

Mathematics Teachers' Conceptions and Instructional Practices  
to Address Student Diversity

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Presented at the National Council of Teachers of Mathematics Research Pre-session,  
Anaheim, CA on April 6, 2005.

## Introduction

Reforms in mathematics education stress the need for problem-solving approaches to promote students' reasoning and communication skills (National Council of Teachers of Mathematics [NCTM], 1989, 2000; National Science Foundation [NSF], 1996). While the achievement gap has narrowed between poor, African American, Latino/a, American Indian students in comparison to affluent, European American and Asian American students in the area of basic skills, the gap continues to widen in assessments that measure more complex mathematical understandings and problem-solving (Tate, 1997). This disturbing pattern connects to the incongruity between this vision of reform and the conceptions and practices of many practicing teachers of mathematics (Battista, 1994).

Mathematics education reform documents promote the need for teachers to be prepared to teach in more culturally sensitive and responsive ways (NCTM, 1989, 1991, 2000). Teachers face a growing ethnic, socio-economic, and linguistic diversity among students in their classrooms. As teacher educators, this requires preparing prospective teachers of mathematics to implicitly and explicitly incorporate socially, culturally, and politically equitable instructional strategies in their classrooms, that is, "teach for diversity" (Rodriguez & Kitchen, 2005). The goal of this work is to promote achievement among culturally, linguistically, and socio-economically diverse students.

Essentially, prospective teachers of mathematics are being asked to teach in ways that they may never have experienced, while supporting the mathematics learning of all their students, many of whom may not share similar academic, racial, cultural or linguistic backgrounds. Mathematics teacher educators have two enormous challenges:

to prepare prospective teachers of mathematics to implement reform-oriented and standards-based curriculum and instruction, and to teach for diversity. Implementing challenging, standards-based curriculum and instruction that is accessible for all students requires transforming mathematics education away from functioning to legitimate a privileged few through selecting, sorting, and certifying students (Apple, 1985). For the reform vision to become a reality, the professional culture of mathematics education requires extensive changes in teachers' deeply held conceptions and practices about both mathematics curriculum, mathematics instruction, and the students that they teach (Richardson & Placier, 2001; Schifter & Fosnot, 1993; Thompson & Zeuli, 1999; Wilson & Berne, 1999), as well as about student diversity (Rodriguez & Kitchen, 2005; Secada, 1992; Rousseau & Tate, 2003; Tate, 1997). This requires change on multiple levels: what teachers do in the classroom, what teacher educators emphasize in their programs, and what mathematics education researchers examine as part of their research agenda on teacher conceptions and practice.

In this presentation, we begin by giving a brief overview of the research literature on the conceptions and instructional practices of teachers of mathematics. When possible, we critically review this literature and ask how it can be extended to consider students of diverse linguistic and cultural backgrounds. Next, we discuss some of the pioneering work that examines the roles of race, culture, and language in mathematics learning and teaching. This work provides the foundation to discuss the challenges in preparing prospective teachers of mathematics to teach for diversity. We finish by giving an overview of the research to be undertaken by the Center for the Mathematics

Education of Latinos/as (CEMELA) faculty, graduate students, and post-doctoral fellows on teachers' conceptions and instructional practices vis-à-vis student diversity.

### Mathematics Teachers' Conceptions and Instructional Practices

In the past several decades, notions about effective teachers' conceptions (beliefs and knowledge) and practices with regards to mathematics content (Ma, 1999), instruction (Ball & Cohen, 1999), and assessment (Kulm, 1991; 1994) have dramatically changed. In this section, literature on teachers' conceptions and practices of mathematics content, instruction, and assessment is reviewed. Throughout this review, we highlight those conceptions and practices that align with standards-based curriculum and instruction that reform documents recommend (see NCTM, 1989, 1991, 2000; NSF, 1996). We will also illustrate that the role of diversity including race, culture, and language continues to be marginalized and/or under-theorized in the teacher cognition and practice literature in mathematics education (Rousseau & Tate, 2003; Tate & Rousseau, 2002).

