Unit 5 – Area

What Is Area?

Overview: Participants determine the area of a rectangle by counting the number of square units needed to cover the region. Group discussion deepens participants' understanding of area (number of square units needed to cover a given region) and connects the formula for the area of a rectangle to the underlying array structure.

Objective: **TExES Mathematics Competencies**
III.013.D. The beginning teacher computes the perimeter, area, and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).
V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

Geometry TEKS
b.3.D. The student uses inductive reasoning to formulate a conjecture.
e.1.A. The student finds area of regular polygons and composite figures.

Background: No prerequisite knowledge is necessary for this activity.

Materials: index cards, patty paper, straightedge

New Terms: area

Procedures:

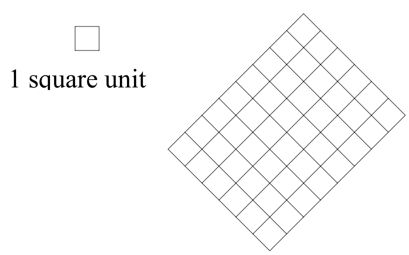
Background information: In order to accurately count the units of a given space, it is necessary to mentally organize the space in a structured manner. However, research referenced in Schifter, Bastable, and Russell (2002), on how children learn the concept of area supports the theory that the structure of the rectangular array is not intuitively obvious to children. When asked to cover a rectangular region, children progress from incomplete or unsystematic coverings to individually drawn units to the use of a row or column iteration. Gradually they will rely less on drawing and move towards multiplication or repeated addition. Covering a rectangular region with unit squares helps children understand area measure, but they must ultimately be able to formally connect area, linear measurement, and multiplication in order to truly understand the area formula $A = b \cdot h$. According to Schifter, Bastable, and Russell (2002), the drawing, filling, and counting that children use in this developmental process are both motor and mental actions that coordinate to organize spatial structuring.

What implications does this research have for secondary teachers? Many of our students come to us with an understanding of area at the Visual Level of the van Hiele model of geometric development. They recognize figures by their shape and understand area as the blank space within the boundaries. Others have moved to the Descriptive Level, which implies that they can mentally see the rectangular array which overlays the shape. This activity asks participants to draw the grid, rather than giving them a pre-structured grid in order for them to experience the type of activity necessary to move a student from the Visual to the Descriptive Level. Secondary students functioning on the Relational Level are able to compare linear dimensions with grid areas and apply formulas with understanding. Teachers must be aware that although students may become proficient at rote application of formulas, they may not be functioning at the Relational Level. If they have not been given sufficient opportunity to understand the principles behind the formulas, they will have difficulty modifying a procedure to fit a particular situation as is necessary to find areas of composite figures and shaded regions.

Distribute index cards to participants and ask them to write a response to the question "What is area?". Indicate that they will have an opportunity to share and revise their responses at the conclusion of this activity. Then, allow time for participants, working independently or in pairs, to complete the activity using the patty paper or straight edge to determine the number of square units needed to completely cover the rectangular region.

Note that the given square units are not convenient measures, such as 1 cm^2 or 1 $in.^2$. Consequently, participants will be less likely to simply measure the rectangle and use the area formula without having the experience of drawing the units. Whether participants mark off the units on two adjacent sides of the rectangle and multiply or actually draw in one or more rows and columns of units, they will be counting the units by considering how many rows of squares are needed to cover the region.

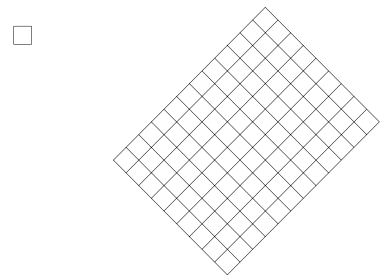
1. Determine the number of square units needed to cover this rectangular region.



The rectangle measures 6 ·8 square units. Therefore, it will take 48 square units to cover the rectangle.

2. Determine the number of square units needed to cover this rectangular region. (Same rectangle, different square unit)

1 square unit



The rectangle measures 9 · 12 square units. Therefore, it will take 108 square units to cover the rectangle.

Was it necessary to draw all 108 square units to determine that it would take 108 units to cover the rectangular region?
No. After drawing one row and one column of square units the total number of squares can be obtained by considering how many rows and columns of squares will be needed to cover the entire region.

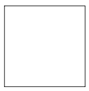
Has the activity caused you to reconsider your definition of area?
Some participants may have responded to the question "What is area?" by stating that area is the amount of space covered by a particular region. It is important to make the distinction that area is the number of square units needed to completely cover a particular region. If the same figure is measured in different units, the number representing the area of the region will be different, but the area will remain constant.

A more abstract definition of area, provided by Michael Serra (Serra, 2003) states that area is a function that assigns to each two-dimensional geometric shape a nonnegative real number so that (1) the area of every point is zero, (2) the areas of congruent figures are equal, and (3) if a shape is partitioned into sub regions, then the sum of the areas of those sub regions equals the area of the shape.

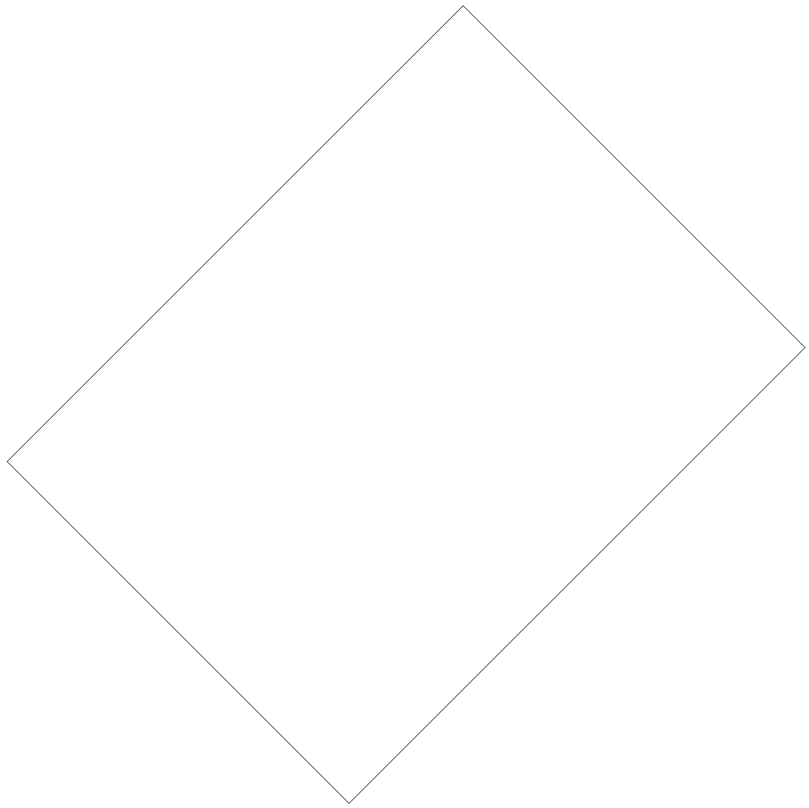
If a figure is rotated so that a different side is considered the base, will the area formula necessarily give the same result?
Yes. Surprisingly, the answer to this question is not evident to all students. According to the work of Clements and Battista referenced in Schifter, Bastable, and Russell (2002), orientation, the position of objects in space in relation to an external frame of reference, is for some children a part of their definition of a particular shape. If secondary students have not had adequate experience manipulating by rotating, flipping or sliding shapes, they may be working at the Visual Level with an inadequate understanding of shape. In developing an understanding of area, students should observe that rotations, reflections and translations preserve area while dilations do not.

What Is Area?

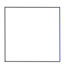
1. Determine the number of square units needed to cover this rectangular region.



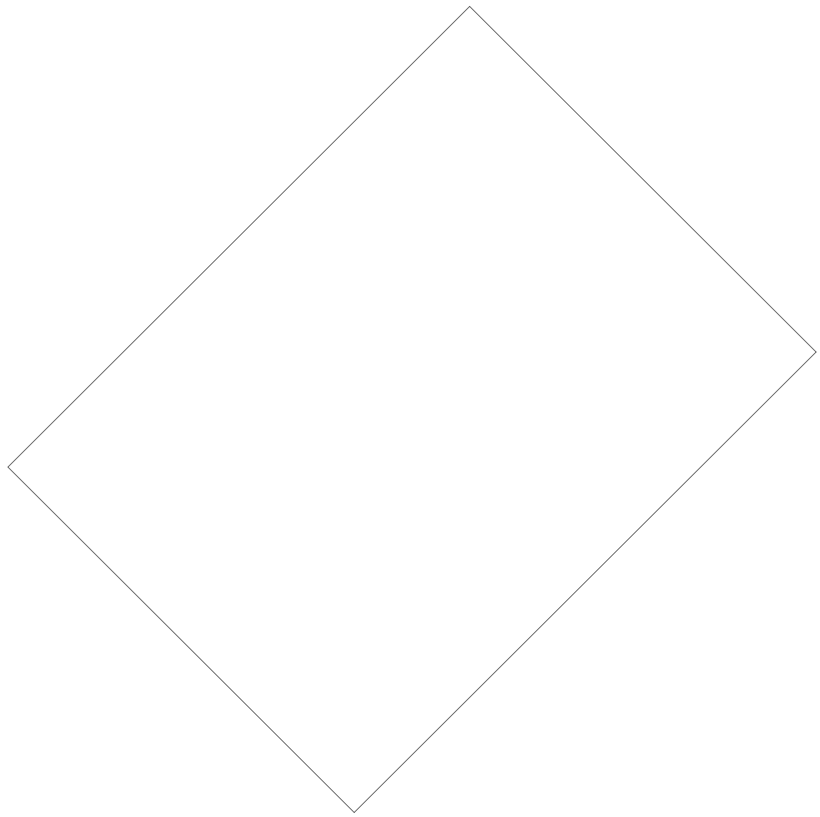
1 square unit



2. Determine the number of square units needed to cover this rectangular region. (Same rectangle, different square unit)



1 square unit



Investigating Area Formulas

Overview: Participants cut and rearrange two triangles, a parallelogram, and a kite to form rectangles with the same areas. Examination of the points at which figures must be cut will lead to a deeper understanding of the formula for the area of each figure.

Objective: **TExES Mathematics Competencies**
III.011.A. The beginning teacher applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another and to find and evaluate solutions to problems.
III.013.C. The beginning teacher uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).
V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

Geometry TEKS
b.2.A. The student uses constructions to explore attributes of geometric figures and to make conjectures about geometric relationships.
b.3.C. The student demonstrates what it means to prove mathematically that statements are true.
d.2.C. The student develops and uses formulas including distance and midpoint.

Background: Participants should know the formula for the area of a rectangle and be able to identify the base and altitude of a triangle, the base and altitude of a parallelogram, and the diagonals of a kite.

Materials: transparency sheets, colored pencils, glue or tape, patty paper, scissors

New Terms:

Procedures:

Participants sit in groups of 3-4 to work collaboratively. However, each participant should cut and glue his/her own figures.

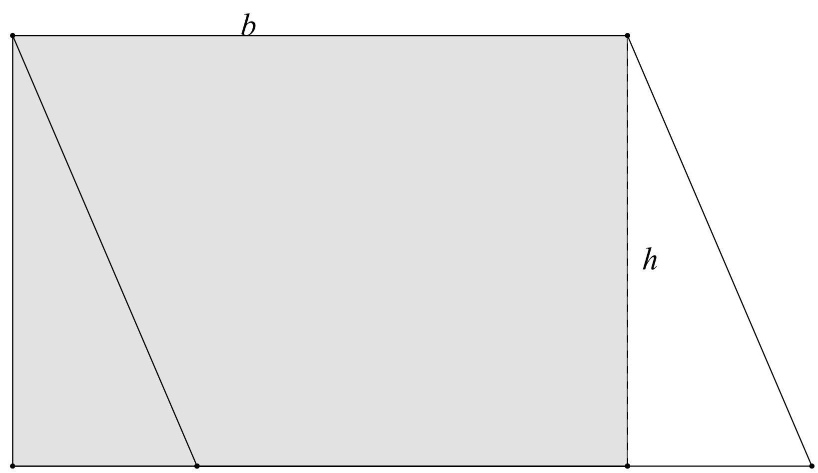
Can any parallelogram, triangle or kite be cut and rearranged to form a rectangle of the same area?
Allow time for discussion within groups and then ask two or three groups to share their responses with the entire group. It is fairly obvious that any parallelogram can be cut to

form a rectangle of equal area, but the group may or may not be able to reach a consensus on the triangle and kite.

