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Dynamic inconsistency and choice

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Abstract In this paper, we analyze an intra-personal game where a decision-maker is summarized by a succession of selves. Selves may (or may not) have conflicting interests, and earlier selves may have imperfect knowledge of the preferences of future selves. At date 1, self-1 chooses a menu, at date 2, the preferences of self-2 realize and self-2 chooses an item from the menu. We show that equilibrium choice is consistent with either a preference for flexibility, a preference for betweenness or a preference for systematic restriction. Overall, the analysis reconciles the decision–theoretic approach of choice over time with the game–theoretic multiple-selves approach.

Keywords Time-inconsistency · Multiple selves · Preference for flexibility · Set-betweenness · Preference reversals

1 Motivation

From a general perspective, it is natural to describe an individual as an entity with multiple intra-personal conflicts both within and between periods of time. Various literatures point us to that direction. Psychologists and behavioral economists emphasize that behavior (or choice) exhibits contradictions that cannot be reconciled with a no-conflict benchmark. Also, neuroscientists demonstrate that brain processes lead-

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ing to choices are complex and sometimes in competition. This evidence of course poses an important methodological problem for any discipline that intends to build parsimonious models of decision-making. The objective of such a model is to summarize the intricacies of decision-making into a simple version capable of generating observed behavior. In fact, it might not matter to understand how the brain itself works in detail to capture behavior. Said differently, processes that lead to choice might well be summarized into an "appropriate small" set of abstract assumptions (or axioms) for decision-making. There are several coexisting views about the methodology we should adopt to represent choice and preferences. At this stage of our understanding of choice and brain processes, there is no unambiguous answer.

One view supports the methodology used in decision theory. It consists in representing the individual as one entity capable of making coherent choices and, it focuses on general properties of choice putting aside the underlying mechanisms leading to it. The objective is to provide an axiomatization of choices as well as utility representations. A different view is to rely on evidence from psychology, and more recently the brain sciences, to determine the various conflicting mechanisms present at the time of a decision. According to this view, the one-entity representation is a big stretch, and likely the origin of the discrepancies between predicted and observed choices. While the advocates of the first view believe that understanding how the brain works does not matter for (economic) choice, the advocates of the second believe it does.

The purpose of this analysis is not to add controversy, but rather to bring the two main philosophies together. We want to show that a specific extension of the basic model of decision-making might be relevant to explain and predict choice. To be more precise, we consider the following thought experiment. In the first period, a decision maker, hereafter DM, chooses a menu consisting of a list of goods (or bundles). In the second period, he chooses an element from the menu. The problem we address is therefore similar to the one analyzed in Kreps (1979), hereafter [K], and in Gul and Pesendorfer (2001), hereafter [GP] among others. This literature is decision–theoretic and provides axioms consistent with different properties of choices over menus that might occur in that experiment. Interestingly, behavior satisfying these properties cannot emerge if the individual is one entity with rational preferences over the set of final consumptions. To be able to focus on representations in which the individual is still one entity, the authors revisit and extend the concept of preferences, and in particular *the set over which they are defined*. Those articles will be reviewed later. In this paper, we revisit the integrity of the decision maker instead. More precisely, we model the decision maker as a succession of selves with possibly conflicting goals. We also allow for the possibility that the individual is not fully aware of his future goals. Still, we restrict the attention to a minimal type of intra-personal conflict: at each period, a self is in command and makes the decisions of that period given his preferences over the set of final consumptions. Furthermore, we assume that the conflict is resolved via a non-cooperative play between selves. In other words, each self is acting so as to maximize his own objective and anticipates other selves do the same. Overall, the only departure with respect to the standard rational model is the presence of different temporal perspectives. No extra bounded rationality is imposed on the model. It is then possible to pinpoint the exact effect on behavior of our alternative modelling.

The two assumptions of our theory—namely that the individual can be seen as a succession of sub-entities and that those sub-entities play non-cooperatively—can be defended on neuroscientific grounds. However, this *is not* the motivation of this analysis. The premise is simply that observed behavior reveals the existence of internal conflicts between objectives. A natural way of modelling such conflicts is to represent the individual as a collection of sub-entities, each pursuing one objective.[1] We want to understand what can be gained from such an enterprise.

Our study makes two methodological contributions. First, by modelling the conflict of preferences explicitly, it is possible to generate preferences over menus consistent with a *preference for flexibility* or a *preference for betweenness*. Those properties of behavior arise at equilibrium of a non-cooperative play between selves. In other words, we can relate our analysis to both [GP] and [K] and provide a unified approach of decision-making within the standard experiment considered in the decision–theoretic literature. Second, the model is rich enough to generate a new type of preference over menus. We call it a *strict preference of restriction* and it occurs when DM is averse to combinations of menus.

The model goes beyond these methodological contributions as it also allows us to investigate the conditions under which each of these properties emerges, a question central here but not in the earlier literature. We show that uncertainty about future preferences is a necessary condition to obtain a preference for flexibility, or a strict preference for restriction. We also show that each particular preference over menus emerge if a particular type of preference reversal between date 1 and date 2 prevails. Allowing self-2 to choose from a larger set results in profitable choices when the preferences of both selves are aligned. This gives rise to a preference for flexibility. By contrast, it results in detrimental choices when the preferences of selves are opposed, which generates a preference for restriction. In the intermediate situation where the conflict is typically mild, a preference for betweenness occurs. Finally, we show that temptation and self-control is just one possible interpretation for the emergence of a preference for betweenness. In particular, the notion of costly self-control does not need to be part of the interpretation. Also, the preference can emerge in settings where neither temptation nor self-control issues are relevant.

Our paper is not the first to depart from the traditional decision–theoretic format to address the same problem as [K] and [GP]. First and foremost, the papers reviewed below are interested in temptation and self-control. They all offer a complementary (sometimes in competition) view on [GP]. Instead, one objective of the current paper is precisely to provide results that are relevant for other classes of situations as well. From a methodological viewpoint, our approach is also different. For instance, Bénabou and Pycia (2002) show that the representation obtained in [GP] emerges at the Nash equilibrium of a particular conflict between a planner and a doer. Here instead, we study the overall mapping between a class of conflicts and the possible preferences over menus. Also, Chatterjee and Krishna (2008) consider a situation where an alter ego sometimes

[1] Of course, this modelling raises the question of which welfare criterion should be used for the individual. We will not address this issue. It should nevertheless be clear that, if the individual is best represented by a collection of sub-entities (that is, if choice is best predicted under that modelling), then this representation should be adopted and the issue of welfare should be addressed eventually.

takes control of consumption decisions. Therefore, the correct choice is made (according to a normative ordering) only with some probability. Assuming DM maximizes the corresponding expected utility, the authors determine axioms for preferences over menus that rationalize this utility.[2] In this paper, we focus on the relationship between the conflict and the properties of choices over menus, rather than utility representations of those properties. Last, Nehring (2006) proposes a model where DM chooses a menu ex-ante as well as a choice disposition ex-interim. The choice disposition is then used to pick an item ex-post. The choice of the disposition is based on a second-order preference over the pair constituted of the disposition and the choice it induces. Therefore, there is an implicit conflict of preferences but the key of the analysis is that DM can take an action to minimize it. In our case, DM can only choose a menu and an item in the menu, and he does not get to affect his dispositions.

The paper is organized as follows. Section 2 presents the model. Section 3 discusses the methodology and reviews the related literature. Section 4 characterizes the properties satisfied by the induced preference over menus under certainty and uncertainty about future motivations. In Sect. 5, we identify the conflicts that are necessary to observe each given property. Section 6 concludes.