***Mathematics content.*** Thompson (1992) identified varying views of the nature of an academic discipline as an isolated body of discrete skills and rules (instrumentalist), as a body of connected and unified knowledge (Platonist), or as a discipline of inquiry that is continuously expanded by human creation (problem-solving). These distinctions have also been cast as a duality between absolute (e.g., instrumentalist) and fallible (e.g., a problem-solving) views that take shape in beliefs about mathematical knowledge and legitimate mathematical activities (Romberg, 1992; Thompson, 1992).

Teachers who embrace an instrumentalist view often present mathematics as a sequence of fixed skills or concepts (Brown, Collins, & Duguid, 1989). Mastery of prerequisite skills is deemed necessary for subsequent learning. This view assumes that “rules are the basic building block of all mathematical knowledge and all mathematical behavior is rule governed” (Thompson, 1992, p. 136). The teacher’s role is to demonstrate procedures, and the students’ role is to practice them.

Teachers who adhere to a Platonist view of mathematics emphasize the logic that connects concepts. These relationships are assumed to be fixed and often require explanations by the teacher. Although emphasis on students’ reasoning is included, this view models a top-down approach in which instruction begins with the knowledge of the expert, rather than that of the learner (Hiebert & Carpenter, 1992).

Teachers who adopt a problem-solving or inquiry view of mathematics see their role as posing questions and challenging students to think and reason. Instruction is student-centered, beginning with an understanding of the learner. This entails developing the teacher’s capacity to interpret the learner’s math/science ideas and misconceptions (Ball & Cohen, 1999). Students are encouraged to actively engage in mathematics learning by constructing their own meanings of the content. This view is compatible with mathematics reform, as well as with a constructivist view of learning.

Recent research has identified two other categories of beliefs: subject matter beliefs and domain-specific beliefs. Törner (2002) identifies subject-matter beliefs as analogous to subject matter knowledge. A teacher can hold a specific belief about a specific mathematical topic such as function (Lloyd & Wilson, 1998; Szydlik, 2000) or proof (Mingus & Grassl, 1999; Knuth, 2002). Törner also identified domain-specific

beliefs. Domain-specific beliefs parallel different fields in mathematics such as algebra, geometry, stochastics, and calculus. Research on both subject matter and domain-specific beliefs has enriched our understandings about teachers' mathematical beliefs and complicated implementing mathematics education reform (Aguirre, 2002; Lloyd & Wilson, 1998; Nathan & Koedinger, 2000a, 2000b).

For example, Aguirre (2002) found that when confronted with district reforms that increased the mathematics requirement to graduate from high school to three years of college preparatory mathematics, "reform-oriented" high school mathematics teachers with Platonist views of mathematics, held domain-specific beliefs about algebra, geometry, and statistics that were in conflict with the reform policy. Some teachers distinguished algebra from geometry and statistics as being the most abstract and the least accessible domain for students to learn. As one teacher expressed it, "there are all kinds of math. And there are lots of parts of math that the kids, ALL kids, get. And I'm finding that the algebra is really the hardest thing for them to learn, for an awful lot of kids to learn" (Aguirre, 2002, p.178). Algebra was a domain that some students could learn while geometry and statistics were domains that all students could learn. Increasing the level of content for all students to learn also increased the responsibilities for mathematics teachers to teach. This increase in responsibility for all students to learn advanced levels of mathematics proved problematic for many of the teachers even though their instructional philosophies and curriculum practices aligned to some of the main tenets of mathematics reform documents.

***Mathematics instruction.*** Thompson (1992) showed a consistent relationship between teachers' beliefs and instructional practices in mathematics. Teachers who

embrace the instrumentalist view teach in a manner that contrasts most distinctly with the ideals of mathematics reform and teaching mathematics for understanding (Hiebert & Carpenter, 1992; Lampert & Ball, 1999; NCTM, 1989, 1991, 2000; NSF, 1996; Romberg, 1992; Schifter & Fosnot, 1993). Since mathematics is characterized as static and predetermined in the instrumentalist philosophy, those who adhere to this view emphasize mathematical facts and pursue the drill-and-practice approach to teaching (Schifter & Fosnot, 1993). Such mathematics instruction is characterized by a focus on teaching mathematical procedures and a preoccupation on students memorizing facts.