Briefly describe the activity. Each of the figures on the activity sheet has the same area as the rectangle. Using a colored pencil, trace the parallelogram on patty paper. Then, using the least number of cuts possible, cut the parallelogram and rearrange the pieces to form a rectangle of equal area. The rectangle will help you determine where to cut. Lay the patty paper tracing over the rectangle and slide it around to decide where to cut. Using a different colored pencil, draw the cut line on the patty paper figure and then cut. Assemble the pieces to form a rectangle and glue it next to the original parallelogram. Repeat the process for each of the figures.

While the groups are working, assign each of the figures to a different participant to draw on a transparency for use during the group discussion. When most participants have completed the task, reconvene as a large group for discussion.

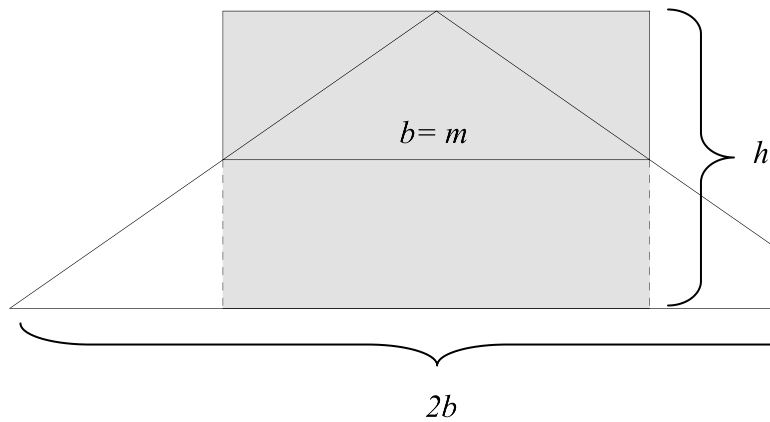
Parallelogram:



Can we express the dimensions of the rectangle in terms of the dimensions of the parallelogram?
Yes. The base of the rectangle is the base of the parallelogram. The height of the rectangle is the height of the parallelogram.

What does this tell us about the formula for the area of the parallelogram?
Since the area of the parallelogram is equal to the area of the rectangle, the area of the parallelogram is $b \cdot h$.

Obtuse Triangle:



How would you describe the location of the cut lines on the obtuse triangle?
The cuts must pass through the midpoints of the sides of the triangle as shown.

What do we call the segment that connects the midpoints of the sides of a triangle?
The midsegment

What do we know about the midsegment of a triangle?
The midsegment is parallel to the base and one half the length of the base of the triangle.

Can we express the dimensions of the rectangle in terms of the dimensions of the triangle?
Yes. The length of the base of the rectangle is equal to the length of the midsegment of the triangle, *m*. The height of the rectangle, *h*, is the height of the triangle.

Can we use this information to derive the formula for the area of a triangle?
Since we know

Area of triangle = Area of rectangle

$= b \cdot h$

By substitution,

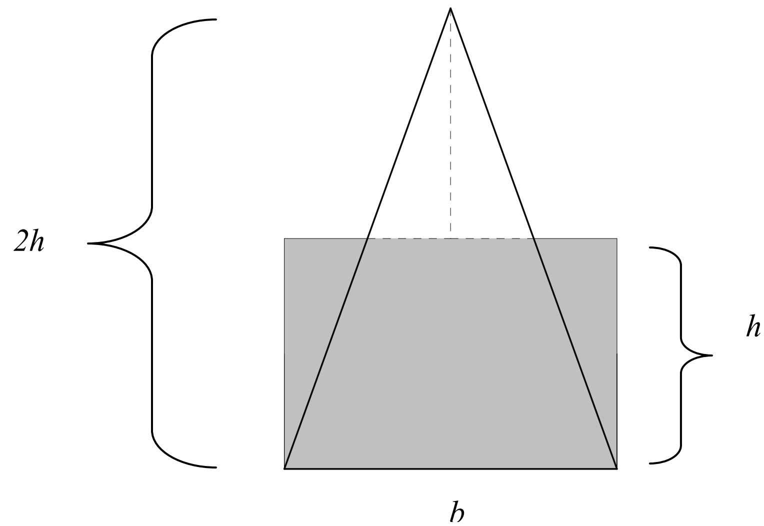
Area of triangle = $m \cdot h$

By the definition of a midsegment,

Area of triangle = $\frac{1}{2}(2b) \cdot h$

$= \frac{1}{2}$ (base of triangle) (height)

Acute Triangle:

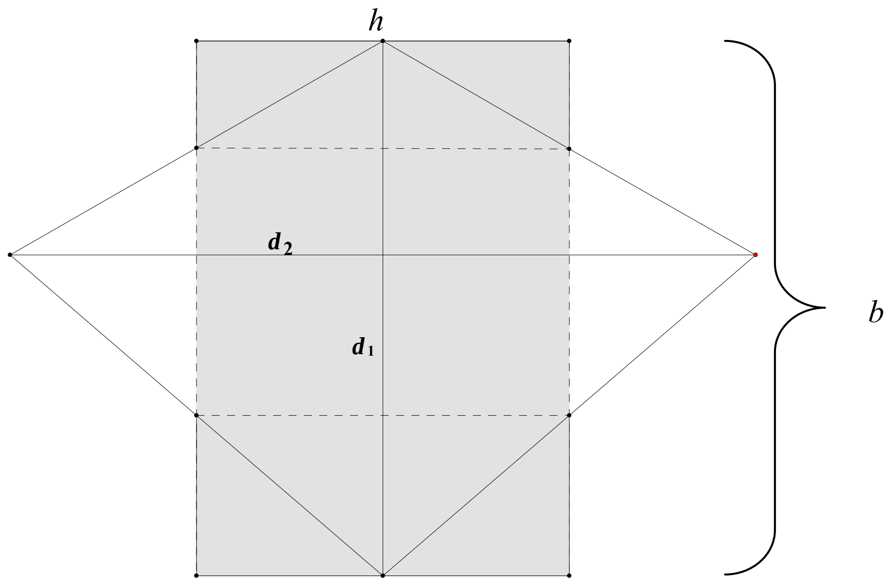


How would you describe the location of the cut lines on the acute triangle?
One cut line goes through the midsegment of the triangle and one cut line is the altitude joining the midsegment to the opposite vertex of the triangle.

Can we use this information to derive the formula for the area of a triangle?
Since we know

$$\begin{aligned}\text{Area of triangle} &= \text{Area of rectangle}\\ &= b\cdot h\\ &= b\cdot\frac{1}{2}(2h)\\ &= \frac{1}{2}\text{ (base of triangle) (height of triangle)}\end{aligned}$$

Kite:



How can we use the formula we have derived for the area of a triangle to derive the formula for the area of the kite?

d_1 lies on the line of symmetry for the kite. The two triangles formed by the line of symmetry, d_1, are congruent.

$\frac{1}{2}(d_2)$ is the length of the altitude of each triangle, and $\frac{1}{2}(d_2) = h$

Area of one triangle $= \frac{1}{2}(d_1) \cdot h = \left(\frac{1}{2}\right)(d_1)\left(\frac{1}{2}\right)(d_2)$

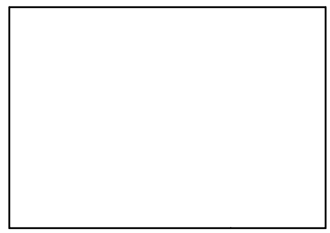
Area of kite $= 2(\frac{1}{2})(d_1)(\frac{1}{2})(d_2) = \frac{1}{2}(d_1 \cdot d_2)$

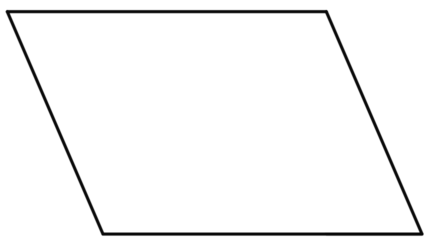
$= \frac{1}{2}d_1 \cdot d_2$

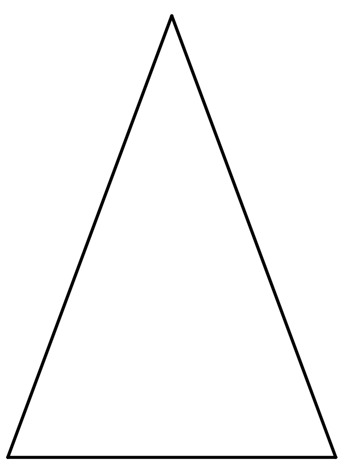
Success in this activity indicates that participants are working at the Relational Level because they must discover the relationship between the area rule for a rectangle and the area rule for a parallelogram, triangle, or kite. While a participant at the Descriptive Level will be able to cut the figures to form the rectangle of the same area, he/she will need prompting to explain "why it works" using informal deductive arguments.

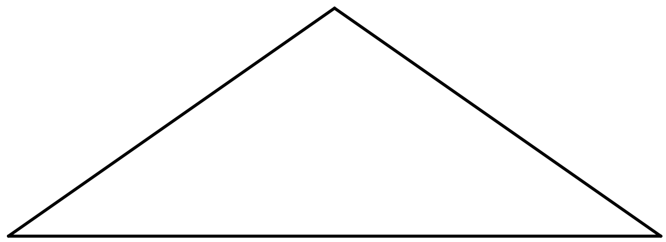
Investigating Area Formulas

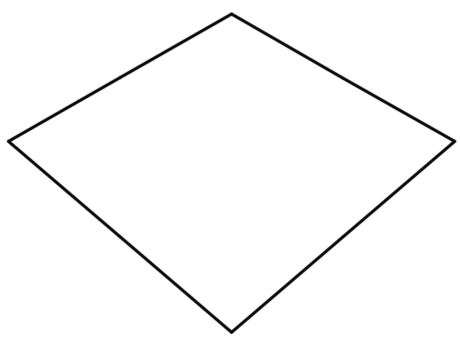
Trace the parallelogram, triangles, and kite on patty paper. Then cut and arrange the pieces of each figure to form a rectangle congruent to the given rectangle. Glue each new figure next to the original figure from which it was made. Label the dimensions of the rectangle *b* and *h*. Determine the area for each figure in terms of *b* and *h*.











Area of Trapezoids

Overview: Participants find many different ways to derive the formula for the area of a trapezoid.

Objective: **TExES Mathematics Competencies**

III.011.A. The beginning teacher applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another and to find and evaluate solutions to problems.

III.013.C. The beginning teacher uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).

V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

Geometry TEKS

b.2.A. The student uses constructions to explore attributes of geometric figures and to make conjectures about geometric relationships.

b.3.C. The student demonstrates what it means to prove mathematically that statements are true.

d.2.C. The student develops and uses formulas including distance and midpoint.

Background: Participants need to know the formulas for areas of triangles, rectangles, and parallelograms. A clear understanding of the midsegment of a trapezoid is a prerequisite.

Materials: patty paper, scissors

New Terms:

Procedures:

Distribute the activity sheet and allow about 10 minutes for participants to work on it individually so that each will have an opportunity for independent discovery. Within their group of four, allow another 15 minutes for participants to share all of the methods for deriving the formula for the area of a trapezoid that they have discovered. Then jigsaw the groups and allow about 20 minutes for the new groups to share their work. During the final 15 minutes of the activity, participants return to their original groups and share any new method they have seen during the jigsaw.

