(u_1^n)$. In this perspective, the KL divergence is a measure of the quality of the fit between $p(u_1^n)$ and $q(u_1^n)$, and the upper bound is equal to the actual entropy rate when the fit is ideal, that is when $q(u_1^n)=p(u_1^n)$.

Assuming that U is ergodic, one can invoke the Shannon-McMillan-Breiman theorem and the chain rule, thus writing for the expectation appearing in (11)

$$\overline{H}(U)=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\log_2\left(\frac{1}{q(u_k\mid u_0^{k-1})}\right), \tag{12}$$

where u_1^n is generated according to the actual multivariate mass probability $p(u_1^n)$, and the initial condition u_0 is given. The bound can be extended to the conditional entropy rate in a straightforward manner.

4. Lower Bound

Assume discrete input alphabet. The lower bound below the information rate is

$$H(X)-\overline{H}(X\mid Y)\leq I(X;Y),$$

where the familiar notation is used for the conditional entropy rate and for the mutual information rate. The upper bound

$$\overline{H}(X\mid Y)\geq H(X\mid Y)$$

is obtained from (12) as

$$\overline{H}(X\mid Y)=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\log_2\left(\frac{1}{q(x_k\mid x_1^{k-1},y_1^n)}\right). \tag{13}$$

To obtain a tight bound, one has to work out an auxiliary mass probability that closely approximates the actual mass probability. Aiming to simple yet effective methods, we observe that the portion of joint sequence (x_1^{k-1}, y_1^{k-1}) can strongly contribute to a data-aided approximation to the wanted probability, while the portion y_k^n gives a weaker contribution of non-data-aided type. The stronger part of the non-data-aided contribution comes from sample y_k, therefore the auxiliary probability that is hereafter considered is based only on (x_1^{k-1}, y_1^k), while y_{k+1}^n is ignored. The auxiliary conditional probability proposed here is

$$q(x_k \mid y_1^k, x_1^{k-1}) = \int_{-\infty}^{\infty} q(\phi_k, x_k \mid y_1^k, x_1^{k-1})\, d\phi_k$$

$$= \int_{-\infty}^{\infty} p(x_k \mid y_k, \phi_k) q(\phi_k \mid y_1^k, x_1^{k-1})\, d\phi_k \tag{14}$$

$$\propto \int_{-\pi}^{\pi} p(y_k \mid x_k, \tilde{\phi}_k)\, p(x_k) \sum_{i=-\infty}^{\infty} q(\tilde{\phi}_k + 2\pi i \mid y_1^k, x_1^{k-1}) d\tilde{\phi}_k, \tag{15}$$

where in (14) we have exploited the fact that X_k is conditionally independent of (Y_1^{k-1}, X_1^{k-1}) given (Y_k, Φ_k), and in (15) we used the change of variable $\phi_k = \tilde{\phi}_k + 2\pi i$ and the fact that the channel transition probability $p(y_k \mid x_k, \phi_k)$ is a periodic function of period 2π of variable ϕ_k. The only distribution in (15) that cannot be computed directly from the channel model is

$$q(\phi_k \mid y_1^k, x_1^{k-1}) \propto q(\phi_k \mid y_1^{k-1}, x_1^{k-1})\, p(y_k \mid \phi_k)$$

$$= q(\phi_k \mid y_1^{k-1}, x_1^{k-1}) \cdot \sum_{x_k \in \mathrm{X}} p(y_k \mid \tilde{\phi}_k, x_k)\, p(x_k),$$

where the auxiliary probability $q(\phi_k \mid y_1^{k-1}, x_1^{k-1})$ adopted here is such that

$$\sum_{i=-\infty}^{\infty} q(\tilde{\phi}_k + 2\pi i \mid y_1^{k-1}, x_1^{k-1}) = g(0, \sigma_k^2; [\tilde{\phi}_k - \hat{\phi}_k]_{[-\pi,\pi)}),$$

where $g(\eta, \sigma^2; u)$ is a Gaussian distribution with mean η and variance σ^2 over the space spanned by u, and $[x]_{[-\pi,\pi)}$ is the modulo reduction of x in the fundamental interval $[-\pi, \pi)$. Note that this choice allows to easily compute the integral in (15) by discretization of the fundamental interval $[-\pi, \pi)$.