An example of teachers who embrace the Platonist view of mathematics is provided in Liping Ma's book, Knowing and Teaching Elementary Mathematics (1999). Ma provides numerous examples in her book of how Chinese teachers promote conceptual understanding of mathematics, though their instruction is characterized as top-down and authoritarian. A theme that runs throughout Ma's book is that teachers must possess a deep and broad knowledge of mathematics to make conceptual connections between mathematical ideas. According to Ma, "limited subject matter knowledge restricts a teacher's capacity to promote conceptual learning among students" (p. 36). Ma's research demonstrates that a strong understanding of mathematics is necessary to teach in a manner consistent with the Platonist or problem-solving view.

Teachers whose primary objective is to advance mathematical problem solving demonstrate the style of teaching and learning that reformers advocate – "doing of mathematics" (Davis & Hersh, 1980; Ernest, 1991; Lakatos, 1976; Tymoczko, 1986). From this view, "learning is primarily a process of concept construction and active interpretation – as opposed to the absorption and accumulation of received items of

information” (Schifter & Fosnot, 1993, p. 8). Pedagogy inspired by this view engages students in posing and solving problems, making and proving conjectures, exploring puzzles, sharing and debating ideas, and contemplating the beauty of ideas in an academic discipline. Students engaged in such active mathematical learning develop “mathematical power” (NCTM, 1989, 1991; Parker, 1993).

In classrooms in which the focus is on mathematical exploration, less emphasis is placed on showing students how to solve problems. Students who were provided support to develop their own mathematical strategies to solve computational problems performed significantly better than those who were regularly taught to memorize algorithms to solve similar problems (Kamii, Lewis, & Livingston, 1993). Given the opportunity, students devised computational procedures that directly modeled the actions and relations of a word problem (Carpenter, Fennema, & Franke, 1996; Fennema & Franke, 1992; Resnick, 1992).

Interestingly, teachers who may want to promote mathematical problem solving might not be able to do so because of a lack of conceptual mathematical knowledge (Ma, 1999). For instance, teachers may introduce manipulatives during mathematics instruction, but teachers with limited mathematical knowledge often use manipulatives only to illustrate procedural-level knowledge (e.g., that 1 ten equals 10 ones). Thus, although teachers may want to adopt a Platonist or a problem-solving view, their lack of conceptual knowledge of mathematics inhibits their capacity to align their teaching with their philosophical view.

***Mathematics assessment.*** Assessments relying primarily on paper-and-pencil drills and the evaluation of recall of isolated facts align with the instrumentalist view of



mathematics. Alternative assessments that do not simply assess facts and skills in isolation, but that also require students to apply their knowledge in real-life contexts align more with the Platonist or problem-solving view. According to Kulm (1994):

Alternative assessment approaches that include open-ended questions, presentation of solutions in both written and oral form, and other performances send very different messages to students about what is important in mathematics learning. The thinking and reasoning approaches and the way mathematical thoughts are presented can receive high marks even if the answer may not be complete or correct. The shift from an emphasis on producing correct answers to the expectation that students think and communicate is a major one for many students and teachers. (p. 6)

This shift on emphasis corresponds to the philosophical change necessary for an instrumentalist to become a Platonist or problem-solving view. Such a shift is supported by researchers who advocate revising assessment practices to bring about changes in instruction based on how children learn (O'Day & Smith, 1993). Another goal of alternative assessment is to promote higher order thinking among students (Kulm, 1991). In addition, alternative assessment approaches and the use of multiple assessment formats require students to communicate their thinking and elicit a range of student responses (Wiggins, 1993).