Scaffolding questions to help participants who are having difficulty finding multiple solutions:

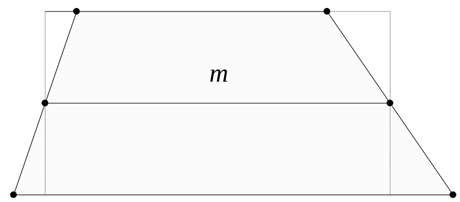
Can you use a technique from the previous activity on areas of triangles and parallelograms?
In the previous activity we saw that the midsegment could be used to cut and rearrange a triangle into a rectangle of the same area.

Can a trapezoid be divided into figures whose area formulas are known?
Yes, it can be divided into a rectangle and two triangles, three triangles, or four triangles.

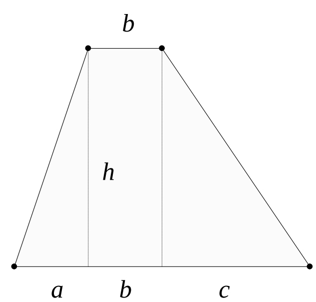
Can you create a larger figure in which the trapezoid is one of the composite parts?
Yes, the trapezoid can be copied and rotated 180^{o} to create a parallelogram whose area is equal to twice the area of the original trapezoid. A triangle can be drawn such that a rectangle is formed from the triangle and the original trapezoid.

Answers will vary. Seven derivations are given below.

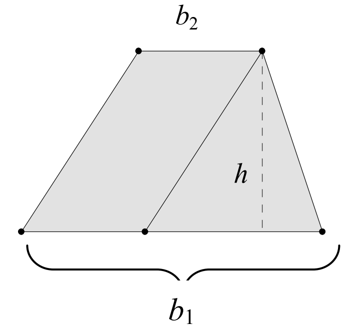
1. *Base of rectangle = Midsegment of trapezoid*
   Height of rectangle = Height of trapezoid
   Area of trapezoid = Area of rectangle =
   Midsegment of trapezoid · *height of trapezoid*



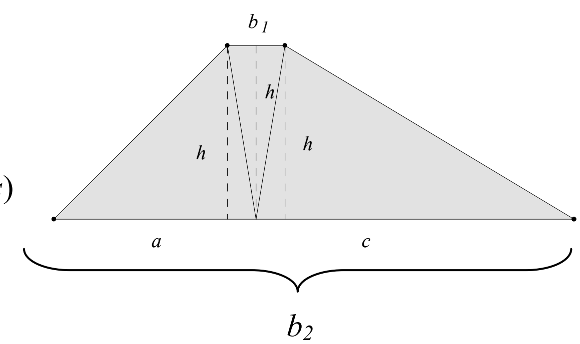
2. $$\text{Area of triangle 1} = \frac{1}{2}ah$$
$$\text{Area of triangle 2} = \frac{1}{2}ch$$
$$\text{Area of rectangle} = bh$$
$$\text{Area of trapezoid} = \frac{1}{2}ah + \frac{1}{2}ch + h$$
$$= h\left(\frac{1}{2}a + \frac{1}{2}c + b\right)$$
$$= \frac{1}{2}h(a + c + 2b)$$
$$= \frac{1}{2}h[(a + c + b) + b]$$
$$= \frac{1}{2}(\text{height})(\text{sum of bases})$$



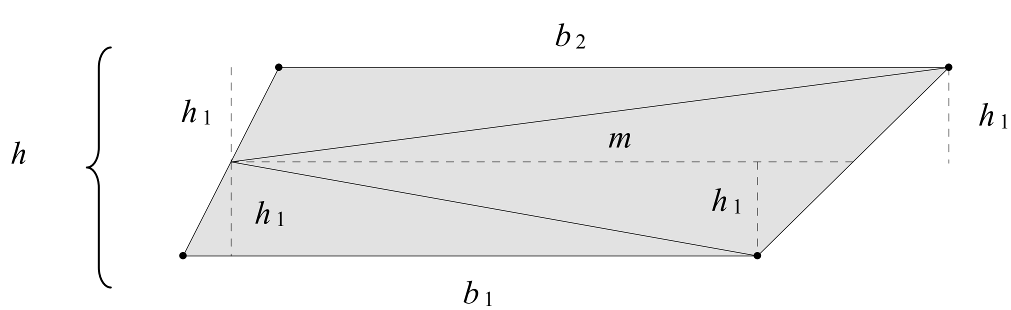
3. $$\begin{aligned} \textit{Area of trapezoid} &= \textit{Area of parallelogram} + \textit{area of triangle} \\ &= h \cdot b_2 + \frac{1}{2} h\,(b_1 - b_2) \\ &= h \cdot b_2 + \frac{1}{2}\, h \cdot b_1 - \frac{1}{2}\, h \cdot b_2 \\ &= \left(\frac{1}{2}\right) h\,(2b_2 + b_1 - b_2) \\ &= \left(\frac{1}{2}\right) h\,(b_1 + \mathrm{b}_2) \\ &= \left(\frac{1}{2}\right)(\textit{height})\,(\textit{sum of bases}) \end{aligned}$$



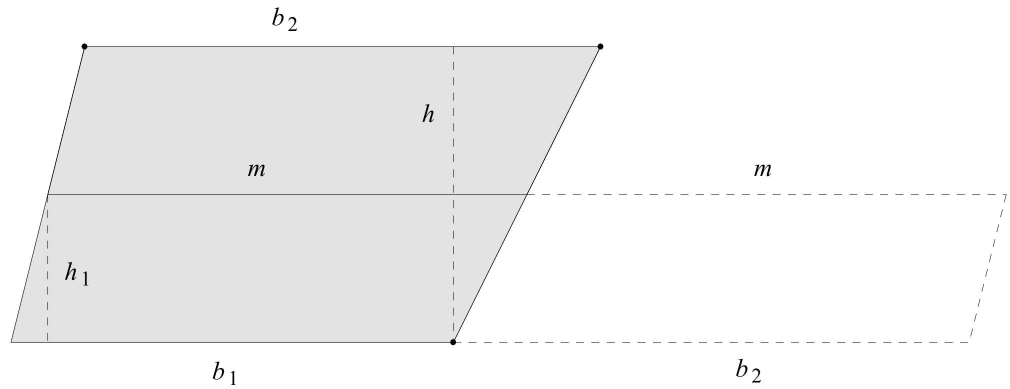
4. $$\begin{aligned} \textit{Area of trapezoid} &= \left(\frac{1}{2}\right) h\, a + \left(\frac{1}{2}\right) h \cdot b_1 + \left(\frac{1}{2}\right) hc \\ &= \left(\frac{1}{2}\right) h\,(a + b_1 + c) \\ &= \left(\frac{1}{2}\right) h\,(b_1 + b_2) \\ &= \left(\frac{1}{2}\right) (\textit{height})\,(\textit{sum of bases}) \end{aligned}$$



5. $$\begin{aligned} \textit{Area of trapezoid} &= \left(\frac{1}{2}\right) h_1 \cdot b_1 + \left(\frac{1}{2}\right) h_1 \cdot b_2 + \left(\frac{1}{2}\right) h_1 \cdot m + \left(\frac{1}{2}\right) h_1 \cdot m \\ &= \left(\frac{1}{2}\right) h_1\,(b_1 + b_2) + h_1 \cdot m \\ &= h_1 \cdot m + h_1 \cdot m \\ &= 2hm \end{aligned}$$



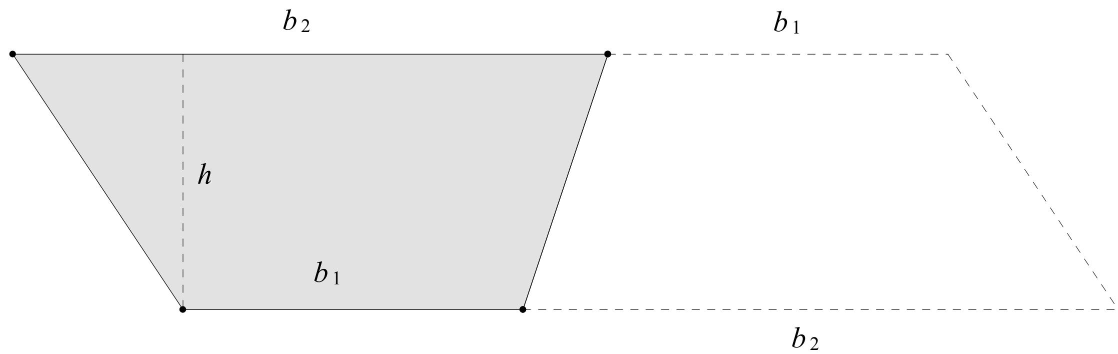
6. *Area of trapezoid* = *Area of parallelogram* = $2 \cdot m \cdot h_1 = 2m \cdot \left(\frac{1}{2}\right) h = mh$



7. *Area of trapezoid* $= \left(\frac{1}{2}\right)$ *Area of parallelogram*

$= \left(\frac{1}{2}\right) h\,(b_1 + b_2)$

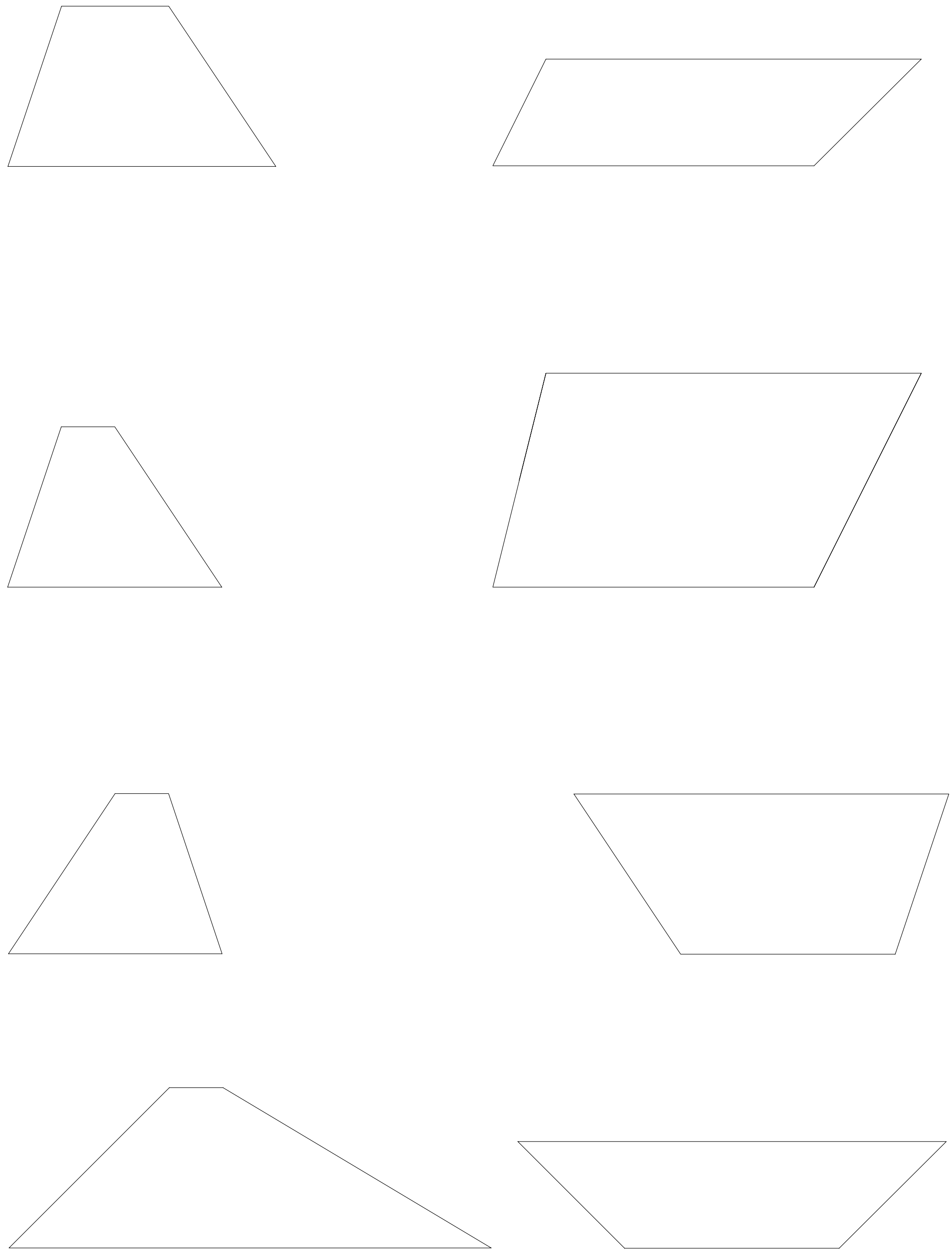
= (*midsegment of trapezoid*) (*height of trapezoid*)



Success in this activity indicates that participants are working at the Relational Level because they must show the relationships among area rules and give informal deductive arguments to justify the rule determining the area of a trapezoid.