2 A simple model of intra-personal conflicts

2.1 The model

The problem we analyze is the following. The finite set of final consumptions is denoted by X, and M is the set of all non empty subsets of X. Typical elements of X (resp. M) are denoted by x (resp. m). At date $t = 1$ (ex-ante), a decision maker (DM, he) chooses an element m in M, and at date $t = 2$ (ex-post), he chooses $x \in m$. We want to think of X as a set of items or bundles that are consumed at date $t = 2$. Then, M is simply the set of all possible menus of those items. So far, the experiment is the same as in [K] and [GP]. We assume also that an event affecting ex-post choices might occur between dates 1 and 2 (ex-interim). We will provide a more specific description of this event below.

The decision-maker. DM is a collection of self-1, who is required to choose $m \in M$, and self-2, who makes the final choice $x \in m$.

Ex-interim event. There exists a finite set of states $S = \{a, b, \dots, \overline{s}\}$. An element of S is denoted by s and realizes with probability p_s. The true state is unknown ex-ante and is chosen by nature before the final choice is made. Therefore self-2 knows s when making a decision. The state affects the preferences of self-2, but not the available alternatives.

Orderings and preferences over X. Each self-i in $\{1, 2\}$ can rank *unambiguously* all the alternatives in X. Let us denote by $\succeq_1$ and $\succeq_{2s}$ the complete and transitive preference relations that summarize the preferences of self-1 and self-2 in state s, respectively. For all $k \in \{1, 2a, 2b, \dots, 2\overline{s}\}$, the preference relation $\succeq_k$ can be rep-

[2] In fact, it corresponds to a particular intra-personal conflict between periods where the conflict happens only with a probability. We show here that this particular conflict implies set-betweenness.

resented by a utility function $u_k : X \to R$. We may refer to the difference between utilities ex-ante and ex-post as a preference reversal.[3]

Orderings and preferences over lotteries. Consider the set Q of probability measures q on X and $\succ_q$ the preference relation of self-1 on Q. We assume that the two following axioms are satisfied:

Independence axiom: $q \succ_q q'$ implies that $\mu q + (1-\mu)q'' \succ_q \mu q' + (1-\mu)q''$ for all $q, q', q'' \in Q$ and $\mu \in (0, 1]$.

Archimedean axiom : if $q \succ_q q' \succ_q q''$, then there exist $\mu, \nu \in (0, 1)$ such that $\mu q + (1-\mu)q'' \succ_q q' \succ_q \nu q + (1-\nu)q''$ for all $q, q', q'' \in Q$.

Therefore, and as is well known, $\succ_q$ can be represented by expected utility. Besides, given the representation is relevant at date 1, that is when self-1 is in charge of the decision, the ordering over final consumption corresponds to $\succ_1$.

The intra-personal game. At date 2, self-2 is left with the subset of alternatives m and chooses the element

$$x^s(m) = \arg\max_{x \in m} u_{2s}(x)$$

if state s realized interim. At date 1, self-1 anticipates and evaluates this choice. Formally, choosing m is equivalent to choosing the lottery $l(m)$ where $x^s(m)$ is obtained with probability p_s. Preferences over menus are characterized in our first result.

Lemma 1 *There exists an induced preference $\succeq_m$ over menus $m \in M$ that can be represented by the utility function $V : X \to R$ where*

$$\forall m \qquad V(m) = E_s u_1(x^s(m)) \quad \text{with} \quad x^s(m) = \arg\max_{x \in m} u_{2s}(x).$$

The dynamic choice problem boils down to a sequential game between selves. The multiple-selves model captures the existence of an intra-personal conflict between periods. Objectives at date 1 are different from objectives at date 2. When making a decision, self-1 anticipates what self-2 will end up choosing and he evaluates this choice with his own preferences.

In what follows, we consider situations with and without ex-interim event. In this last case, it just means that there is only one possible state at date 2. To economize on notations, we will denote the utility at date 2 simply by u_2 and the optimal choice at date 1 by $x^*(m)$.

Let us emphasize the fact that in our setting, each self is rational vis-a-vis all the elements relevant for the decision: self-1 anticipates that self-2 has a different objective and will act accordingly. In particular, the model of preferences cannot be interpreted as a model where measures of utility differ over time in a non-anticipated way. Also, as already noted in the introduction, we make a precise assumption regarding the relationship between selves: they play a *non-cooperative* game. This assumption captures our interest for conflicts. We want to understand how choice is generated when the conflict exists and parties optimize (rationally) upon it.

[3] Uncertainty is also a feature in Chatterjee and Krishna (2008). Their setting is comparable to this analysis in the particular case $S = \{a, b\}$, $\succeq_{2a} = \succeq_1$ and $\succeq_{2b} \neq \succeq_1$.

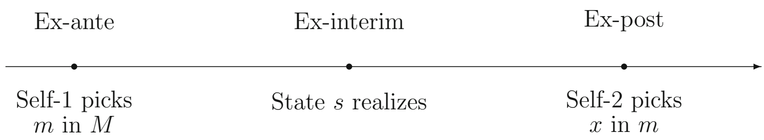

Fig. 1 Timing

The model and timing are summarized in Fig. 1.

2.2 The nature of conflicts

Our analysis focuses on a minimal intra-personal conflict. It corresponds to two distinct, yet formally equivalent, possible situations.

- ☐ *Pure intra-personal conflicts between periods*. The first possible situation is such that DM does not have any intra-personal conflict within periods. In other words, DM can be seen as one entity at each point in time, but his objectives change over time. This corresponds to the interpretation generally given to hyperbolic discounting models.[4]
- ☐ *Intra-personal conflicts within periods and delegation*. DM exhibits intra-personal conflicts within periods but the decision is delegated in such a way that the entity in charge at each period does not need to agree or compromise with any other entity. This corresponds to a particular case of a dual-self model.[5] For instance, self-2 is a short-term self who is interested in immediate satisfaction (also called doer, or agent in the literature), while self-1 is a long-term self who cares also about long-term effects (also called planner or principal). Our model applies generally to any situation in which the long-term self acts only ex-ante to restrict the choice set and the short-term self takes the ultimate decision conditional on how pleasurable options are from his own perspective.

These two possibilities are illustrated in the next two figures (Fig. 2).

In the second case, our model does not preclude the existence of a short-term self at date 1 who disagrees with the decision over menus; or the existence of a long-term self at date 2 who disagrees with the ultimate choice. Overall, DM might “feel” the choice is not right at any point in time but, given only one entity is in control of the decision, DM ends up making the choice supported by that entity.

[4] See the seminal article by Strotz (1955) as well as Laibson (1997). These models make also precise assumptions on preference orderings and choice sets. In particular, to address this case properly we would need to consider experiments where consumption takes place at least in two periods.

[5] See Thaler and Shefrin (1981), Shefrin and Thaler (1988), Bénabou and Pycia (2002), Bernheim and Rangel (2004), Fudenberg and Levine (2006) and Brocas and Carrillo (2008a,b) for examples of analyses where DM is modelled as a dual entity in each possible period of time.