***The roles of race, class, and language: a critique.*** It is clear from the research that beliefs and knowledge of content and how to teach that content are strongly linked to classroom practice and student learning in interesting and complex ways. What remains

under-researched in the literature on teacher cognition and practice is explicit attention paid to the role of race, culture, and language on teacher cognition development about mathematics instruction (Rousseau & Tate, 2003). Below are two recent examples in the literature that beg for more exploration of and/or problematization of the role race, culture, and language plays in mathematics teaching and learning. Both come from studies that exemplify the importance of teachers developing deep understandings of the content, beliefs that are consistent with the problem-solving view of mathematics, and practices that are strongly aligned with the current mathematics reform recommendations (NCTM, 2000).

Magdalene Lampert's book (2001), Teaching Problems and the Problems of Teaching is an exquisite example of a scholar analyzing and critically reflecting on teacher thinking (her own) and instructional practice. Her descriptions are vivid and the questions she considers both as she is teaching and later on in her journal reflections illustrate the robust, multidimensional, and complex thinking processes that influence her instruction. She considers an amazing array of factors that include content, instruction, learning, and students. Reading her book was valuable from a research standpoint in thinking about how teachers' think, from a teacher education standpoint (i.e. an assigned reading for a mathematics methods course), and from a practical standpoint (i.e. "an easy and pleasureable read" – like novels can be). However, there were several places in the book in which Lampert identified the racial/cultural membership of her students, but did not fully interrogate what that meant to her in her thinking about how students learn mathematics. For example, below are a series of questions she asked herself in relation

to an exchange among students about a puzzling solution provided by a student named Richard.

“I had seen enough of Catherine’s performance since September to imagine that if I did tell her to continue, she would provide not only the correct solution but also a clear and correct explanation about why her solution made sense. But what effect would that have on Richard? How would it affect Catherine if I did not let her continue? What effect would either course of action have on what the rest of the class could learn about math, as well as what they could learn about Richard and about Catherine or about racial and gender groups to which these students—an African American boy and a Caucasian girl—belonged? I would need to act without knowing the answers to these questions, but I might learn something from what I do (p. 16).”

There are numerous such examples in which students’ cultural/racial backgrounds are divulged, but not fully explored in relation to the classroom interactions. It is clear that Lampert fully recognizes that students’ racial/cultural background contribute to the learning opportunities she creates for and with her students. The nature and extent of that contribution is never fully analyzed in her book. While it can be argued that she raises important questions about equity and student diversity in her thinking about her mathematics teaching, it can be equally argued that if she raises those questions, it calls for an example of her deep and complex thinking around race, culture, and mathematics, including how her own racial/cultural identity shapes and is shaped by her interaction with students.

A more recent example of how race, culture, and language are acknowledged but not included as an integral part of research on teacher cognition is the recent “Learning to

teacher secondary Mathematics” conceptual framework offered by researchers Peressini, Borko, Romagnano, Knuth, and Willis (2004). These researchers offer a conceptual framework on teacher cognition grounded in situated learning to examine beginning/pre-service secondary mathematics teachers’ learning development and practices. In the following, they described a hallmark component of their framework: “A situative perspective suggests that knowledge and beliefs, the practices they influence, and the influences themselves, are inseparable from the situations in which they are embedded (p. 73).” In particular, they look at three “domains of knowledge” relevant to instructional practice: 1) mathematics content (specifically function, rate, and proof); 2) mathematics-specific pedagogy (mathematical tasks and orchestration of classroom discourse); and professional identity (conceptions of self as teacher).