Recalling (8), estimate $\hat{\phi}_k$ can be obtained as

$$\hat{\phi}_k = \mathbf{b}^T \cdot \hat{\mathbf{s}}_k. \tag{16}$$

The estimate of the state $\hat{\mathbf{s}}_k$ is worked out by a predictive Kalman filter, that is

$$\hat{\mathbf{s}}_k = E\{\mathbf{s}_k \mid y_1^{k-1}, x_1^{k-1}\},$$

and the covariance matrix of the estimation error is

$$\mathbf{P}_k = E\{(\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^T \mid y_1^{k-1}, x_1^{k-1}\}. \tag{17}$$

Thanks to (16) and (17) the variance of the estimation error for $\hat{\phi}_k$ is $\sigma_k^2 = \mathbf{b}^T \mathbf{P}_k \mathbf{b}$. The mean vector $\hat{\mathbf{s}}_k$ and the covariance matrix $\mathbf{P}_k$ can be computed in a recursive manner thanks to the update equations of the Kalman filter. Since the channel model (1) is a nonlinear function of the phase ϕ_k, and hence of the state $\mathbf{s}_k$, the Kalman filter has to be linearized around the current state estimate [20, Ch. 13.1]. The error that drives the Kalman filter is the one produced by the phase detector of classical data-aided carrier recovery, that is

$$e_k = \Im\{y_k x_k^* e^{-j\hat{\phi}_k}\} = \Im\{y_k x_k^* e^{-j\mathbf{b}^T \cdot \hat{\mathbf{s}}_k}\}, \tag{18}$$

where $\Im\{\cdot\}$ and the superscript * denote the imaginary part and the complex conjugation, respectively. Assuming that the phase error $\phi_k - \hat{\phi}_k$ is small, error (18) can be linearized as

$$e_k \approx \phi_k - \hat{\phi}_k + z_k = \mathbf{b}^T \cdot (\mathbf{s}_k - \hat{\mathbf{s}}_k) + z_k,$$

where z_k is assumed to be white Gaussian noise with zero mean and time-varying variance $\sigma_{z,k}^2 = |x_k|^2 (2\text{SNR})^{-1}$.

The estimate $\hat{\mathbf{s}}_k$ and error's covariance matrix $\mathbf{P}_k$ are computed for $k \geq 0$ according to the iterative equations

$$\hat{\mathbf{s}}_{k+1} = \mathbf{F}\hat{\mathbf{s}}_k + \mathbf{g}_k e_k$$

$$\mathbf{P}_{k+1} = \mathbf{F}\mathbf{P}_k (\mathbf{F} - |x_k|^2 \mathbf{g}_k \mathbf{b}^T)^T + \mathbf{Q}$$

where $\mathbf{g}_k$ is the Kalman gain at time k

$$\mathbf{g}_k = \mathbf{F}\mathbf{P}_k \mathbf{b}(|x_k|^2 \mathbf{b}^T \mathbf{P}_k \mathbf{b} + (2\text{SNR})^{-1})^{-1} = \frac{\mathbf{F}\mathbf{P}_k \mathbf{b}}{|x_k|^2 \sigma_k^2 + (2\text{SNR})^{-1}},$$

matrix $\mathbf{F}$ has been defined in (7), and the unique nonzero entry in position $(1,1)$ of matrix $\mathbf{Q}$ is γ^2. Initial values can be set as $\hat{\mathbf{s}}_0 = \mathbf{0}_{N+1}$ and $\mathbf{P}_0$ as a zero matrix except for the entry in position $(1,1)$ that is set to a large value, e.g., 10.

5. Simulation Results

In this section, the new lower bound is compared to the actual channel capacity of Wiener phase noise channel and of the second-order phase noise channel of [18].