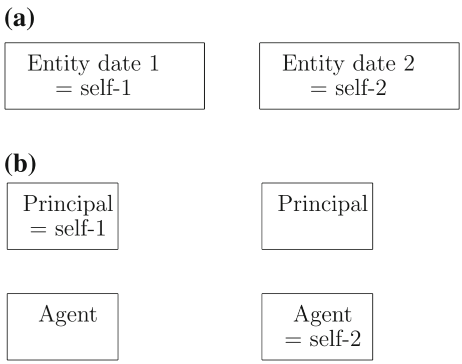

Fig. 2 **a** DM with pure intra-personal conflicts between periods. **b** DM with intra-personal conflicts within periods and delegation

3 Properties of preferences over menus

The objective of the analysis is to study the induced preference over menus when the conflict is modelled explicitly. In particular, if we observe that $m \succeq_m m'$, we want to determine how DM ranks $m \cup m'$. Given our game–theoretic approach, the properties of $\succeq_m$ emerge at equilibrium of the intra-personal game. The corresponding representation $V(\cdot)$ is an *equilibrium*-induced utility representation. Let us emphasize that, preferences over menus are not the starting point of the analysis as in the standard decision-theoretic approach, they are instead the result. To clarify our methodology, we first review the decision–theoretic approach.

3.1 Conflicts within the one-entity paradigm

Traditional decision theory relies on the one-entity paradigm: the individual is not only one entity at each point in time, but also one entity across periods of time.

□ *Typical consistent behavior*. Suppose preferences are defined over X and DM is one entity with an unambiguous ordering over that set. Then, for each possible menu m, there exists one preferred element, and obviously,

$$m \succeq_m m' \Rightarrow m \sim_m m \cup m' \qquad (\mathcal{A})$$

property $(\mathcal{A})$ means that if DM prefers menu m to menu m', then he is indifferent between having to pick an item from m or from $m \cup m'$. [6]

However, casual evidence indicates this property of behavior is sometimes violated. Two main categories of situations have been analyzed in the literature.

[6] Conversely, if behavior satisfies $(\mathcal{A})$, then DM's preferences can be summarized by a complete and transitive relation over X. Overall, a preference over items can be represented by a utility function over items in X if and only if preferences over menus in M satisfy $(\mathcal{A})$.

□ *Preference for flexibility* [K]. Orderings over menus reflect a preference for flexibility. Consider two menus n and n', the ordering is such that:[7]

$$n \subseteq n' \Rightarrow n' \succeq_m n$$

This axiom (also called monotonicity) means that DM prefers to have more choices. [K] shows that this type of preferences can be rationalized by a set of states Σ and a state-dependent utility function $v(x, \sigma)$ such that $W(n) = \sum_\sigma \max_{x \in n} v(\sigma, x)$. DM is one entity with preferences over the pairs (x, σ) and an induced preference over menus represented by $W(\cdot)$. Preferences at date 2 are summarized by an ordering on $X \times \Sigma$. Preferences at date 1 over such elements are summarized by the same ordering.[8] In what follows, we will use a restated version of the above axiom. Let $m \equiv n$ and $n' = m \cup m'$, the axiom above implies that $m \cup m' \succeq_m m$. Similarly, if $n = m'$, we have $m \cup m' \succeq_m m'$. Overall, we can restate the axiom as:

$$m \succeq_m m' \Rightarrow m \cup m' \succeq_m m \succeq_m m' \qquad (\mathcal{A}^{\mathrm{F}})$$

□ *Temptation and self-control* [GP]. Choices over menus satisfy set-betweenness.

$$m \succeq_m m' \Rightarrow m \succeq_m m \cup m' \succeq_m m' \qquad (\mathcal{A}^{\mathrm{SB}})$$

Temptation refers to the fact that m is preferred to $m \cup m'$: some element in m' is referred to as "tempting". DM exerts self-control when he refrains to choose such element when $m \cup m'$ is offered. However, the overall utility derived by consuming a "non-tempting element" is not the same when there is a tempting element in the choice set. [GP] show that choices over menus satisfy $(\mathcal{A}^{\mathrm{SB}})$ if and only if there exist $v_1(\cdot)$ and $v_2(\cdot)$ such that $W(m) = \max_{x \in m} v_1(x) + v_2(x) - \max_{y \in m} v_2(y)$. In that situation, $v_2(\cdot)$ is interpreted as a ranking under "temptation" and $v_1(\cdot)$ a ranking under "commitment". When evaluating a menu, DM maximizes his commitment utility $v_1(x)$ minus the cost of commitment $\max_{y \in m} v_2(y) - v_2(x)$. In that case, DM is one entity with preferences over elements (x, m) that can be represented by a utility function of the form $W^*(m, x) = v_1(x) + v_2(x) - \max_{y \in m} v_2(y)$.[9] Preferences at date 2 are summarized by an ordering on $X \times M$. Preferences at date 1 over such elements are summarized by the same ordering. The fact that preferences are menu-dependent

[7] To simplify the exposition, we do not emphasize the second axiom necessary to get the representation:

$$n \subseteq n' \text{ and } n \sim_m n' \Rightarrow n \cup n'' \sim_m n' \cup n'' \; \forall n''$$

It says that if more choices are of no value, then adding more options does not make those choices more valuable.

[8] This model has been reinterpreted as a model of unforeseen contingencies. See also Dekel et al. (2001) for an extension of K in which DM is allowed to have a preference for commitment in some contingencies.

[9] Additional axioms on preferences over (x, m) must be added to get the representation, which we skip for the sake of brevity.

accommodate for the notion of self-control: if the individual ends up choosing x in m and $m \cup m'$, the cost of self-control differs in both cases.

We will take one main lesson from these studies.

Lemma 2 *In the one-entity approach, choices over menus satisfy* (i) ($\mathcal{A}^F$) *if preferences over items are state-dependent;* (ii) ($\mathcal{A}^{\text{SB}}$) *if preferences over items are menu-dependent.*

Therefore, the ranking over alternatives in X is necessarily ambiguous. By ambiguous, we mean that it is not the same for DM to consume the same item in different situations. This is how the model captures the underlying conflict of objectives. However, to keep dynamic consistency, it is necessary to have an unambiguous ranking over an extended choice set. In the case of [GP], the individual at date 1 and the individual at date 2 agree on the best choice to make if menu m is offered. In the case of [K], both agree on the best choice to make if state σ realizes. As a result, preferences are *ambiguous* on X but *unambiguous* over the extended choice sets $X \times M$ and $X \times \Sigma$ respectively. By extending the choice set, it is possible to define a ranking on which both perspectives (at date 1 and at date 2) agree. By construction, this departs from our setting because we impose preferences are unambiguous on X for each self. Given the conflict, it is not possible to get them to agree unless the conflict vanishes.

3.2 A few definitions

Note that, in our framework, $m \succeq_m m'$ means that self-1 prefers the elements that are eventually chosen in m to those that are eventually chosen in m'. For instance, when there is no uncertainty (there is only one state), this corresponds to the case in which self-1 prefers $x^*(m)$ to $x^*(m')$. In other words, from a situation where self-1 selects m (in which case, self-2 will pick $x^*(m)$), adding m' amounts to increasing the choice set with some non preferred items from the perspective of self-1. Conversely, from a situation where self-1 selects m' (in which case, self-2 will pick $x^*(m')$), adding m amounts to increasing the choice set with some preferred items from the perspective of self-1. When there is uncertainty about future preferences, $m \succeq_m m'$ means that self-1 prefers on average the elements that are chosen in m. However, it does not imply that self-1 prefers the eventual choice in m in each state. In that case, adding m' to m amounts to adding items that are not preferred in a subset of states. For future references, we introduce the following definitions.