Area of Trapezoids

How many ways can you derive the formula for the area of a trapezoid?



Area of Circles

Overview: Participants develop the concept of area of circles by drawing a circle and then cutting it into sectors. The sectors are rearranged to form a shape that resembles a parallelogram.

Objective: **TExES Mathematics Competencies**
III.011.B. The beginning teacher applies formulas for perimeter, area, surface area, and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.
III.013.D. The beginning teacher computes the perimeter, area, and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc, length, area of sectors).
V.018.E. The beginning teacher understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).
V.019.C. The beginning teacher translates mathematical ideas between verbal and symbolic forms.
V.019.D. The beginning teacher communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).

Geometry TEKS
b.3.C. The student demonstrates what it means to prove mathematically that statements are true.
b.4. The student selects an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.
d.2.C. The student develops and uses formulas including distance and midpoint.
e.1.B. The student finds areas of sectors and arc lengths of circles using proportional reasoning.

Background: Participants need to understand area as a covering of a space, pi, and the circumference of a circle.

Materials: cups (preferably large plastic cups), glue or tape, graphing calculator, colored markers, patty paper, scissors

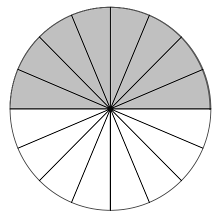
New Terms: circumference, sector

Procedures:

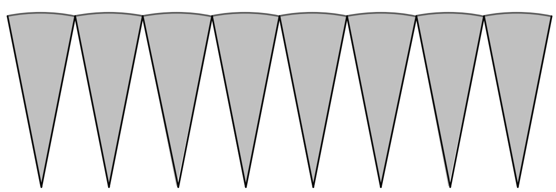
Participants work in pairs for both parts of this activity.

Part 1: Developing the Formula for the Area of a Circle

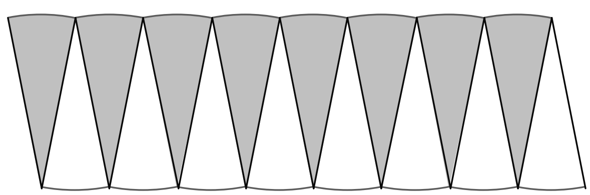
Using a piece of patty paper, trace the circular rim of the plastic cup. Fold the patty paper to locate the center of the circle. Fold the circle in half four additional times to create sixteenths. Firmly crease the folds. Using a marker, color half of the circle in one color and the other half of the circle in a different color.



Cut out the 16 sectors of the circle. Paste the sectors in one color upside down on a sheet of paper.



Paste the sectors in the other color between the sectors in an interlocking manner, forming a parallelogram shaped figure.



Possible scaffolding questions to help participants develop the formula of area:

How is the sum of the sectors related to the area of the circle?
The sum of the areas of the sectors is equal to the area of the circle.

Identify the length of the parallelogram.
The length is half of the circumference of the circle. Because the circumference is $C = 2\pi r$, and the length of the parallelogram is half of the circle, the length is πr. Label the length πr.

Identify the width of the parallelogram.
The width is the same as the radius of the circle. Label it *r*.

What is the area of the parallelogram-shaped figure?
The area is $(\pi r)(r) = \pi r^2$.

Is the area of a circle a function of the radius?
Yes; the area depends on the length of the radius.

Part 2: Applying the Formula for the Area of a Circle

1. The circumference of a circle and the perimeter of a square are each 20 in. Which has the greater area, the circle or the square? How much greater?

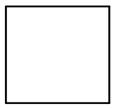
$P = 20\ in$

$P = 4s$

$20 = 4s$

$5 = s$

$A = 5^2 = 25\ in^2$



$C = 2\pi r$

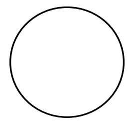
$20 = 2\pi r$

$r \approx \frac{10}{\pi}$

$A = \pi r^2$

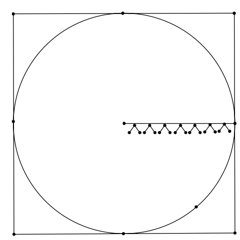
$A = \pi \times \left[\frac{10}{\pi}\right]^2$

$A \approx 31.8\ in^2$



The circle has the greater area. The approximate difference is $31.8 in^2 - 25 in^2 = 6.8\ in^2$.

2. A rice farmer uses a rotating irrigation system in which 8 sections of 165-foot pipe are connected end to end. The resulting length of sprinkler pipe travels in a circle around a well, which is located in the center of the circle.



a. How many acres of the square field are irrigated? (one acre is 43,560 ft^2)
There are 8 sections of 165 feet pipe, or 1320 feet of pipe, which is the radius of the irrigated field. The area is $\pi \cdot 1320^2$ *or* $1742400\pi\ ft^2$ *or* $5473911.0\ ft^2$. *To find the number of acres, divide the total square feet by the 43,560* ft^2 *in one acre.*
$\frac{5473911}{43560} \approx 125.7 acres$.

b. Broccoli is often planted in the corners of the field, which are not irrigated. How many acres of broccoli are planted in this field?

Area of square – Area of circle = Area of corners

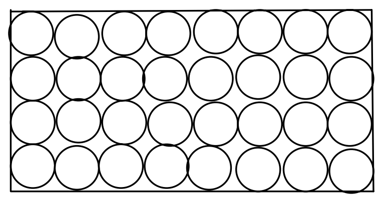
The diameter of the circle is the length of the side of the square. The diameter is 2640 ft. The area of the square is 2640^2 or 6969600 is ft^2.

Area of square – Area of circle = Area of corners

Using Substitution, $6969600 - 5473911 = 1495689\ ft^2$

Changing the area to acres, $\dfrac{1495689}{43560} \approx 34.3\ acres$

3. Ron opened a new restaurant. He asked his wait staff to determine the greatest number of cylindrical glasses that would fit on a 24 in. by 12 in. tray. Each glass has a radius of 1.5 inches. Being a math geek, he began to wonder how much area was not being used on the tray. Make a drawing of the arrangement that fits the most glasses on the tray. Make a prediction of how many glasses will fit on the tray. What is the area on the tray **not** used by the cylindrical glasses?



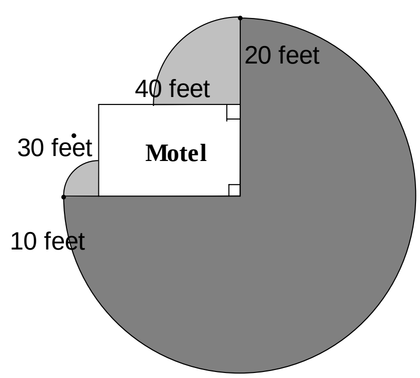
$Area\ of\ glass = \pi(1.5)^2 \approx 7.1\ in^2$

$Area\ of\ 32\ glasses = 32(7.1) \approx 227.2\ in^2$

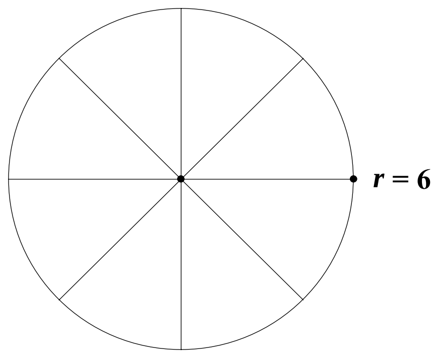
$Area\ of\ tray = 24(12) = 288\ in^2$

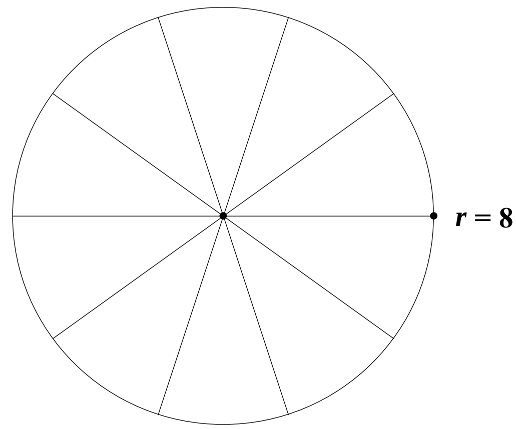
$Area\ of\ difference = 288\ in^2 - 227.2\ in^2 = 60.8\ in^2$

4. When Ruth's family moved during her sophomore year of high school, they stayed in a motel for 3 weeks. Their dog, Frosty, was tied at the corner of the motel on a 50 foot rope. If the motel were a 40 ft by 30 ft rectangle, how many square feet could Frosty wander? (Give answer in terms of π.)
The dog could run in a three quarters circle having a radius of 50 feet.
At both ends of the building his path forms a quarter circle, one with a radius of 20 feet, and the other with a radius of 10 feet. The dog can wander 2000π square feet.



5. Circle Pizzeria is changing the size of its circular pizza from 12 inches to 16 inches and increasing the number of slices per pizza from 8 to 10. What is the percent increase of the size of each new slice? (Round answer to the nearest tenth of a percent.)





12 inch pizza **16 inch pizza**

Area of 12 inch pizza $= \pi\, 6^2 = 36\pi$

Area of 16 inch pizza $= \pi\, 8^2 = 64\pi$

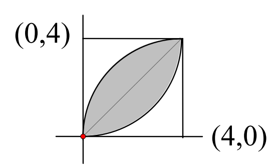
$$\frac{36\pi}{8\ slices} = 4.5\pi\ in.^2\ per\ slice$$

$$\frac{64\pi}{10\ slices} = 6.4\pi\ in.^2\ per\ slice$$

$$6.4\pi - 4.5\pi = 1.9\pi\ difference$$

$$\frac{1.9\pi}{4.5\pi} \approx 42.2\%\ increase$$

6. Circles of radius 4 with centers at (4,0) and (0,4) overlap in the shaded region shown in the figure. Find the area of the shaded region in terms of π.
   One way to find the area of the shaded region is to draw the diagonal of the square.
   $\frac{1}{2}$ *Area of shaded region = Area of quarter circle – Area of triangle*



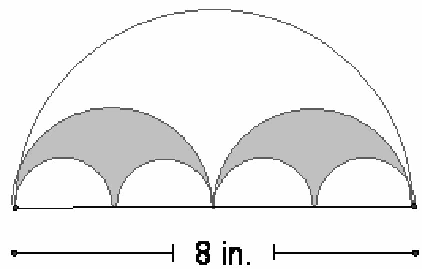
$$\frac{1}{4}\pi r^2 - \frac{1}{2}bh$$

Area of sector: $\frac{1}{4}\pi 16 - \frac{1}{2}16$

$$2(4\pi - 8)$$

$$8\pi - 16\ units^2$$

7. In the figure shown, all arcs are semicircles, and those that appear to be congruent are. What is the area of the shaded region? (Give answer in terms of π.)



$$2(\textit{Area midsized semi-circle}) = 2\left(\frac{1}{2}\pi \cdot 2^2\right) = 4\pi$$

$$4(\textit{Area small semi-circle}) = 4\left(\frac{1}{2}\pi \cdot 1^2\right) = 2\pi$$

Area midsized semi-circles. Area small semi-circles $= 4\pi - 2\pi = 2\pi$

Remind participants to add the new terms circumference and sector to the glossaries.

Part 1: Success in this part of the activity indicates that participants are working at the Relational Level because they must discover the relationship between the formula for the area of a parallelogram and the formula for the area of a circle.