In the case of Wiener phase noise, the actual channel capacity is worked out by the computationally demanding trellis-based method of [14]. Specifically, the actual channel capacity has been obtained using a large number of states in the lower bound and in the upper bound of [14]. The number of states is so large that the upper bound and the lower bound become virtually indistinguishable, leading to the actual channel capacity. Fig. 2 reports the results obtained with 4-QAM and with two values of γ. Specifically, $\gamma = 0.125$ is the largest value obtained in the experimental results of [4] and can be regarded as a case of strong phase noise in cases of practical interest. Although less realistic, also the huge $\gamma = 0.5$ is studied, to show the limits of the proposed method. For $\gamma = 0.125$ the lower bound is virtually indistinguishable from the actual capacity in a wide range of information rate, say, below 1.5 bit/2D. This range is the one spanned by codes with rate lower than 0.75, that are codes of large practical interest. For information rate greater than 1.5 bit/2D the bound looses accuracy. This is because we have not exploited the conditioning on y_{k+1}^{n}, which, at high SNR, could potentially bring a non-negligible contribution to the accuracy of the fit between the auxiliary probability and the actual probability. Although being fairly close to the actual capacity, the lower bound is less accurate for $\gamma = 0.5$, because, with a so large value of γ, frequent cycle slips affect the performance of Kalman carrier recovery. A similar analysis holds for the results obtained with 16-QAM and reported in Fig. 3. Also in this case, the lower bound virtually gives the actual channel capacity for $\gamma = 0.125$ and coding rate below 0.75.

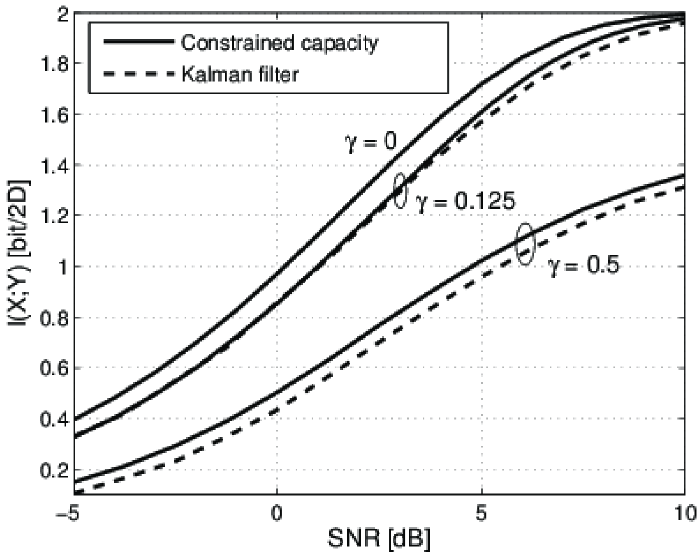

Figure 2. Actual channel capacity (solid line) and lower bound (dashed line) for 4-QAM, Wiener phase noise, and two values of γ.

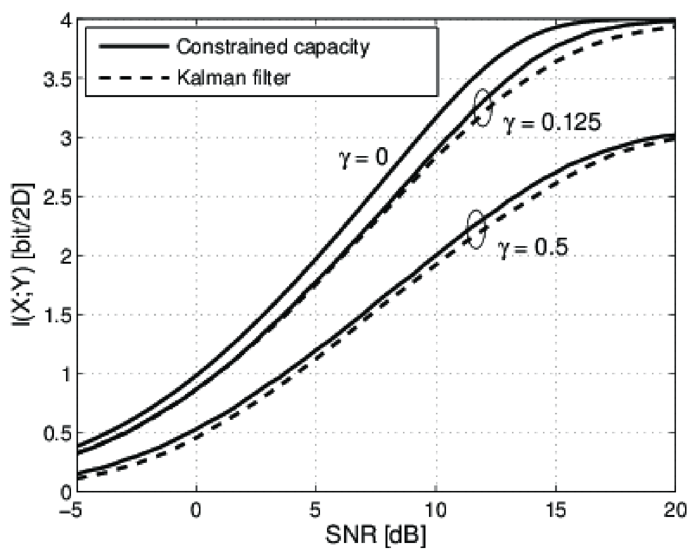

Figure 3. Actual channel capacity (solid line) and lower bound (dashed line) for 16-QAM, Wiener phase noise, and two values of γ.