Definition 1 DM exhibits a preference for small choice sets when

$$m \succeq_m m' \quad \Rightarrow \quad m \succeq_m m \cup m' \qquad (\mathcal{P}^{\text{s}})$$

A preference for small choice sets is a preference over menus such that DM reveals to be weakly worse-off if non-preferred alternatives from the perspective of self-1 are added.

Definition 2 DM exhibits a preference for large choice sets when

$$m \succeq_m m' \quad \Rightarrow \quad m \cup m' \succeq_m m \qquad (\mathcal{P}^{\text{l}})$$

A preference for large choice sets is such that DM weakly prefers to allow himself to pick from a larger set even if a priori this includes non preferred items from the perspective of self-1.

Definition 3 DM exhibits a preference for restricted choice sets when

$$m \succeq_m m' \quad \Rightarrow \quad m' \succeq_m m \cup m' \qquad (\mathcal{P}^{\mathrm{r}})$$

A preference for restricted choice sets is such that DM weakly prefers to allow himself to pick from a small set even if a priori this excludes preferred items from the perspective of self-1.

Definition 4 DM exhibits a preference for expanded choice sets when

$$m \succeq_m m' \quad \Rightarrow \quad m \cup m' \succeq_m m' \qquad (\mathcal{P}^{\mathrm{e}})$$

A preference for expanded choice set is a preference over menus such that DM reveals to be weakly better-off if preferred alternatives from the perspective of self-1 are added.

Note that the combination of ($\mathcal{P}^{\mathrm{e}}$) and ($\mathcal{P}^{\mathrm{l}}$) amounts to a preference for flexibility ($\mathcal{A}^{\mathrm{F}}$): DM weakly prefers to allow himself to pick from a larger set independently of how good or bad the extra items are from the perspective of self-1. Similarly, the combination of ($\mathcal{P}^{\mathrm{s}}$) and ($\mathcal{P}^{\mathrm{e}}$) amounts to set-betweenness ($\mathcal{A}^{\mathrm{SB}}$). For future reference, we introduce the following preference property.

property DM exhibits a strong preference for restriction when both ($\mathcal{P}^{\mathrm{s}}$) and ($\mathcal{P}^{\mathrm{r}}$) are satisfied. Formally,

$$m \succeq_m m' \;\Rightarrow\; m \succeq_m m' \succeq_m m \cup m' \qquad (\mathcal{A}^{\mathrm{SR}})$$

In that case, self-1 prefers not to get additional choices at all, and DM will restrict his choices no matter what.

4 Equilibrium-induced preferences

We now characterize the equilibrium-induced preferences in the presence of our conflict. In what follows, we consider menus m and m' such that $m \succeq_m m'$, and we derive the properties of the preferences over the joint menu $m \cup m'$.

4.1 Consistency as a limit case

In this section, we show that our model encompasses the standard model of decision-making when the conflict vanishes.

In the case of no-conflict, our model collapses into a simple description stating that the individual is one entity with unambiguous preferences over the consumption set X. These preferences are not changing over time or states. Formally, $S = \{a\}$ and preferences over the lifetime are summarized by a single preference ordering transitive and complete $\succeq (= \succeq_1 = \succeq_{2a} \equiv \succeq_2)$. That preference ordering is represented by a utility function $u(\cdot)$ $(= u_1(\cdot) = u_{2a}(\cdot) = u_2(\cdot))$. At stage 2, self-2 chooses $x^*(m) = \arg\max_{x \in m} u(x)$ when he is left with menu m. At stage 1, we have $V(m) = u(x^*(m))$. Given the absence of conflict, this can be rewritten as $V(m) = \max_{x \in m} u(x)$.

Lemma 3 *If $\succeq_1 = \succeq_{2s}$ for all s, then the induced preference ordering over menus satisfies* ($\mathcal{A}$).

When the conflict vanishes, the induced preference over M cannot conflict with the preference over X. In particular, the model predicts that if $\tilde{x}$ is the best element in X, menus containing it are preferred to menus that do not contain it; and menus that contain it are all equivalent to DM, who will eventually pick $\tilde{x}$. The main lesson is that in the limit case in which intra-personal conflicts vanish, DM acts as if he were a single entity with uniquely defined complete and transitive preferences over the set of items X. Our approach is therefore capable of generating "rational" decision-making.

4.2 Preferences under certainty

We now analyze the more interesting case where preference orderings differ. Still, we restrict the attention to choices at dates 1 and 2 when there is only one state, namely $\succeq_{2s} = \succeq_2$ for all s. Then self-1 would like self-2 to pick an element $x \in m$ but anticipates that self-2 might pick $y \neq x$ if m is available tomorrow. Preferences over menus are characterized in the next result.

Proposition 1 *Suppose $\succeq_{2s} = \succeq_2$ for all s and $\succeq_1 \neq \succeq_2$. For all m and m', preferences over menus satisfy only a version of* ($\mathcal{A}^{\text{SB}}$). *Formally*

$$m \succ_m m' \Rightarrow \quad m \sim_m m \cup m' \succ_m m' \quad \text{or} \quad m \succ_m m \cup m' \sim_m m'. \tag{1}$$

$$m \sim_m m' \Rightarrow \quad m \sim_m m \cup m' \sim_m m' \tag{2}$$

Notice that (1) and (2) imply that the preferences over menus satisfy two different properties. First, DM is worse-off if non preferred choices from the perspective of self-1 are added to his choice set and the preference over menus satisfy ($\mathcal{P}^{\text{s}}$). Second, DM is better-off if preferred choices from the perspective of self-1 are added to his choice set and the preference over menus satisfy ($\mathcal{P}^{\text{e}}$). However, DM is always indifferent between having the joint menu $m \cup m'$ and one of the two separate menus m or m'. Said differently, only one property is satisfied strictly.

The intuition is simple. Suppose $X = \{x, y\}$ and therefore $M = \{\{x\}, \{y\}, \{x, y\}\}$. Suppose also that $\{x\} \succeq_m \{y\}$. When self-1 does not restrict the choices of self-2 and therefore offers $\{x, y\}$, then there are two cases. If self-2 prefers x, the decision is congruent with the preferences of self-1 who obtains his highest level of utility. In that case, self-1's satisfaction is the same as if he offers $\{x\}$. If by contrast, self-2 prefers y, then self-1 derives the same (low) utility if he offers $\{y\}$ or $\{x, y\}$. Overall, in the presence of conflicts but under certainty, DM's behavior satisfies set-betweenness ($\mathcal{A}^{\text{SB}}$). However, it is only a version of that axiom.

There are three important aspects in this result. First, y can be interpreted as a tempting element only in the second case where self-2 prefers y. In other words, a tempting element is an element that is relatively less desirable ex-ante but relatively more desirable ex-post. The notion of temptation is explicit and emerges from a conflict between orderings. Second, commitment refers to the inability of self-1 to impose his preferred choices on self-2. Then, he needs to restrict choices as a commitment device against future detrimental decisions. Restricting the set of options is required when selves disagree, that is when there is a tempting element in the choice set. Third, whenever self-2 prefers the tempting element, he succumbs no matter what. In other words, there is no self-control in the environment of Proposition 1. The "right" choice will never be made when the tempting element is offered to self-2.

This is due to the "delegation" aspect of the decision. Self-1 may prefer to exercise self-control but self-2 is in charge of the decision: either self-1 does not exists anymore when the decision is made (in the case with pure intra-personal conflicts between periods), or it cannot retain control over the decision (in the case with intra-personal conflicts within periods and delegation).

4.3 Preferences under uncertainty

Suppose now that a state realizes ex-interim. To simplify the exposition, let us assume without loss of generality that $S = \{a, b\}$ and let us denote simply by p the probability that state a realizes. We have the following result.