The researchers provided two examples to illustrate different dimensions of their framework. One of the examples involved an analysis of a first year middle school teacher named Adam. The school in which Adam taught was described as “suburban” with 600 students, “a fourth of whom were Hispanic.” Contrary to his student-teaching experience in a more affluent school (racial/ethnic demographics were not given) and in a classroom that emphasized mathematical discourse, observations of Adam’s first year teaching did not align with standards-based instructional practices that promoted problem-solving and mathematical communication. Adam believed that it was important for students to construct their own understanding versus a teacher telling the student how to do a problem. However, his department chair and principal held different beliefs about mathematical learning that focused on the teacher telling and repeated practice by students to learn the material (i.e. beliefs consistent with the instrumentalist view of

mathematics). He also confronted student beliefs about mathematics in the form of resistance to activities using manipulatives, working in collaborative groups or making student presentations. Lastly, Adam felt that the district's emphasis on standardized tests, "made it impossible for him to take the time to help his students develop conceptual understanding" (p. 88).

The researchers argued that these demands (time, students, school norms/expectations) were "at odds with his developing professional identity (p. 89)." The roles of race, culture, and language were not addressed in the analysis. By characterizing the school as "a fourth of whom are Hispanic," the researchers introduce an element of the sociocultural context that explicitly connects to race, culture, and language. Why include this characteristic? What does it contribute to the analysis of Adam's learning to be a secondary mathematics teacher? Some possible connections that remained unexamined include what might be the philosophical underpinnings of the department and school norms around mathematics teaching and learning and beliefs about students, particularly Latino/a students, and their capacities to learn—"subtractive schooling" (Valenzuela, 1999) or schools/departments not "organized for advancement" (Non OFA departments) (Gutierrez, 1996; 2002). In addition, if Adam struggled to foster mathematical discourse in his class, what are his beliefs and knowledge about mathematical discourse particularly as it relates to Latinos/as, some who may be learning English? Lastly, the researchers did not identify Adam's own cultural/racial/linguistic background. If it is important to identify a specific student population of the school by ethnic background (regardless of the problematic use of the term Hispanic), is it not also important to identify the teacher's cultural/racial/linguistic background? How does the

new teacher's cultural/racial/linguistic background contribute to the development of his professional identity as a secondary mathematics teacher? While the framework makes an important contribution to what we know about how secondary mathematics teachers learn to teach, aspects of the social cultural context that the situative perspective calls for remain unaddressed.

Both examples illustrate important research on teacher cognition and practice that illuminate our understanding about mathematics teaching and how one learns to teach, but comes up short in critically examining the role of culture, race, and language in mathematics teaching. cursory references without analytical interrogation provides an incomplete picture of teacher cognition and practice. While equity in mathematics teaching and learning is explicitly addressed in mathematics reform documents, more explicit attention to issues of equity (e.g. race, culture, language) needs to be included in research on teacher cognition as well as teacher education.

The next section describes some research in a small but growing body of work that critically examines issues of culture, race, and language in mathematics instruction. These works serve as a foundation to address teacher conceptions and practice, particularly at the pre-service level in an effort to better prepare beginning teachers to work successfully in culturally and linguistically diverse settings.

#### Addressing Prospective Teachers' Conceptions and Practices for Diverse Classrooms

The vision of mathematics education reform aiming at "academic achievement for all students" requires integrating disciplinary knowledge with knowledge of student diversity (McLaughlin, Shepard, & O'Day, 1995). Unfortunately, the existing

knowledge base for promoting academic achievement with a culturally and linguistically diverse student population is limited and fragmented, in part because disciplinary knowledge and student diversity have traditionally constituted separate research agendas (Lee, 1999). Although reform documents highlight “mathematics for all” (NCTM, 1989, 2000; NSF, 1996) as the principle of equity and excellence, they do not provide a coherent conception of equity or strategies for achieving it (Eisenhart, Finkel, & Marion, 1996; Lee, 1999).

The multicultural education literature, on the other hand, emphasizes issues of cultural and linguistic diversity and equity, but with little consideration of the specific demands of the different academic disciplines (Banks, 1993; Ladson-Billings, 1994). Nevertheless, studies considering the interaction between academic disciplines and students’ linguistic and cultural practices are emerging in mathematics (e.g., Adler, 1995, 1998; Brenner, 1998; Khisty, 1995; Moschkovich, 1999). Other scholars have employed a multidisciplinary framework to investigate the intersection of students’ language, culture, and family as a means to support high achievement in mathematics (Celedón-Pattichis, 2004; Civil & Andrade, 2002; Gutiérrez, 2002; Lipka, 1994).