Figures 4 and 5 report the results obtained with the second-order phase noise model of (9). Now the state space is multidimensional, and quantizing a multidimensional state space according to the trellis-based approach of [13, 14] would lead to an exponential increase of the number of states of the trellis, making computation unfeasible. Therefore, in order to work out an approximation to channel capacity, we adopt the particle filter of [19]. In the simulation results, 10^4 particles are used. Basically, this means that 10^4 filters are used in parallel. The results of Figs. 4 and 5 show that our proposed bound is even closer to the actual channel capacity with second-order phase noise than with first-order phase noise, at least with the parameters (9).

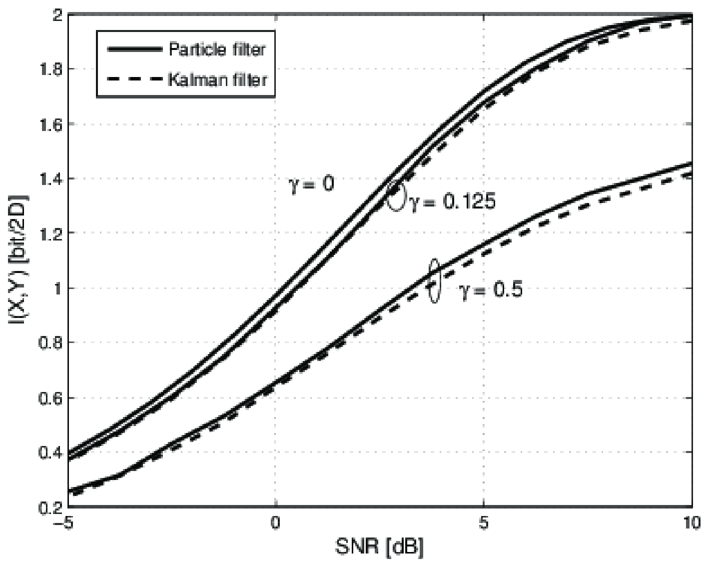

Figure 4: Channel capacity obtained by the particle method (solid line) and lower bound (dashed line) for 4-QAM and second-order phase noise for two values of γ.

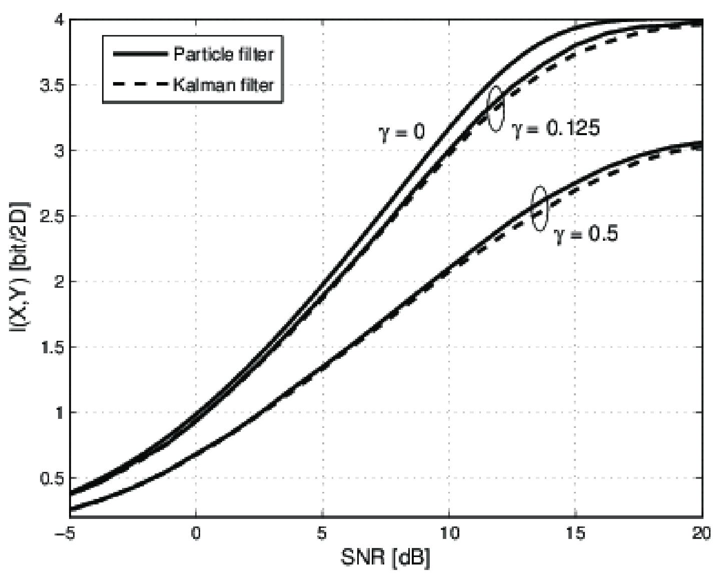

Figure 5. Channel capacity obtained by the particle method (solid line) and lower bound (dashed line) for 16-QAM and second-order phase noise for two values of γ.

6. Conclusion

In the paper, a new lower bound below the information rate transferred through the phase noise channel with higher-order memory has been presented. The results, compared to the actual channel capacity obtained with the computationally demanding methods of [14] and [19], show that the bound is accurate in many cases of practical interest.

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