Proposition 2 *Suppose $S = \{a, b\}$, $\succeq_1 \neq \succeq_{2a}$, $\succeq_1 \neq \succeq_{2b}$ and $\succeq_{2a} \neq \succeq_{2b}$. For all m and m', preferences over menus may satisfy ($\mathcal{A}^{\text{SB}}$), ($\mathcal{A}^{\text{F}}$) or ($\mathcal{A}^{\text{SR}}$).*

Proposition 2 shows that when preferences are uncertain at date 2, the decision maker might want either to enlarge or reduce his choice set. The logic of the result is simple. Any menu m generates a lottery for self-1: with probability p, he gets $x^a(m)$ and with probability $1 - p$, he gets $x^b(m)$. Choosing between two menus consists in choosing between two corresponding lotteries. Combining menus is equivalent to combining lotteries, and combining lotteries amounts to giving the possibility to self 2 to switch from $x^a(m)$ to $x^a(m')$ in state a and/or from $x^b(m)$ to $x^b(m')$ in state b. From the perspective of self-1, such switches are either beneficial or detrimental. Self-1 trades off the effect of switches with the likelihood they occur to decide whether to restrict or enlarge the choice set. Suppose that by offering $m \cup m'$, DM ends up choosing $x^a(m)$ in state a and $x^b(m')$ in state b. Then, there are three cases.

First, if self-1 strictly prefers choices in m, then he is better-off if he is left with choices in m only: when m' is added to m, welfare is reduced in state b (choice is $x^b(m')$). Then, preferences satisfy ($\mathcal{P}^{\text{s}}$). Also, DM prefers to be offered $m \cup m'$ rather than m'. The reason is simply that, when m is added to m', welfare is increased in state a (choice is $x^a(m)$). Then, preferences satisfy ($\mathcal{P}^{\text{e}}$). Overall, preferences over menus satisfy ($\mathcal{A}^{\text{SB}}$). The concept of temptation is present here also: the desirability of a menu is reduced when elements that are ex-ante relatively less attractive become ex-post relatively more attractive at least in one state. Now, given the existence of different states, sometimes self-2 will prefer and choose y and sometimes he will prefer and choose x. On average, there is a loss if $\{x, y\}$ is offered instead of $\{x\}$ but it is still better from self-1 perspective to offer $\{x, y\}$ rather than $\{y\}$. Overall, and contrary to Proposition 1, it is now possible to get $(\mathcal{A}^{SB})$ to its full extent. The notion of self-control is therefore restored. Yet, in our model, it corresponds to a disposition in a state that leads DM to choose the item that is relatively more attractive ex-ante. Of course, the notion is meaningful if an only if there exist some states in which that item becomes relatively less attractive. Overall, in some states, DM prefers y but not in others: those states are states of self-control.

Second, if self-1 prefers choices in m in state a and in m' in state b, offering $m \cup m'$ precisely offers the possibility to reach the preferred outcome. Restricting the choice set generates a systematic welfare loss. In other words, the preferences over menus exhibit both $(\mathcal{P}^{l})$ and $(\mathcal{P}^{e})$, and DM exhibits a preference for flexibility $(\mathcal{P}^{F})$.

Last, self-1 prefers choices in m' in state a and in m in state b, self-1 strictly prefers to offer m than any other alternative. Indeed, compared to his preferred outcome, offering $m \cup m'$ leads to the opposite. In particular, not only $V(m \cup m') < V(m)$ but also $V(m \cup m') < V(m')$: more choices lead DM to choose an option which is always detrimental from self-1's perspective. In that case, the preferences over menus satisfy both $(\mathcal{P}^{s})$ and $(\mathcal{P}^{r})$, that is a strong preference for restriction $(\mathcal{A}^{SR})$.

4.4 Information and preferences

Combining Propositions 1 and 2, we have shown that a simple model of intra-personal conflicts can generate the entire spectrum of possibilities for preference over menus. The model is rich enough to generate a variety of behaviors, traditionally studied in isolation. In this section, we emphasize the role of uncertainty on equilibrium behavior.

Proposition 3 *Uncertainty about future preferences is a necessary condition to observe preferences over menus that satisfy both* $(\mathcal{A}^{F})$ *and* $(\mathcal{A}^{SR})$. *It is not necessary to observe preferences over menus that satisfy* $(\mathcal{A}^{SB})$.

Removing non-preferred options from the perspective of self-1 is always weakly beneficial when there is no uncertainty: self-1 secures a good option by removing bad options that he expects to be chosen eventually otherwise. Similarly, adding preferred options cannot hurt self-1 in the absence of uncertainty. Overall, $(\mathcal{P}^{s})$ and $(\mathcal{P}^{e})$ are the only two properties that can emerge in the certain environment. Of course, they also emerge when future preferences are unknown: it may be profitable to secure a good option by removing bad options if there is a good chance they will be chosen otherwise $((\mathcal{P}^{s}))$; and adding preferred options may help increase welfare if they are chosen at the correct time $((\mathcal{P}^{e}))$. Overall, uncertainty is not a necessary condition to observe a preference over menus that satisfy $(\mathcal{A}^{SB})$.

Under uncertainty, $(\mathcal{P}^{s})$ and $(\mathcal{P}^{e})$ can be relaxed sometimes. On one extreme, removing non-preferred options from the perspective of self-1 hurts that self. This is the case because some of those options are good in some states provided they are undertaken only in those. If self-2 agrees with this, then he will take the correct decision from the perspective of self-1. Flexibility is good. On the other extreme, adding preferred options from the perspective of self-1 hurts that self. Here some of those options are bad in some states and should be avoided. If self-2 disagrees with this, then he will take the incorrect decision from the perspective of self-1. Restricting choices is beneficial.

This suggests that the desirability of keeping all options open are intrinsically related to uncertainty about future outcomes. However, those prospects must be good enough to counterbalance the natural tendency to restrict choices. In our case, it must be that the uncertainty resolution allows to reallocate second-period choices across states in a way that is profitable for self-1. Similarly, the willingness to reduce options to a minimum arise when prospects are bad enough and it reinforces the tendency to restrict choices. Here, the uncertainty resolution prevents to reallocate second-period choices in a way that is profitable for self-1.

5 Preference reversals and preferences over menus

In this section, we discuss conditions under which each of the possible behavior can emerge in the context of Proposition 2. Precisely, we want to identify the types of preference reversals that are necessary to obtain each possible property.

5.1 Betweenness

As noted earlier, our multiple self approach generates predictions equivalent to those obtained under the one-entity approach. However, in our case it results from a specific preference reversal.

Proposition 4 $\mathcal{A}^{\text{SB}}$ *is an equilibrium property when self-1 prefers elements in* m *in both states and/or when the reversal leads self-2 to pick elements in* m' *in both states.*

Preferring menu m to m' means that the lottery resulting in the payoffs $(x^a(m), x^b(m))$ is preferred to the lottery resulting in the payoffs $(x^a(m'), x^b(m'))$. However, it does not mean that choices in m are systematically preferred in both states. In particular, it might be the case that there are "tempting" elements in both sets depending on which state is prompted. Now, starting from offering m, set-betweenness implies the following. First, either $x^a(m)$ or $x^b(m)$ are changed for either $x^a(m')$ or $x^b(m')$ respectively and, this (these) changes result in a utility loss from the perspective of self-1. Second, this utility loss is bounded in such a way that self-1 is at least weakly better-off than if only m' is chosen. This means in particular that the reversal cannot make self-1 better-off: self-2 cannot pick choices in m' that increase self-1's welfare unless he also picks other choices that reduce this welfare by a bigger amount. In other words, any gain must be compensated by a bigger loss. This also means that the reversal cannot make self-1 worse-off than if only m' is offered: in the worst scenario, self-2 chooses items in m' in both states. It is clear that if self-1 prefers elements in m in both states, any reversal leads to set-betweenness. In the case where $x^a(m)$ is preferred to $x^a(m')$ but $x^b(m)$ to $x^b(m')$, any reversal that preserves one choice in m

either makes self-1 better-off than if m is offered, or worse-off than if m' is offered instead. Overall, set-betweenness emerges only when self-2 chooses both elements in m'.