There is also a developing body of inquiry into the social, cultural, and political context of the teaching and learning of mathematics (see Atweh et al., 2001; Gutstein, 2003; Kitchen, 2005; Secada, 1995; Tate, 1995). Research on teaching in mathematics education that takes seriously the social, cultural, and political context of learning examines how tracking affects learning, whether diverse students have equitable opportunities to learn challenging mathematics, and how race and class play out in the classroom (Tate and Rousseau, 2003). Others apply a social reconstructionist orientation

in their teaching to prepare prospective teachers of mathematics to incorporate equitable and socially just instructional strategies in their classrooms (Dunn, 2005; Leonard & Dantley, 2005).

Transforming mathematics education to value the mathematical preparation of the majority over the achievements of a select group is a daunting challenge. For the most part, prospective mathematics teachers, particularly at the secondary level, are prepared by mathematicians who maintain “an elitist and privileged position by maintaining that mathematics is abstract, objective, and independent of social, cultural and political conditions” (Burton, 1994, p. 73). Because prospective mathematics teachers primarily experience mathematics as devoid of social, cultural, and political considerations (see Hersh, 1979; Lakatos, 1976, for perspectives on how mathematics is in fact value-laden and fallible), it is highly unlikely that multiculturalism and issues related to equity are modeled or discussed in their mathematics course work. In addition, after successfully completing a series of upper-division mathematics courses, prospective secondary-level teachers are among the few who are granted legitimacy, albeit not complete, by “members of the mathematics club” (Burton, 1994, p. 73).

As teacher educators, we typically have limited opportunities to address prospective teachers’ sense of entitlement. Teacher educators should consider the entitlement granted teachers of mathematics, particularly secondary-level mathematics teachers as a potential reason why they may resist efforts to prepare them to teach for diversity. The ever-increasing volume of research on teachers’ resistance to teaching for diversity (see e.g., Gomez & Tabachnick, 1992; Kleinfeld, 1992; Murrell, 1992) demonstrates that this is a complex endeavor.



Many teachers of mathematics also work in schools with racially and ethnically heterogeneous student populations that are located in high-poverty communities. Among the most profound challenges faced by school districts that serve diverse student populations is the recruitment and retention of people of color into the teaching ranks. In 1996, approximately 91% of teachers in the United States were White whereas the percentage of minority teachers in schools has steadily decreased during the past three decades (National Education Association, 1997). The decreasing numbers of teachers of color in the United States contrasts starkly with data that indicate that 50 of the largest 99 school districts in the country in 1992 had majority minority-student populations (National Center for Education Statistics, 1994). Furthermore, many schools that serve diverse student populations have unique sets of problems that distinguish them from their more affluent, suburban counterparts. For example, at high-poverty schools, students often attend classes in dilapidated facilities, and have higher percentages of novice teachers, teachers without a teaching credential and teachers who are teaching subjects in which they have neither a major nor a minor (Ingersoll, 1999; National Research Council, 2001).

The huge disparities between the cultural, ethnic, and racial backgrounds of the majority of teachers and their students demonstrates why colleges of education must make preparing prospective teachers to work effectively with culturally and linguistically diverse students a priority (Nieto, 2000). To prepare prospective teachers to work with diverse student populations, only one course in multicultural education is typically included in the course work of prospective teachers. Yet, as Gomez (1996) wrote, “efforts at multicultural education isolated in single courses or field experiences only

begin to challenge prospective teachers' beliefs about diverse peoples; they seldom address the knowledge, skills, and dispositions required to increase diverse children's learning and achievement" (p. 118). Moreover, course work in multicultural teacher education has been relatively ineffective at challenging prospective teachers' beliefs about racism, White privilege (Sleeter, 1994), and their belief that through hard work, one can be successful in school and in mathematics.