Part 2: Success in this part of the activity indicates that participants are working at the Relational Level because they must solve geometric problems by using known properties of figures and insightful approaches.

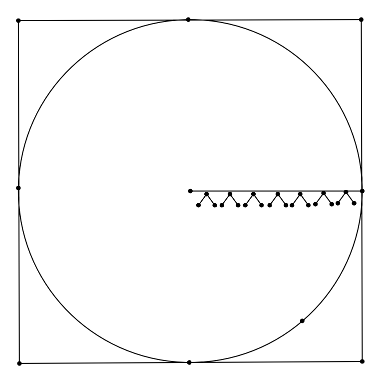
Area of Circles

Solve the following problems.

1. The circumference of a circle and the perimeter of a square are each 20 in. Which has the greater area, the circle or the square? How much greater?

2. A rice farmer uses a rotating irrigation system in which 8 sections of 165 foot pipe are connected end to end. The resulting length of sprinkler pipe travels in a circle around a well, which is located in the center of the circle.

   a. How many acres of the square field are irrigated? (One acre is 43,560 ft^2.)



   b. Broccoli is often planted in the corners of the field, which are not irrigated. How many acres of broccoli are planted in this field?

3. Ron opened a new restaurant. He asked the wait staff how many cylindrical glasses would fit on a 24 in. by 12 in. tray. Each glass has a radius of 1.5 inches. Being a math geek, he began to wonder how much area was not being used on the tray. Make a drawing of the arrangement that fits the most glasses on the tray. Make a prediction of how many glasses will fit on the tray. What is the area on the tray **not** used by the cylindrical glasses?

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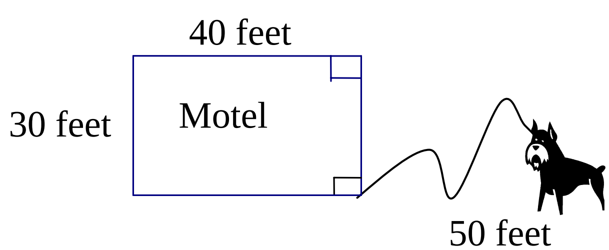
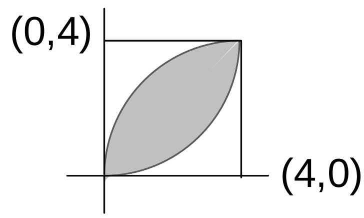
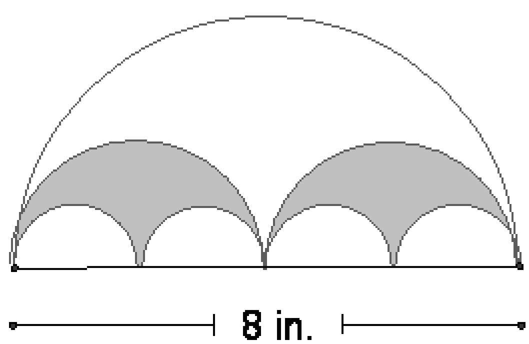


5. Circle Pizzeria is changing the size of its circular pizza from 12 inches to 16 inches and increasing the number of slices per pizza from 8 to 10. What is the percent of increase of the size of each new slice? (Round answer to nearest tenth of a percent.)

6. Circles of radius 4 with centers at (4,0) and (0,4) overlap in the shaded region shown in the figure. Find the area of the shaded region in terms of π.



7. In the figure shown, all arcs are semicircles, and those that appear to be congruent are. What is the area of the shaded region? (Give answer in terms of π.)



Applying Area Formulas

Overview: Participants use the problem solving process to find the area of composite figures (composite of triangles, quadrilaterals and circles).

Objective: **TExES Mathematics Competencies**

III.011.B. The beginning teacher applies formulas for perimeter, area, surface area, and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems).

III.013.D. The beginning teacher computes the perimeter, area, and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).

V.018.E. The beginning teacher understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).

V.019.C. The beginning teacher translates mathematical ideas between verbal and symbolic forms.

V.019.D. The beginning teacher communicates mathematical ideas using a variety of representations (e.g., numerical, verbal, graphical, pictorial, symbolic, concrete).

Geometry TEKS

b.4. The student selects an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

d.2.C. The student develops and uses formulas including distance and midpoint.

e.1.A. The student finds the area of regular polygons, and composite figures.

Background: Participants need to know the area formulas and how to connect the formulas to models of composite figures (composite of triangles, quadrilaterals and circles).

Materials: calculator

New Terms: composite figures

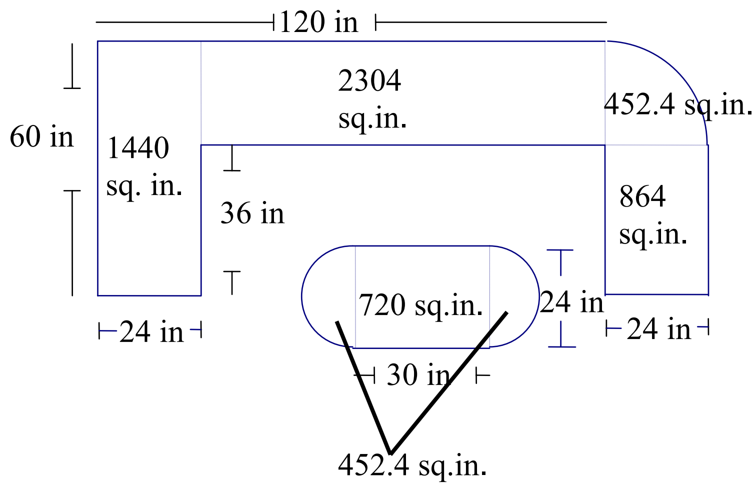
Procedures:

Write the word “composite” on the overhead and brainstorm some meanings of this word. The Merriam-Webster Collegiate Dictionary, from *http://www.yourdictionary.com,* defines *composite*, when used as an adjective, as “consisting of separate interconnected parts.” Remind the participants to add the new term to their glossaries.

What are some real world examples of composite figures?
Possible answers are:
Photography: an image or scene made up of two or more original images placed side by side, overlapped, or superimposed.
Automotive: any material that consists of two or more substances bonded together for strength, such as fiberglass.
Architecture: architectural drawings, consisting of more than one room or combination of shapes.

Participants work collaboratively on problems.

1. An interior designer created a plan for the kitchen counters and an island to be located in the middle of the kitchen, as shown below. The opposite sides of the counter are parallel and the intersecting straight lines are perpendicular. The curved part of the countertop is a quarter circle. The island has parallel sides, and the curved end is a semicircle. What is the total area of the tops of the counter space and island? Use the π key on the graphing calculator instead of $\frac{22}{7}$ or 3.14. Round calculations to the nearest tenth.



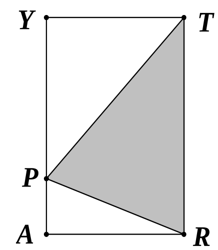
The total areas of the tops of the counter and the island are 6232.8 sq.in.

$$\frac{6232.8}{144} \approx 43.3\, sq.ft.$$

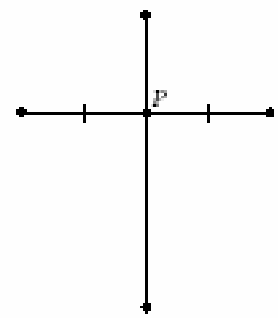
The counter tops and the island in the kitchen will be covered with granite. Granite costs $14.38 per square foot. Find the cost of the countertops and island for this kitchen? Round the cost to the nearest penny.

$$43.3\, sq.ft. \cdot \frac{\$14.38}{sq.ft.} = \$622.654 \approx \$622.65$$

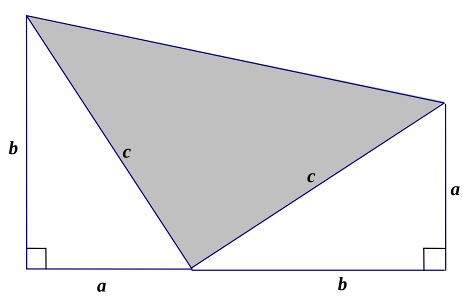
2. P is a random point on side $\overline{AY}$ of rectangle $ARTY$. The shaded area is what fraction of the area of the rectangle? Why?
   The altitude of the triangle, h, is equal to the height of the rectangle. The base of the triangle is equal to the base of the rectangle. So, the area of the triangle = $\frac{1}{2}bh$ *or* $\frac{1}{2}$ *area of the rectangle.*



3. Kit and Kat are building a kite for the big kite festival. Kit has already cut his sticks for the diagonals. He wants to position P so that he will have maximum kite area. He asks Kat for advice. What should Kat tell him? This problem is taken from *Discovering Geometry: An Investigative Approach, Practice Your Skills*, 3rd Edition, ©2003, p.49, used with permission from Key Curriculum Press.
   The location of the point of intersection of the two diagonals does not change the area, because the area of the kite is $\frac{1}{2}d_1d_2$.

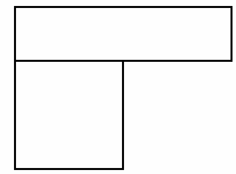


4. A trapezoid has been created by combining two congruent right triangles and an isosceles triangle, as shown. Is the isosceles triangle a right triangle? How do you know? Find the area of the trapezoid two ways: first by using the trapezoid area formula, and then by finding the sum of the areas of the three triangles. This problem is taken from *Discovering Geometry: An Investigative Approach*, 3rd Edition, ©2003, p.420, 21, used with permission from Key Curriculum Press.



The isosceles triangle is a right triangle because the angles on either side of the right angle are complementary. If you use the trapezoid area formula, the area of the trapezoid is $\frac{1}{2}(a + b)(a + b)$. *If you add the areas of the three triangles, the area of the trapezoid is* $\frac{1}{2}c^2 + ab$.

5. The rectangle and the square have equal area. The rectangle is 12 ft by 21 ft 4 in. What is the perimeter of the entire hexagon in feet?
   The area of the rectangle is 256 ft^2. The area of the square is 256 ft^2. The length of the side of the square is 16 ft. The perimeter of the composite figure is $98.\overline{6}$ *ft.*



Success in this activity indicates that participants are working at the Relational Level because they must solve geometric problems by selecting known properties of figures or formulas and deductive reasoning to solve problems.

Applying Area Formulas

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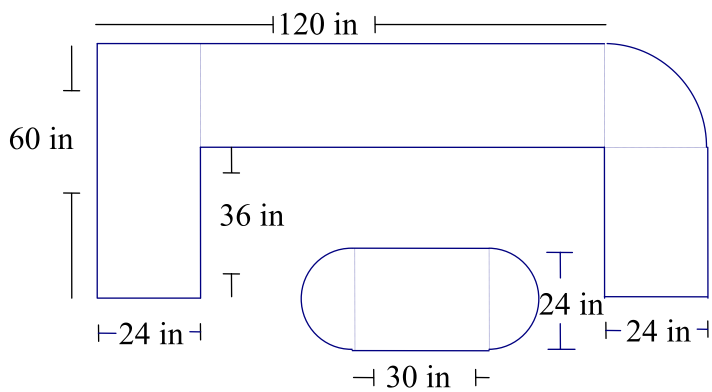
   The counter tops and the island in the kitchen will be covered with granite. Granite costs $14.38 per square foot. Find the cost of the countertops and island for this kitchen? Round the cost to the nearest penny.



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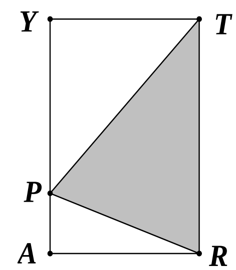

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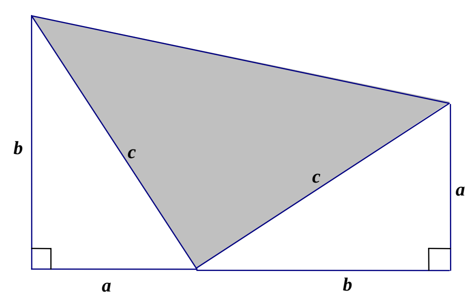
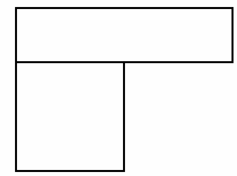


5. The rectangle and the square have equal area. The rectangle is 12 ft by 21 ft 4 in. What is the perimeter of the entire hexagon in feet?