It is possible to provide an interpretation in terms of temptation and self-control, as in [GP]. However, there are a few differences. The logic of the reasoning in [GP] is as follows: given ($\mathcal{A}^{\mathrm{SB}}$), it has to be the case that the individual likes and dislikes some elements at the same time and prefers to avoid them to some extent. Temptation and self-control come as natural interpretations. Self-control results in a loss of utility at the time of consumption, loss that is anticipated ex-ante. In our model, the observation of the choice over menus suggests an underlying model that rationalizes the interpretation. First, temptation is a result of the environment. It comes from the fact that items might be valued differently ex-ante and ex-post. A tempting element is simply an item that is relatively less desirable ex-ante and relatively more desirable ex-post. Second, self-control is a disposition affecting utility representations in some states in a favorable way from the perspective of self-1. Now, given the disposition is not necessarily present ex-post, there is an implicit cost for self-1 of not restricting the choice set. If y is tempting and both x and y are offered, then self-1 suffers a utility loss each time the "bad" state is prompted and the "bad" choice is made. However, self-control is not costly per se in our setting: x is chosen when a state of self-control realizes, and in that case, it is optimal (and not costly) to choose x also for self-2. This leads to an immediate conclusion.

Corollary 1 *It is possible to generate a behavior consistent with* ($\mathcal{A}^{\mathrm{SB}}$) *when the notion of costly self-control does not have a bite. Costly self-control does not need to be part of the representation or the interpretation.*

According to our result, preferences over menus will exhibit set-betweenness in situations where the concepts of temptation or self-control do not have a bite. For instance, suppose DM either has a preference for French food or for Italian food. At date 1, French food is more appealing to him and he must decide whether to make a reservation in a French Brasserie or an Italian Pizzeria. At date 2, either he will have a preference for French food (state a) or for Italian food (state b). If he makes a reservation in the Brasserie, he'll have his best choice from the perspective of date 1. However, if he makes a reservation in the Pizzeria, he'll have his worst choice from the perspective of date 1. For self-1, restricting to the Brasserie is preferred to restricting to the Pizzeria. If he leaves both choices open, he will end-up going to the Brasserie in state a and to the Pizzeria in state b. He is better-off if he makes a reservation at the Brasserie today, but it is better to leave the two possibilities available rather than booking at the Pizzeria. In terms of preferences, it might be the case that preferences change over time and DM evaluates options conditional on his current mood only; or it can be the case that self-1 is a long-term self who cares about many aspects of the choice, while self-2 is a short-term self and cares only about current appetite.

Remark 1 Ex-post costly self-control. The analysis suggests that our model cannot capture what we "feel" as a cost of self-control or, more generally, any cost associated to picking x instead of y when y is available *at the date of consumption*. To get a representation that accounts for ex-post costly self-control, preferences must be such that picking x is costly when y is available. For instance, if $\{x\}$ is offered, self-2 picks x and gets utility $w_2(x)$ but if $\{x, y\}$ is offered, self-2 picks x and gets utility $z_2(x, y) < w_2(x)$. The cost of self-control is $w_2(x) - z_2(x, y)$. To get such representation at date 2, the preferences of self-2 need to depend on the menu that is offered. This menu dependency may come from an intra-personal conflict within period 2. Enriching our basic model to account for extra conflicts would help to restore ex-post costly self-control.

Remark 2 Ex-ante costly self-control. This corresponds to a case in which it is costly to affect favorably future choices. Formally, suppose that if $\{x, y\}$ is offered, self-2 picks x only with some probability. Exerting costly self-control means taking a costly action to increase the probability of choosing x, that is the probability that a given state where x is chosen realizes. This is reminiscent of Nehring (2006), Benhabib and Bisin (2005), and Fudenberg and Levine (2006). Doing the right thing is costly. To get this effect in our setting, we need to assume that the probability distribution of the states depends on an action $e \in E$ and, self-1 incurs a cost of exerting that action. Then, self-1 must have preferences on $X \times E$. If we introduce this feature, we are in an almost identical setting as Nehring (2006): preferences of self-1 are second-order preferences.[10]

5.2 Flexibility

Proposition 2 shows that ($\mathcal{A}^{\mathrm{F}}$) can be obtained when the preferences of self-2 are uncertain at date 1. Said differently, a necessary condition for the property to emerge is the state-dependency of self-2 preferences combined to the uncertainty about the realization of the states. It occurs in the presence of a particular preference reversal.

Proposition 5 *$\mathcal{A}^{\mathrm{F}}$ is an equilibrium property when*

(i) *self-1 prefers the choice in m in one state and the choice in m' in the other and*
(ii) *if m' is added to m, self-2 keeps self-1 preferred choice in m and switches to self-1 preferred choice in m'.*

This situation corresponds to a case where the preferences of self-1 and self-2 are "congruent" and it is therefore optimal to keep options open. In particular, a preference for large choice sets $\mathcal{P}^1$ will not emerge when self-1 prefers systematically choices in m. This is the case because his welfare cannot be improved by adding elements systematically less preferred. A preference for large choice sets can emerge only when self-1 benefits from choices in m' sometimes and suffers from them otherwise. In that case, it is optimal to enlarge systematically the choice set when self-2 switches to beneficial choices and not to detrimental ones.

This reversal is satisfied in the following example. Suppose that self-1 must decide at date 1 between several shopping malls offering restaurants and theaters. Self-2 will

[10] The model of second-order preferences is dynamically inconsistent, as the present model. In particular, whenever the choice is formally equivalent to GP, the individual is inconsistent.

then have to decide whether to go to a particular restaurant or to the theater. Suppose that the preference orderings are such that

$$\text{“casual meal''} \succeq_1 \text{“sophisticated meal''} \succeq_1 \text{“action movie''} \succeq_1 \text{“drama''}$$
$$\text{“casual meal''} \succeq_{2a} \text{“action movie''} \succeq_{2a} \text{“drama''} \succeq_{2a} \text{“sophisticated meal''}$$
$$\text{“sophisticated meal''} \succeq_{2b} \text{“action movie''} \succeq_{2b} \text{“drama''} \succeq_{2b} \text{“casual meal''}$$

Self-1 is considering offering one of the two menus $m = \{$“casual meal”, “action movie”$\}$ and $m' = \{$“sophisticated meal″, “drama″$\}$ (which correspond for instance to two possible malls). At date 2 and in state a, he will choose to have a casual meal in m and to watch a drama in m'. In state b, he will choose to watch the action movie in m and to have a sophisticated dinner in m'. If all options are open at date 2, DM will end-up having a casual dinner in state a and a sophisticated dinner in state b. These are the best options from the perspective of self-1 and it is optimal to be flexible.