Despite the difficulties associated with preparing teachers to work in multicultural settings, many scholars have identified instructional approaches to prepare prospective teachers to teach for diversity. For example, according to Zeichner (1996), continued guided reflection on the part of prospective teachers on the backgrounds and life experiences of students different from them will "result in shifts in attitudes, beliefs, dispositions, and theories that govern teachers' practices" (p. 161). Sleeter (1995) required her students to engage in research in their communities in which students "are not investigating characteristics of groups but rather comparing at least two groups' access to society's resources, such as housing or health care; or they can investigate media images that help shape our belief systems" (p. 25).

For progressive educators, a potential role of the mathematics education reform movement is to promote a more egalitarian and democratic society in which all students have the opportunity to develop mathematical literacy. There are a few examples of movements that promote mathematical literacy for those who have historically been excluded in mathematics. For instance, Robert Moses' Algebra Project (Moses & Cobb, 2001) has worked to "drive a broad math literacy effort [for] the Black and poor students and the communities in which they live, the usually excluded" (p. 19). The goal of

Moses' Algebra Project starkly contrasts with the traditional role of mathematics to sort people, reinforcing society's power structure that facilitates the selection of elites in society (D'Ambrosio, 1983; Gerdes, 1988).

To realize the vision of the mathematics education reform movement to support equitable instructional formats requires policy makers and reform advocates to also pay attention to the real barriers to reform, particularly in schools serving high-poverty, diverse communities (Kitchen, 2003). In particular, the mathematics education community must prioritize improving teachers' work conditions as a means to support teachers as they implement standards-based reforms and teach for diversity.

Lastly, preparing teachers to mathematically challenge all of their students requires mathematics teacher educators to actively model the sort of respectful and trusting relationships with prospective mathematics teachers that we are asking them to pursue with their students (Kitchen, 2005). Without trust and respect, prospective teachers may not be open to activities that challenge them to examine their beliefs about who can be successful in mathematics. Even worse, they may even actively resist our attempts to model the value of multicultural perspectives and issues of equity. Such resistance could ultimately lead to behaviors that do little to support the learning of students who have historically been underrepresented in mathematics. Given the ever-increasing volume of research on teachers' resistance to teaching for diversity, it is clear that this work is complex and that teachers will need ongoing professional development to build capacity to teach for diversity (Jordan, 1995; Scott, 1995; Tatum, 1992; Rodriguez & Kitchen, 2005).

Towards this goal of increasing the capacity of teachers with knowledge, beliefs and instructional practices that serve to teach for diversity, the National Science Foundation supported Center for the Mathematics Education of Latinos/as (CEMELA) has designed a comprehensive research agenda with mathematics, culture and language as the explicit foci of work.

### The CEMELA Research Agenda

The research agenda for CEMELA addresses a pressing problem – the mathematics education of low-income, Latino students. A portion of that research will broaden the understanding of the mathematics education research community on the issues, concerns, and questions raised in the preceding sections of this paper. Specifically, this research will focus on student learning, teacher conceptions and instructional practice. CEMELA’s research focusing on student learning will consist of studies that address aspects of the broad question, “What is the nature of Latino students’ mathematical understanding and language use in multiple settings?” The goal of these studies is to provide a research base for curricular and pedagogical recommendations for mathematics instruction. Studies in this arena will include those that examine

- Latino students’ participation in mathematics classrooms in Mexico and in the United States to better understand immigrant students’ transition from learning mathematics in Spanish to learning mathematics in English;
- The interactions among language, culture, and standards-based mathematics curricula and the impact these materials have on Latino student’s learning of mathematics; this will also include investigating the relationship between reading

comprehension and mathematics learning with the use of these standards-based materials; and

- Students' use of multiple classroom resources (e.g., Spanish texts, Spanish speaking aides, peers, representations for concepts).