What Is Surface Area?

Overview: Participants determine the surface area of a rectangular prism by counting and then finding the sum of the number of square units needed to cover each face. Group discussion deepens participants' understanding of surface area and connects the formula for surface area to the net of the solid.

Objective: **TExES Mathematics Competencies**
III.013.E. The beginning teacher analyzes cross-sections and nets of three-dimensional shapes.
V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

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b.2.B. The student makes and verifies conjectures about angles, lines, polygons, circles, and three-dimensional figures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.
b.3.D. The student uses inductive reasoning to formulate a conjecture.
d.1.B. The student uses nets to represent and construct three-dimensional objects.
d.2.C. The student develops and uses formulas including distance and midpoint.
e.1.A. The student finds area of regular polygons and composite figures.

Background: No prerequisite knowledge is necessary for this activity.

Materials: centimeter grid paper, linking cubes, scissors, straightedge, tape

New Terms: lateral surface area, net, total surface area

Procedures:

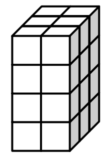
Distribute the activity sheet Intro to Nets to each participant. Tell participants that they have a two-dimensional paper figure that, when cut out and folded, will yield a solid object. Ask participants to cut out the figure, fold it, and create the solid.

Describe the solid.
The solid is a triangular prism. It has two congruent, parallel triangular faces that are the bases of the prism, and each lateral face is a rectangle. By definition, this is a triangular prism.

Distribute the activity sheet, linking cubes, straightedge, and centimeter grid paper to participants. Ask participants to answer the question **"What is surface area?"**

Indicate that they will have an opportunity to share and revise their responses at the conclusion of this activity.

1. Build the solid below using linking cubes.



2. Cut rectangles from centimeter grid paper so that one rectangle will cover each face of the solid.

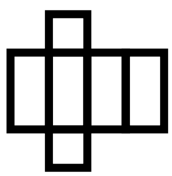
Participants should create and cut out six rectangles that will match the faces of the solid. They should have three pairs of congruent rectangles.

3. Cover the solid with the grid paper rectangles, matching each rectangle with the appropriate face. Tape the rectangles together.

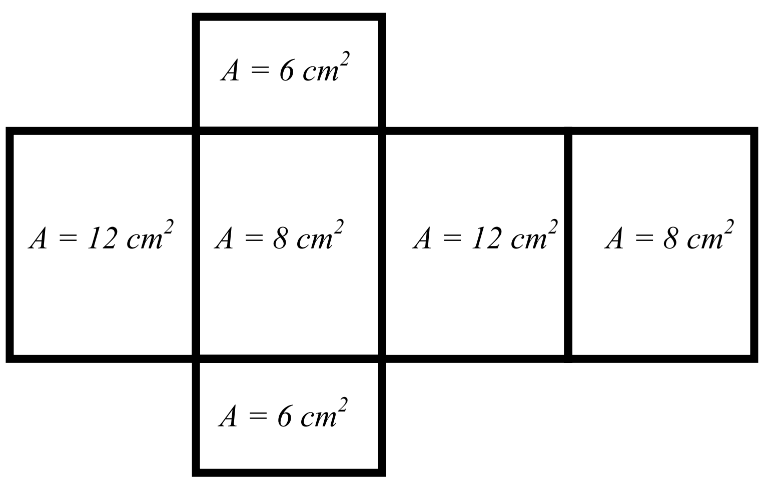
   The rectangles should completely cover the solid without overlapping.

4. Cut through enough tape connections so that the paper will unfold into a two-dimensional figure that if refolded would form the original solid. This pattern is called a net. Sketch the net here.

Participants should cut only enough tape connections to unfold the paper while keeping each rectangle connected to at least one other rectangle. A possible net is shown below. Define net as a two-dimensional pattern that can be folded to form a solid.



5. Determine the number of square centimeters on each face of the net. Net is not drawn to scale.

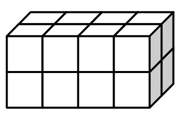
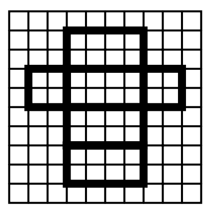
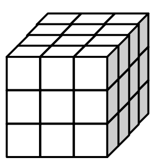
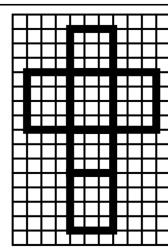
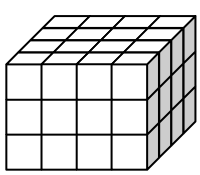
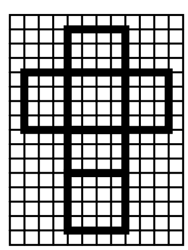


6. Find the sum of the areas of the six faces. This represents the total surface area of the solid.

   Total Surface Area = $6\ cm^2 + 6\ cm^2 + 8\ cm^2 + 8\ cm^2 + 12\ cm^2 + 12\ cm^2$
   Total Surface Area = $52\ cm^2$

   Define *total surface area* (surface area) as the sum of the areas of the faces and curved surfaces of a solid.
   Define *lateral surface area* of a prism as the sum of the areas of the faces excluding the area of the two parallel faces, bases, of the solid used in naming the prism. Refer back to the Intro to Nets activity. The lateral surface area of a triangular prism is the sum of the three parallelograms, rectangular faces, forming the lateral faces of the solid.

7. Complete the table below.

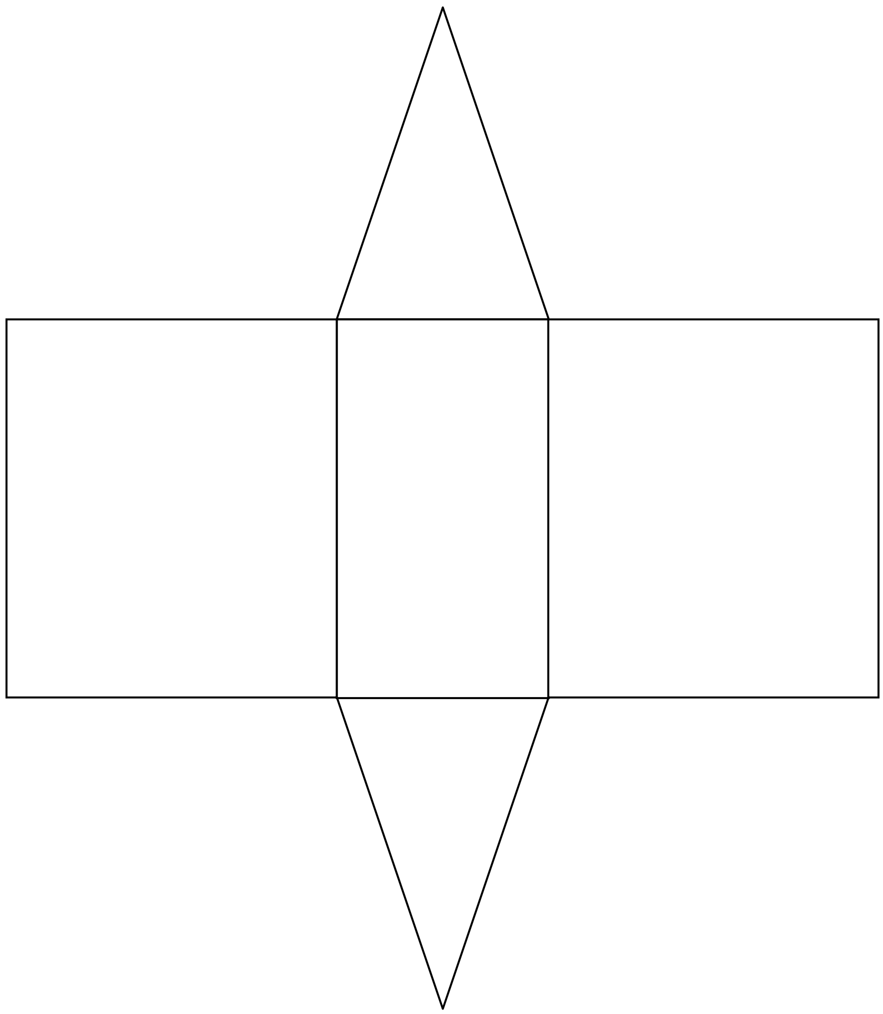
Solid	Net	Total Surface Area
		*40* cm^2
		*66* cm^2
		*80* cm^2

8. Describe a method for finding the total surface area of a solid figure.
   Answers will vary but should include adding the areas of all the surfaces of a solid, noting that rectangular prisms have three sets of congruent pairs of rectangular faces. The congruent pairs lie on parallel planes.

Remind participants to add the terms net, lateral surface area, and total surface area to their glossaries.

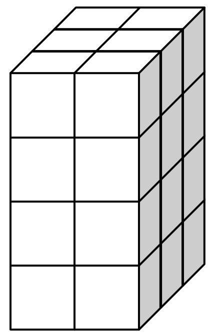
In the introductory activity Intro to Nets participants work at the Visual Level. In the activity What is Surface Area?, participants inductively determine the properties of the solid which relate to its surface area. Success with this activity indicates that participants are working at the Descriptive Level with respect to surface area of rectangular prisms.

Intro to Nets



What Is Surface Area?

1. Build the solid below using linking cubes.



2. Cut rectangles from centimeter grid paper so that one rectangle will cover each face of the solid.

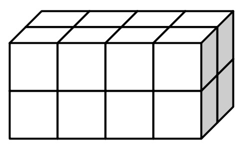
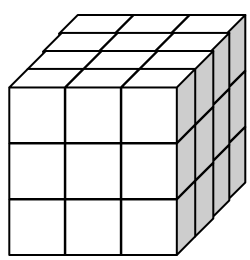
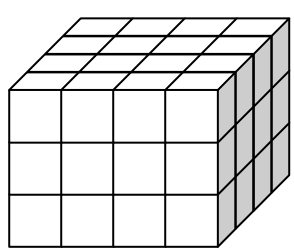
3. Cover the solid with the grid paper rectangles, matching each rectangle with the appropriate face. Tape the rectangles together.

4. Cut through enough tape connections so that the paper will unfold into a two-dimensional figure that if refolded would form the original solid. This pattern is called a net. Sketch the net here.

5. Determine the number of square centimeters on each face of the net.

6. Find the sum of the areas of the six faces. This represents the total surface area of the solid.

7. Complete the table below.

Solid	Net	Total Surface Area
		
		
		

8. Describe a method for finding the total surface area of a solid figure.

What Is Volume?

Overview: Participants determine the volume of a rectangular prism by constructing the solid from a net then counting the number of cubes needed to fill the solid. Group discussion deepens participants' understanding of volume (the amount of space occupied by a solid measured in cubic units) and connects the formula for volume to the net of the solid.

Objective: **TExES Mathematics Competencies**

III.011.B. The beginning teacher applies formulas for perimeter, area, surface area, and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.

III.013.E. The beginning teacher analyzes cross-sections and nets of three-dimensional shapes.

V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

Geometry TEKS

b.2.B. The student makes and verifies conjectures about angles, lines, polygons, circles, and three-dimensional figures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

b.3.D. The student uses inductive reasoning to formulate a conjecture.

d.1.B. The student uses nets to represent and construct three-dimensional objects.

d.2.C. The student develops and uses formulas including distance and midpoint.

e.1.A. The student finds area of regular polygons and composite figures.

Background: No prerequisite knowledge is necessary for this activity.

Materials: centimeter cubes, straightedge, centimeter grid paper, scissors, tape

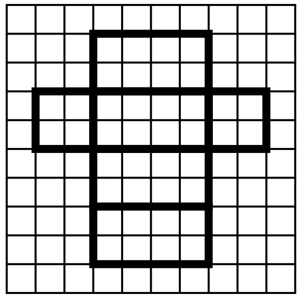
New Terms:

Procedures:

Distribute the activity sheet, centimeter cubes, straightedge, and centimeter grid paper to participants. Ask participants to answer the question **"What is volume?"** Groups should record their responses to this question for future revision at the conclusion of this activity.