5.3 Restriction

An interesting feature of our model is that preferences over menus might reflect a strong preference for restriction. Such behavior has not been analyzed in the one entity approach. As in the case of flexibility, a necessary condition for this to occur is to have uncertain preferences at date 2. As explained before, choosing a menu amounts to choosing a lottery. If self-1 prefers elements that are eventually chosen in m in state a and elements that are eventually chosen in m' in state b, then combining both menus can be detrimental. If the preferences of self 2 are opposite, DM ends up choosing elements in m' in state a and elements in m in state b. It is therefore better to offer only one menu and get the preferred choices in a subset of states.

The preference reversal between periods 1 and 2 is strong. If self-2 could choose items in the different states, he would pick systematically the opposite of what self-2 will eventually pick. Formally, whenever m and m' are offered, self-2 will choose between $x^a(m)$ and $x^a(m')$) in state a and between $x^b(m)$ and $x^b(m')$ in state b. The conflict is the strongest when rankings disagree in both states. Whenever this happens, it is always best to restrict choices in such a way that at least one preferred option is taken in one state. An alternative way to phrase these properties is as follows.

Proposition 6 $\mathcal{P}^{\mathrm{SR}}$ *is an equilibrium property when*

(i) *self-1 prefers the choice in m in one state and the choice in m' in the other and*
(ii) *if m' is added to m, self-2 keeps self-1 non preferred choice in m and switches to self-1 non preferred choice in m'.*

This result says in particular that a strong preference for restriction will not emerge when self-1 always prefers choices in m. From his perspective, the worst scenario occurs when self-2 reverses all choices to those in m', but then he obtains the same welfare as if only m' is offered. Therefore, a preference for restriction can emerge only when self-1 benefits from choices in m' sometimes and suffers from them otherwise. However, it is optimal to restrict choices when self-2 switches to detrimental choices and not to beneficial ones. Note that this is the reverse logic of a preference for flexibility.

Let us consider an example of such a situation. Again, assume that self-2 will have to decide whether to go to a restaurant or to the theater. There are four possible choices: a meal in a casual restaurant, a meal in a sophisticated restaurant, an action movie or a drama. Today, self-1 is in the mood of going to the restaurant but he prefers simple alternatives. His preference ordering is such that

$$\text{“casual meal”} \succeq_1 \text{“sophisticated meal”} \succeq_1 \text{“action movie”} \succeq_1 \text{“drama”}$$

Tomorrow, his preferences will be different and will depend on whether the state realized is a or b. More specifically,

$$\text{“drama”} \succeq_{2a} \text{“casual meal”} \succeq_{2a} \text{“sophisticated meal”} \succeq_{2a} \text{“action movie”}$$
$$\text{“action movie”} \succeq_{2b} \text{“sophisticated meal”} \succeq_{2b} \text{“casual meal”} \succeq_{2b} \text{“drama”}$$

Suppose that self-1 is considering offering one of the two following menus: $m = \{\text{“casual meal”, “action movie”}\}$ and $m' = \{\text{“sophisticated meal”, “drama”}\}$. At date 2, and in state a, DM will choose to have a casual meal in m and to watch a drama in m'. In state b, he will choose to watch the action movie in m and to have a sophisticated meal in m'. If all options are open at date 2, DM will end-up watching a drama in state a and an action movie in state b. None of these options are good from the perspective of self-1 and restricting choices is in fact optimal.

6 Concluding remarks

This analysis is an attempt to show the benefits of introducing game theoretical elements into individual decision-making problems. Observable choices are the results of complex phenomena and possibly intra-personal conflicts.

For that purpose, we have modelled DM as a succession of selves with conflicting interests and considered this simple thought experiment: at date 1, self 1 must choose a menu and at date 2, self 2 chooses an item from the menu. We have assumed that preferences at date 2 are uncertain to reflect the fact that consumption moods or cues might realize ex interim. We have shown that equilibrium choice over menus is consistent with either a preference for flexibility, a preference for betweenness, or a strict preference for restriction. More precisely, if menu m is preferred to menu m', DM may strictly prefer to combine both menus or to restrict choices to m. He may strictly prefer to choose m rather than the combination of both menus, and at the same time he may prefer the combination to m'. Sometimes also DM derives systematically less utility if he combines menus. Overall, our study makes the following methodological contributions. First, it shows that by modelling the conflict of preferences explicitly, it is possible to provide a unified approach of decision-making within the standard experiment considered in [GP] and [K]. Second, the model is capable of generating a new set of choices over menus.

We have identified under which conditions each possible choice can occur. In particular, uncertainty about future preferences is a necessary condition to obtain a preference for flexibility, or a strict preference for restriction. Moreover, the type of the preference reversals between date 1 and date 2 (or, said differently the size of the conflict between self-1 and self-2) generates particular preferences over menus.

We have also shown that interpretations are sometimes misleading. In particular, we have compared in detail our setting to [GP] and found the following. First, set-betweenness is a choice property any time an item is relatively less desirable ex-ante and relatively more desirable ex-post in at least one state. It can be, but does not have to be, interpreted as an instance of temptation and self-control. Second, whenever it is relevant to interpret set-betweenness as an issue of temptation and self-control, we have the following. Temptation is the result of the conflict of preferences between dates 1 and 2. An item is considered tempting only from the perspective of self-1. Self-control is a disposition that affects second-period preferences and prevents DM from consuming tempting elements. Third, the notion of cost of self-control does not emerge naturally and intuitively in our setting. This means that set-betweenness does not need to be associated to costly self-control. As we discussed, it reflects rather a cost for not removing tempting elements that are consumed each time an "out-of-control" disposition is prompted.

Interestingly, observing choices over menus conveys information about DM. Suppose an experimenter offers menus to DM and asks him to choose between them, and assume our model of conflict is correct. When DM exhibits a choice pattern over menus that satisfies properties ($\mathcal{A}^{\mathrm{SR}}$) or ($\mathcal{A}^{\mathrm{F}}$), the experimenter is informed about one important element: DM does not know his future preferences. Moreover, observing ($\mathcal{A}^{\mathrm{F}}$) signals to the experimenter that the conflict of preferences is weak. Self-2 is mostly agreeing with self-1 and the correct options will be chosen if they are available at date 2. Also, anytime ($\mathcal{A}^{\mathrm{SR}}$) is observed, the experimenter knows that the conflict of preferences is strong. Self-2 systematically picks the option self-1 does not want, and DM tends to let only a few options available to the future. Last, in the case of ($\mathcal{A}^{\mathrm{SR}}$), it suggests that the conflict is sufficiently strong so that it is necessary to avoid some options, but at the same time sufficiently mild, so that good options are often taken if they are available.

Another natural issue is whether insisting in modelling preferences within the one-entity paradigm generates a loss, *assuming* the model with intra-personal conflicts is correct. In the case of ($\mathcal{A}^{\mathrm{SB}}$), we have already noted that uncertainty about future preferences is necessary to obtain the property to its full extent, and to have a specific preference reversal. Those features are not part of the representation or the interpretation under the one-entity approach. Assuming the one-entity model is correct, the experimenter will believe DM is tempted by some element and possibly exerts self-control. However, the situation may not include any tempting element, and/or DM is wrongly attributed a costly self-control problem when he is simply solving optimally a trade-off. Removing some elements from X is an objective improvement in the first case; however it is only optimal from one perspective in the other. This suggests that the prescriptions that may be obtained in subsequent analyses building on the different models may be drastically different. Given the possible impact of those results, game-theoretic models of decision-making should not be disregarded.