CEMELA's research focusing on teachers will address the broad question, "What is the nature of teachers' knowledge and use of Latino students' linguistic and cultural backgrounds to create effective mathematics learning environments?" The goal of these studies is to provide insights into the challenges teachers encounter as they integrate cultural identity development, first and second language development (especially written communication) in language-rich, reform-oriented standards-based mathematics curricula. Research topics will include how teachers think about and resolve issues of academic expectations, biases, and mathematical learning demands of reform curricula. This research will broaden current understanding of teachers' content and pedagogical knowledge to include linguistic and cultural variables. Studies in this arena will include those that examine:

- Teachers' integration of mathematics reform practices with effective principles of instruction for Latinos;
- Teachers' discourse in Spanish and in English to mediate learning of high level mathematics; and
- Teachers' conceptions of the integration of race, culture, and language development with mathematics.

CEMELA will conduct this research through several professional development (preservice and inservice) activities, to include cohorts of teachers taking graduate

courses designed to broaden and deepen their understanding of mathematics or participating in study groups that utilize teacher inquiry, action research or a lesson study model. Some studies will examine the impact of the courses on not only teachers' understanding of the mathematics content but also on the interplay of language and cultural factors in their teaching of that content. The study groups will be the sites for examining teacher's conceptions about mathematics, race, culture, and language, instructional practices, and their growth/change in these areas.

Several studies are currently underway. One study focusing on beginning teachers will investigate how cultural identity of students and teachers (e.g. race, class, culture, language) intersects with beginning teachers' developing professional identity to inform their instructional vision and practices for teaching mathematics in the early years of their career. A cohort of pre-service and first year teachers working in predominantly Latino elementary and middle schools will be followed for three years to examine growth and change of beliefs, knowledge, and practices about mathematics, language and culture. CEMELA research also includes studies that focus on in-service teachers conceptions and practices. For example, one CEMELA PI has begun looking at mathematics instruction in a school in Mexico. This instruction will be compared/contrasted with mathematics instruction with Latino second language learners in US schools in five areas: intellectual support for learning, depth of mathematical reasoning, mathematical analysis, mathematical discourse, and engagement. In another study, a weekly CEMELA study group of five elementary school teachers at one school began in February. This group will be one of the sites for examining change in teachers' understanding of the mathematics curriculum being taught and their use of standards-based instructional

practices, all through a lens informed by the students' language and culture. Baseline data is being collected in another study that will investigate the impact of lesson study professional development on a group of middle school teachers. These teachers currently are part of a cohort of teachers taking the first of five mathematics courses. Aspects of this lesson study research will focus on teachers' growth in their fluency using representations in academic tasks with English Language Learners.

### Conclusion

Research demonstrates that teachers' conceptions about mathematics powerfully influence mathematics teaching and learning. Teachers' beliefs and knowledge play a key role in curriculum, instruction, and assessment practices. Teachers can hold a variety of conceptions about mathematics including beliefs that are domain-specific. These conceptions may complicate reform efforts when examined in conjunction with beliefs about student diversity and equity. While mathematics reforms and the current literature base on mathematics teacher conceptions and practices acknowledge the importance of student diversity and equity, few studies explicitly examine the roles of race, culture, and language in mathematics teaching and learning. A more comprehensive and complex understanding of the growth and change of teachers' conceptions and practices about mathematics and student diversity is necessary for mathematics education reform to be successfully implemented. Studies with pre-service teachers have shown that examining issues of equity and diversity, particularly as it relates to race, culture, and language is challenging and often met by teacher resistance.

The foundation for research that focuses on mathematics teaching and learning, language and culture has been set by a small number of scholars many of whom are

affiliated with CEMELA. Building on this foundation, CEMELA is conducting innovative research that critically examines the role of race, culture, and language in the learning and teaching of mathematics. This research will also inform teacher education that deepens teacher knowledge about mathematics, language and culture and prepares them to successfully work in culturally and linguistically diverse schools. By making the integration of mathematics, language, and culture explicit in research and professional development, CEMELA's ultimate goal is to promote Latino/a student advancement and increased achievement in mathematics. ¡Sí se puede!



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