What is Volume?

1. Using centimeter grid paper, build the solid (rectangular prism) from the net below.

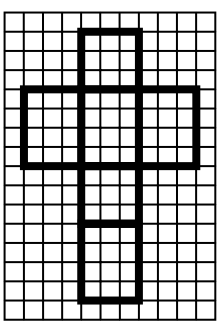
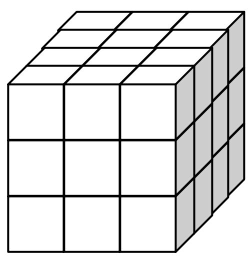
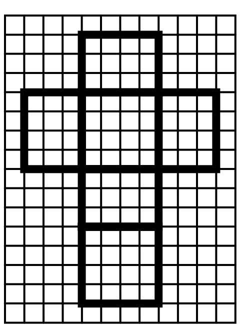
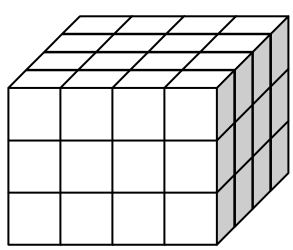


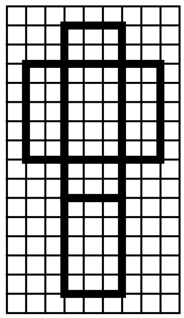
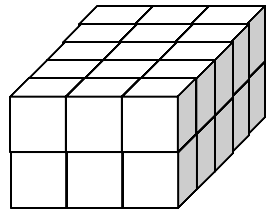
2. Predict how many cubes will fit into the prism.
   Answers may vary.

3. Check your prediction by completely filling the prism with centimeter cubes.
   Participants will find that 16 cubes fill the box.

4. What is the volume of the prism, i.e., what is the total number of cubes necessary to fill the prism?
   The volume of the box is 16 cm^3.

5. Complete the table below.

Net	Solid (Rectangular Prism)	Volume
		*36 cm^3*
		*48 cm^3*

		*30 cm^3*

6. Describe a method to find the number of cubes necessary to fill a rectangular prism without actually filling it.
   Answers may vary. Participants should describe finding the area of the base in square units and multiplying that number by the height of the prism in units, or the formula $V = Bh$.

In groups, participants should review their responses to the question "What is volume?" Close the activity with a whole-group discussion. The following points should be addressed:

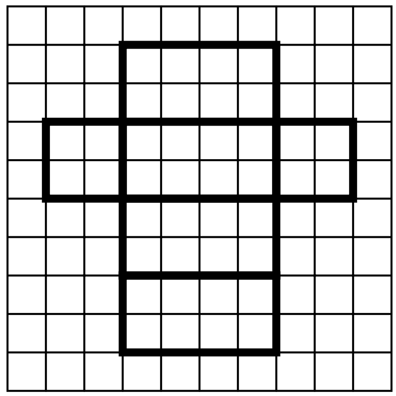
- To find the volume, in cubic units, multiply the number of cubes that completely cover one face of the prism by the number of congruent layers of cubes which completely fill the prism. Or more simply $V = lwh$.

- When the prism is viewed as a net, find three dimensions which make up the rectangular parts of the net. Two of the dimensions are used to find the area of one face; the third dimension determines the third variable in the volume formula.

Participants approach the Relational Level if they are able to describe the concept of volume directly from the net or the 3-dimensional representation. If they need to build the prism to find the volume, they are performing at the Descriptive Level. If they are able to use the formula for volume, $V = lwh$, but are unable to articulate the concept then they are probably at the Visual or Descriptive Level. Use of a formula without conceptual understanding is often misinterpreted to be at the Relational Level. Van Hiele refers to this as "level reduction". Conceptual growth cannot occur without further work at the Descriptive Level.

What is Volume?

1. Using centimeter grid paper, build the solid (rectangular prism) from the net below.

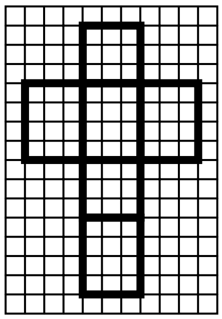
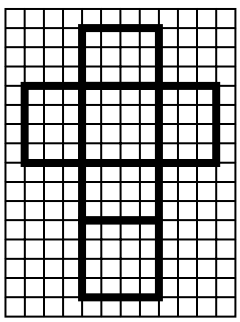
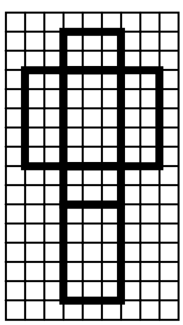


2. Predict how many cubes will fit into the prism.

3. Check your prediction by filling the prism with centimeter cubes.

4. What is the volume of the prism, i.e., what is the total number of cubes necessary to fill the prism?

5. Complete the table below.

Net	**Solid (Rectangular Prism)**	**Volume**
		
		
		

6. Describe a method to find the number of cubes necessary to fill a rectangular prism without actually filling it.

Net Perspective

Overview: Participants construct solids from nets and use nets to explore the effects on the solid's surface area and volume by changing one, two, or three dimensions of a solid.

Objective: **TExES Mathematics Competencies**

II.005.C. The beginning teacher understands when a relation is a function.

II.006.G. The beginning teacher models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

II.004.A. The beginning teacher recognizes and extends patterns and relationships in data presented in tables, sequences, or graphs.

III.011.A. The beginning teacher applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.

III.011.B. The beginning teacher applies formulas for perimeter, area, and surface area, and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.

III.011.C. The beginning teacher recognizes the effects on length, area, or volume when the linear dimensions of plane figures or solids are changed.

III.013.E. The beginning teacher analyzes cross-sections and nets of three-dimensional shapes.

V.018.F. The beginning teacher evaluates how well a mathematical model represents a real-world situation.

V.019.A. The beginning teacher recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).

V.019.D. The beginning teacher communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).

Geometry TEKS

b.3.B. The student constructs and justifies statements about geometric figures and their properties.

d.1.B. The student uses nets to represent and construct three-dimensional objects.

e.1.D. The student finds surface areas and volumes of prisms, pyramids, spheres, cones, and cylinders in problem situations.

f.1. The student describes the effect on perimeter, area, and volume when length, width, or height of a three-dimensional solid is changed and applies this idea in solving problems.

Background: Participants need experience with measurement, nets, surface area, and volume.

Materials: paper, scissors, tape, rulers, centimeter grid paper (optional), centimeter cubes

New Terms:

Procedures:

This activity encourages participants to explore patterns in surface area and volume of rectangular prisms. Often in the middle school classroom, teachers introduce the concepts of volume and surface area of rectangular prisms by using linking cubes and asking the students to count the number of cubes to determine the volume and to count the number of faces to determine the surface area. Participants should be aware that students might develop the misconception of counting the corner cubes twice in order to find the volume of a prism.

Construct rectangular prisms with the following dimensions using centimeter cubes. Complete the table.

Height cm	**Dimensions** $l \times w \times h$	**Process**	**Volume** cm^3
1	$3 \times 2 \times 1$	*3(2)*	*6*
2	$3 \times 2 \times 2$	*3(2) + 3(2)*	*12*
3	$3 \times 2 \times 3$	*3(2) + 3(2) + 3(2)*	*18*
4	$3 \times 2 \times 4$	*3(2) + 3(2) + 3(2) + 3(2)*	*24*

1. When the dimensions of the base, the length, l, and the width, w, are fixed, how does the volume change as the height changes?
   The volume increases as the height increases.

2. What function rule, in terms of the area of the base and the height, could you use to determine the volume? How does this rule relate to the model?
   $V = Bh$*, where B represents the area of the base, which in this case is* $3 \cdot 2 = 6$ *and h represents the height of the prism. Each time the height increases by one unit, another 6 blocks (the area of the base) are added to the solid. Thus, a prism is a stack of layers whose volume is equal to the area of the base times the height of the prism.*

Construct rectangular prisms with the following dimensions using centimeter cubes. Complete the table.

Height cm	**Dimensions** $l \times w \times h$	**Process**	**Lateral Surface Area cm²**
1	$3 \times 2 \times 1$	*(3+2+3+2) = 2(3)(1) + 2(2)(1)*	*10*
2	$3 \times 2 \times 2$	*(6+4+6+4) = 2(3)(2) + 2(2)(2)*	*20*
3	$3 \times 2 \times 3$	*(9+6+9+6) = 2(3)(3) + 2(2)(3)*	*30*
4	$3 \times 2 \times 4$	*(12+8+12+8) = 2(3)(4) + 2(2)(4)*	*40*
n	$3 \times 2 \times n$	*(3n+2n+3n+2n) = 2(3)(n) + 2(2)(n)*	$(2l + 2w)h = Ph$

3. When the dimensions of the base, the length, *l*, and the width, *w*, are fixed, how does the lateral surface area change as the height changes?
   The lateral surface area increases as the height increases; every increase in height of one unit results in an increase in the lateral area of 10 square units. In this case, every increase in height of one centimeter results in an increase in the lateral area of 10 cm².

4. What function rule, in terms of the base and the height, could you use to determine the lateral surface area? How does this rule relate to your model?
   $Lateral\ \ Surface\ \ Area = Ph$*, where P is the perimeter of the base and h is the height of the prism. The lateral faces of a prism are always parallelograms (rectangles if it is a right prism) whose areas can be found by multiplying the length of the base of the parallelogram by the height of the parallelogram. The perimeter of the base of the prism is the sum of the lengths of the bases of these parallelograms.*

Construct rectangular prisms with the following dimensions using centimeter cubes. Complete the table.

Height cm	**Dimensions** $l \times w \times h$	**Process**	**Total Surface Area cm²**
1	$3 \times 2 \times 1$	*10 + 6 + 6*	*22*
2	$3 \times 2 \times 2$	*20 + 6 + 6*	*32*
3	$3 \times 2 \times 3$	*30 + 6 + 6*	*42*
4	$3 \times 2 \times 4$	*40 + 6 + 6*	*52*
n	$3 \times 2 \times n$	*10n + 6 + 6*	$Ph + 2(lw) = Ph + 2B$

5. When the dimensions of the base, length, *l*, and width, *w*, are fixed, how does the total surface area change as the height changes?
   The total surface area increases as the height increases; every increase in height of one unit results in an increase in the total surface area of 10 square units. In this

case, every increase in height of one centimeter results in an increase in the total surface area of 10 cm^2.

6. What function rule, in terms of the base and the height, could you use to determine the total surface area? How does this rule relate to your model?
   $Total\ Surface\ Area = Ph + 2B$. *The total surface area is the sum of the lateral surface area and the areas of both bases of the prism.*

Distribute the activity sheet Net Perspective to each participant.

Participants are working at the Relational Level as they formulate relationships among total surface area, length, width, and height of geometric solids and analyze the effects on the solid's surface area and volume when one, two, or three dimensions of a solid are changed.

Net Perspective

Construct rectangular prisms with the following dimensions using centimeter cubes. Complete the table.

Height cm	**Dimensions** $l \times w \times h$	**Process**	**Volume** $\mathbf{cm^3}$
1	$3 \times 2 \times 1$		
2	$3 \times 2 \times 2$		
3	$3 \times 2 \times 3$		
4	$3 \times 2 \times 4$		

1. When the dimensions of the base, the length, l, and the width, w, are fixed, how does the volume change as the height changes?

2. What function rule, in terms of the area of the base and the height, could you use to determine the volume? How does this rule relate to the model?

Construct rectangular prisms with the following dimensions using centimeter cubes. Complete the table.

Height cm	**Dimensions** $l \times w \times h$	**Process**	**Lateral Surface Area cm^2**
1	$3 \times 2 \times 1$		
2	$3 \times 2 \times 2$		
3	$3 \times 2 \times 3$		
4	$3 \times 2 \times 4$		

3. When the dimensions of the base, the length, l, and the width, w, are fixed, how does the lateral surface area change as the height changes?