However, even if the interpretation differs, which again may have a consequence when it comes to provide prescriptions, both models may be statistically undistinguishable. For instance, in [K], DM does not know his preferences tomorrow but knows how the available items in m will be ranked in each possible state. Then, it is possible to determine which item will be picked in each possible states and what utility level will be ensured at date 2. The utility derived by the menu at date 1 is consistent with date 2 utility levels. In our framework, however, self-1 has his own way of ranking alternatives. He is an expected utility maximizer conditional on his own perspective. This is as if there are two competing orderings over items in states: one is used to predict choice at period 2 and the other is used to assess it at period 1. These two decision-makers are different. However, their choice will only reveal to the experimenter that they make decisions under uncertainty about their future preferences and have either none (as in [K]) or a mild conflict of interest (as in this paper).

Appendix

Proof of Lemma 1. Preferences over lotteries are defined for probability measures on X. To evaluate $l(m)$, we first need to show it is equivalent to such a probability measure.

Let $X(m)$ be the elements of X that are attainable if the menu is m. Formally, $X(m) = \{x \in X | x = x^s(m) \text{ for some } s \in \{a, b, ..., \overline{s}\}\}$. Let also $S(m, x)$ be the set of states in which $x \in X$ is chosen at date 2 when m is chosen at date 1. Formally, $S(m, x) = \{j | x^j(m) = x\}$, and by construction $S(m, x) = \emptyset$ when $x \notin X(m)$. Last let $q(m)$ be the probability measure on X such that $q(m, x) = 0$ for all $x \notin X(m)$ and $q(m, x) = \sum_{s \in S(m,x)} p_s$ for all $x \in X(m)$. We denote this lottery by $\hat{l}(m)$.

Given self-1 is an expected utility maximizer, lottery $\hat{l}(m)$ is evaluated according to $\sum_{x \in X} q(m, x)u_1(x)$. However, by construction $\hat{l}(m)$ is equivalent to $l(m)$ and in particular, $\sum_{x \in X} q(m, x)u_1(x) \equiv \sum_{s \in S} p_s u_1(x^s(m))$. Therefore, there exists an induced preference $\succeq_m$ over menus $m \in M$ that can be represented by the utility function $V : X \to R$ where

$$\forall m \qquad V(m) = E_s u_1(x^s(m)) \quad \text{with} \quad x^s(m) = \arg\max_{x \in m} u_{2s}(x).$$

This completes the proof. □

Proof of Lemma 3. Consider menus m and m'. In the second period we have

$$x^*(m) = \arg\max_{x \in m} u(x) \quad \textit{and} \quad x^*(m') = \arg\max_{x \in m'} u(x).$$

In the first period $V(m) = u(x^*(m))$ and $V(m') = u(x^*(m'))$. Suppose $m \succeq m'$, that is $u(x^*(m)) > u(x^*(m'))$. If self-1 chooses $m \cup m'$, then self-2 picks $x^*(m)$ and therefore $V(m \cup m') = V(m)$. □

Proof of Proposition 1. For all m and m', we have

$$V(m) = u_1(x^*(m)), \qquad V(m') = u_1(x^*(m')).$$

Suppose that $V(m) > V(m')$ and assume that self-2 is now offered the menu $m \cup m'$. There are two cases.

- If $u_2(x^*(m)) > u_2(x^*(m'))$, then self-2 chooses $x^*(m)$ and $V(m \cup m') = V(m)$.
- If $u_2(x^*(m)) < u_2(x^*(m'))$ then self-2 chooses $x^*(m')$ and $V(m \cup m') = V(m')$.

□

Proof of Proposition 2. For all m and m', we have

$$V(m) = pu_1(x^a(m)) + (1-p)u_1(x^b(m)),$$
$$V(m') = pu_1(x^a(m')) + (1-p)u_1(x^b(m')).$$

Suppose that $V(m) > V(m')$ and assume that self-2 is now offered the menu $m \cup m'$. In state a, he picks $x^a(m)$ if $u_{2a}(x^a(m)) > u_{2a}(x^a(m'))$ and $x^a(m')$) otherwise. Similarly, in state b, he picks $x^b(m)$ if $u_{2b}(x^b(m)) > u_{2b}(x^b(m'))$ and $x^b(m')$ otherwise.

1. If $u_{2a}(x^a(m)) > u_{2a}(x^a(m'))$ and $u_{2b}(x^b(m)) > u_{2b}(x^b(m'))$, then $V(m \cup m') = V(m)$.
2. If $u_{2a}(x^a(m)) < u_{2a}(x^a(m'))$ and $u_{2b}(x^b(m)) < u_{2b}(x^b(m'))$, then $V(m \cup m') = V(m')$.
3. If $u_{2a}(x^a(m)) > u_{2a}(x^a(m'))$ and $u_{2b}(x^b(m)) < u_{2b}(x^b(m'))$, then

$$V(m \cup m') = pu_1(x^a(m)) + (1-p)u_1(x^b(m'))$$

   - If $u_1(x^a(m)) < u_1(x^a(m'))$, then $V(m \cup m') < V(m')$
   - If $u_1(x^b(m')) > u_1(x^b(m))$, then $V(m \cup m') > V(m)$
   - If $u_1(x^a(m)) > u_1(x^a(m'))$ and $u_1(x^b(m')) < u_1(x^b(m))$, then $V(m \cup m') \in (V(m'), V(m))$.
4. The same three cases arise when $u_{2a}(x^a(m)) < u_{2a}(x^a(m'))$ and $u_{2b}(x^b(m)) > u_{2b}(x^b(m'))$.

□

References

Benhabib, J., & Bisin, A. (2005). Modelling internal commitment-mechanisms and self-control: A neuroeconomics approach to consumption-savings decisions. *Games and Economic Behavior, 52*, 460–492.
Bernheim, B. D., & Rangel, A. (2004). Addiction and cue-triggered decision processes. *American Economic Review, 4*(5), 1558–1590.
Bénabou, R., & Pycia, M. (2002). Dynamic inconsistency and self control: A planner-doer interpretation. *Economic Letters, 77*(3), 419–424.
Brocas, I., & Carrillo, J. D. (2008a). Theories of the mind. *American Economic Review, Papers and Proceedings, 98*(2), 175–180.
Brocas, I., & Carrillo, J. D. (2008b). The brain as a hierarchical organization. *American Economic Review, 98*(4), 1312–1346.
Chatterjee, K. & Krishna V. (2008). A "Dual Self" representation for stochastic temptation. *American Economic Journal, Microeconomics* (forthcoming).
Dekel, E, Lipman, B. L., & Rustichini, A. (2001). Representing preferences with a unique subjective state space. *Econometrica, 79*(4), 891–934.
Fudenberg, D., & Levine, D. K. (2006). A dual self model of impulse control. *American Economic Review, 96*, 1449–1476.

Gul, F., & Pesendorfer, W. (2001). Dynamic inconsistency and self-control. *Econometrica, 69*(6), 1403–1436.

Kreps, D. (1979). A representation theorem for preference for flexibility. *Econometrica, 47*(3), 565–578.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics, 112*(2), 443–477.

Nehring, K. (2006). *Self-control through second-order preferences*. Mimeo: University of California Davis.

Shefrin, H., & Thaler, R. (1988). The behavioral life-cycle hypothesis. *Economic Inquiry, 26*(4), 609–643.

Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies, 23*, 165–180.

Thaler, R., & Shefrin, H. M. (1981). An economic theory of self-control. *Journal of Political Economy, 89*, 392–406.