4. What function rule, in terms of the base and the height, could you use to determine the lateral surface area? How does this rule relate to your model?

Construct rectangular prisms with the following dimensions using centimeter cubes. Complete the table.

Height **cm**	**Dimensions** $l \times w \times h$	**Process**	**Total Surface Area** $\mathbf{cm^2}$
1	$3 \times 2 \times 1$		
2	$3 \times 2 \times 2$		
3	$3 \times 2 \times 3$		
4	$3 \times 2 \times 4$		

5. When the dimensions of the base, the length, l, and the width, w, are fixed, how does the total surface area change as the height changes?

6. What function rule, in terms of the base and the height, could you use to determine the total surface area? How does this rule relate to your model?

Area Proofs

Overview: Participants use area formulas and deductive reasoning in area problems.

Objective: **TExES Mathematics Competencies**
III.011.B. The beginning teacher applies formulas for perimeter, area, surface area, and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.
III.012.A. The beginning teacher understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
III.013.D. The beginning teacher computes the perimeter, area, and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).
V.018.A. The beginning teacher understands the nature of proof, including indirect proof, in mathematics.
V.018.B. The beginning teacher applies correct mathematical reasoning to derive valid conclusions from a set of premises.
V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

Geometry TEKS
b.1.A. The student develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems.
b.3.B. The student constructs and justifies statements about geometric figures and their properties.
b.3.E. The student uses deductive reasoning to prove a statement.
d.2.C. The student develops and uses formulas including distance and midpoint.
e.1.A. The student finds areas of regular polygons and composite figures.

Background: Participants need to be familiar with the formulas for the area of rectangles, triangles, parallelograms, trapezoids, and kites.

Materials: colored markers, easel paper

New Terms:

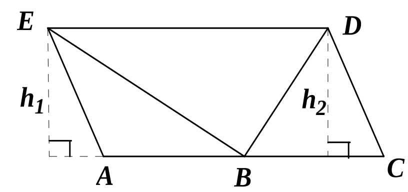
Procedures:

Allow time for participants to work independently on proofs. Then assign one proof to each group to be recorded on easel paper. As time permits, participants can walk the room, observing similarities and differences in the proofs they have done independently and the proofs on display. Proofs can be done informally in paragraph form.

1. Given: B is the midpoint of $\overline{AC}$

   $\overline{ED} \parallel \overline{AC}$

   Prove: Area of ΔABE = Area of ΔBCD

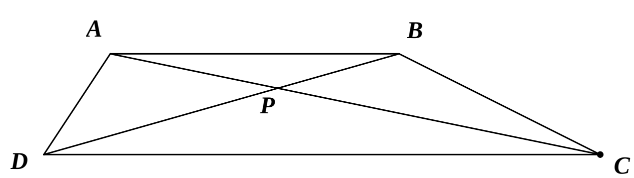


Draw h_1, the height of ΔEAB, and h_2, the height of ΔDBC.

$h_1 = h_2$	($\overline{ED} \parallel \overline{AC}$)
Area of $\Delta ABE = \frac{1}{2} h_1 \cdot AB$ *Area of* $\Delta BCD = \frac{1}{2} h_2 \cdot BC$	(*Area of triangle* $= \frac{1}{2}$ *base* $\cdot$ *height*)
$AB = BC$	(*B is the midpoint of* $\overline{AC}$)
$\frac{1}{2} h_1 \cdot AB = \frac{1}{2} h_2 \cdot BC$	(*Substitution*)
Area of ΔABE = *Area of* ΔBCD	(*Substitution*)

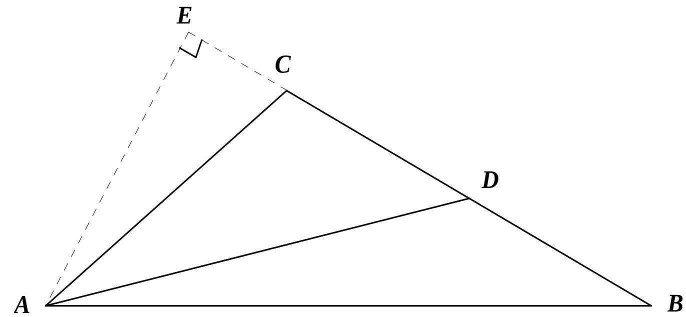
2. Given: Trapezoid $ABCD$ with diagonals intersecting at P

   Prove: Area of ΔAPD = Area of ΔBPC



Area of ΔADC = *Area of* ΔBCD	(ΔADC *and* ΔBCD *have the same base and height*)
Area of ΔADC – *Area of* ΔDPC = *Area of* ΔBCD – *Area of* ΔDPC	(*Subtraction*)
Area ΔAPD = *Area* ΔBPC.	(*Substitution of composite parts*)

3. Given: $\overline{AD}$ is a median of ΔABC
   Prove: Area ΔCAD = Area ΔBAD



Draw $\overline{AE}$, the altitude to base $\overline{CB}$ of Δ ABC.

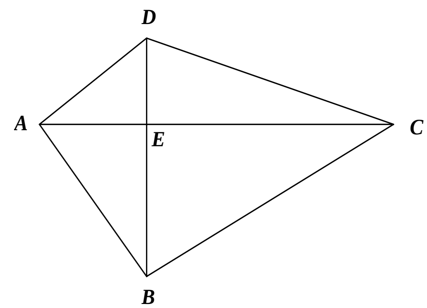
Area of $\Delta CAD = \frac{1}{2} AE \cdot CD$ (*Area formula*)

Area of $\Delta BAD = \frac{1}{2} AE \cdot DB$ (*Area formula*)

$CD = DB$ (*Definition of median*)

Area *of* ΔCAD = *Area of* ΔBAD (*Substitution*)

4. Given: Diagonals $\overline{DB}$ and $\overline{AC}$ of quadrilateral *ABCD* are perpendicular.
   Prove: Area of $ABCD = \frac{1}{2}\ DB \cdot AC$



Area of quadrilateral ABCD = (*Composite parts*)
Area of ΔADC + *Area of* ΔABC

Area of $\Delta ADC = \frac{1}{2} DE \cdot AC$

Area of $\Delta ABC = \frac{1}{2} BE \cdot AC$ (*Area formula*)

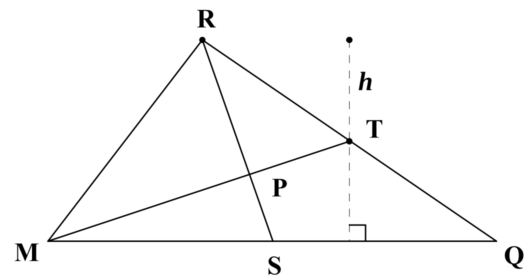
Area of quadrilateral ABCD = (*Substitution*)
$\frac{1}{2} DE \cdot AC + \frac{1}{2} BE \cdot AC$

$\frac{1}{2} DE \cdot AC + \frac{1}{2} BE \cdot AC = \frac{1}{2} \cdot AC\,(DE + BE)$ (*Distributive property*)

$\frac{1}{2} \cdot AC\,(DE + BE) = \frac{1}{2}\ AC \cdot DB$ (*Substitution of composite parts*)

5. Given: ΔMQR with medians $\overline{RS}$ and $\overline{MT}$ intersecting at P

   Prove: Area ΔPMS = Area ΔPRT



Area of $\Delta TMQ = \frac{1}{2}\left(MQ \cdot \frac{1}{2}h\right)$	(*Area formula*)
Area of $\Delta SRQ = \frac{1}{2}\left(\frac{1}{2} \cdot MQ\right)h$	(*Area formula*)
Area of ΔTMQ = *Area of* ΔSRQ	(*Substitution*)
Area of ΔTMQ – *Area of quadrilateral TPSQ* = *Area of* ΔSRQ – *Area of quadrilateral TPSQ*	(*Subtraction*)
Area of ΔPMS = *Area of* ΔPRT	(*Substitution of composite parts*)

Success in this activity indicates that participants are working at or approaching the Deductive Level. Whereas at the Relational Level deductive argument is informal, proof at the Deductive Level requires the use of an axiomatic system with formal definitions and postulates.

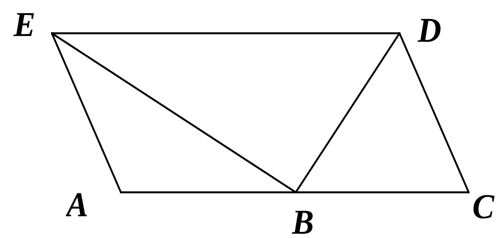
Area Proofs

Use area formulas and the properties of triangles and quadrilaterals to prove each of the following area relationships. Use an informal presentation, such as a flow chart proof or paragraph proof, to demonstrate your deductive reasoning.

1. Given: B is the midpoint of $\overline{AC}$

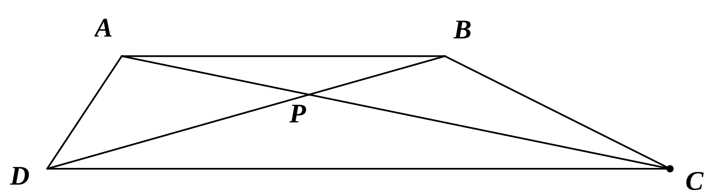
$\overline{ED} \parallel \overline{AC}$

Prove: Area of ΔABE = Area of ΔBCD

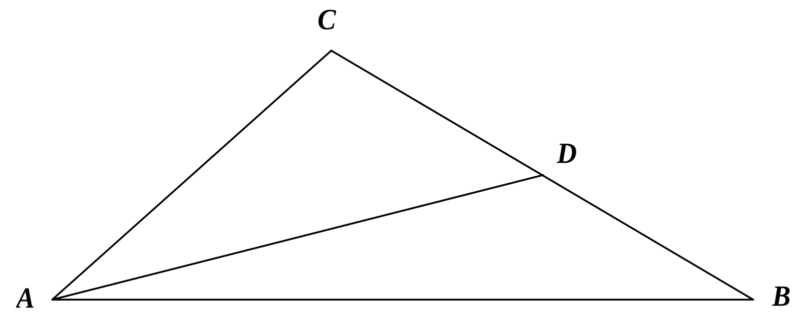


2. Given: Trapezoid $ABCD$ with diagonals intersecting at P

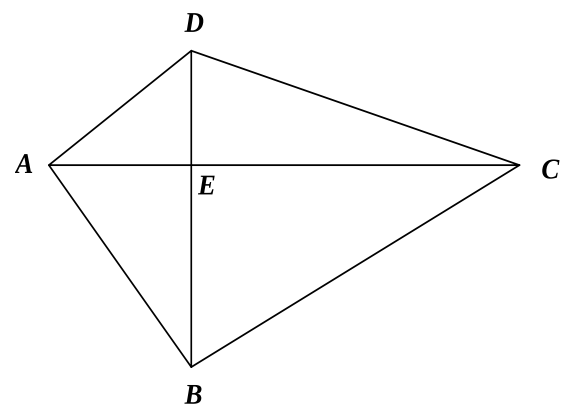
Prove: Area of ΔAPD = Area of ΔBPC



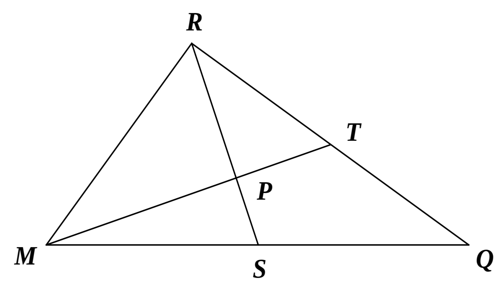
3. Given: $\overline{AD}$ is a median of ΔABC
   Prove: Area of ΔCAD = Area of ΔBAD



4. Given: Diagonals $\overline{DB}$ and $\overline{AC}$ of quadrilateral $ABCD$ are perpendicular.
   Prove: Area of $ABCD = \frac{1}{2}\overline{DB} \cdot \overline{AC}$



5. Given: ΔMQR with medians $\overline{RS}$ and $\overline{MT}$ intersecting at P
   Prove: Area of ΔPMS = Area of ΔPRT



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